10-301/601 Introduction to Machine Learning

Machine Learning Department
School of Computer Science
Carnegie Mellon University

(Logistic Regression)
+ Feature Engineering
+ Regularization

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Lecture 10
Feb. 21, 2022

Q&A

Q: How do I avoid submitting a misaligned written homework on Gradescope?

A: Great question, since this will help avoid AI-assisted grading incorrectly grading your submission.

Here are some tips:

1. Check that your PDF has the same number of pages as our PDF.
2. Check that your submission boxes match on Gradescope. Be sure to check at least one box on each page.
3. Attend our special OH on how to use LaTeX!

Q&A

Q: I think you graded seven different questions incorrectly. Should I put them all in on Gradescope regrade request and submit that?

A: Please, no. I'd encourage you to watch this tutorial video from Gradescope so you know how to use this important tool.

https://help.gradescope.com/article/8hchz9h8wh-student-regrade-request

Reminders

- **Homework 4: Logistic Regression**
  - Out: Fri, Feb 18
  - Due: Sun, Feb. 27 at 11:59pm

LOGISTIC REGRESSION REMARKS

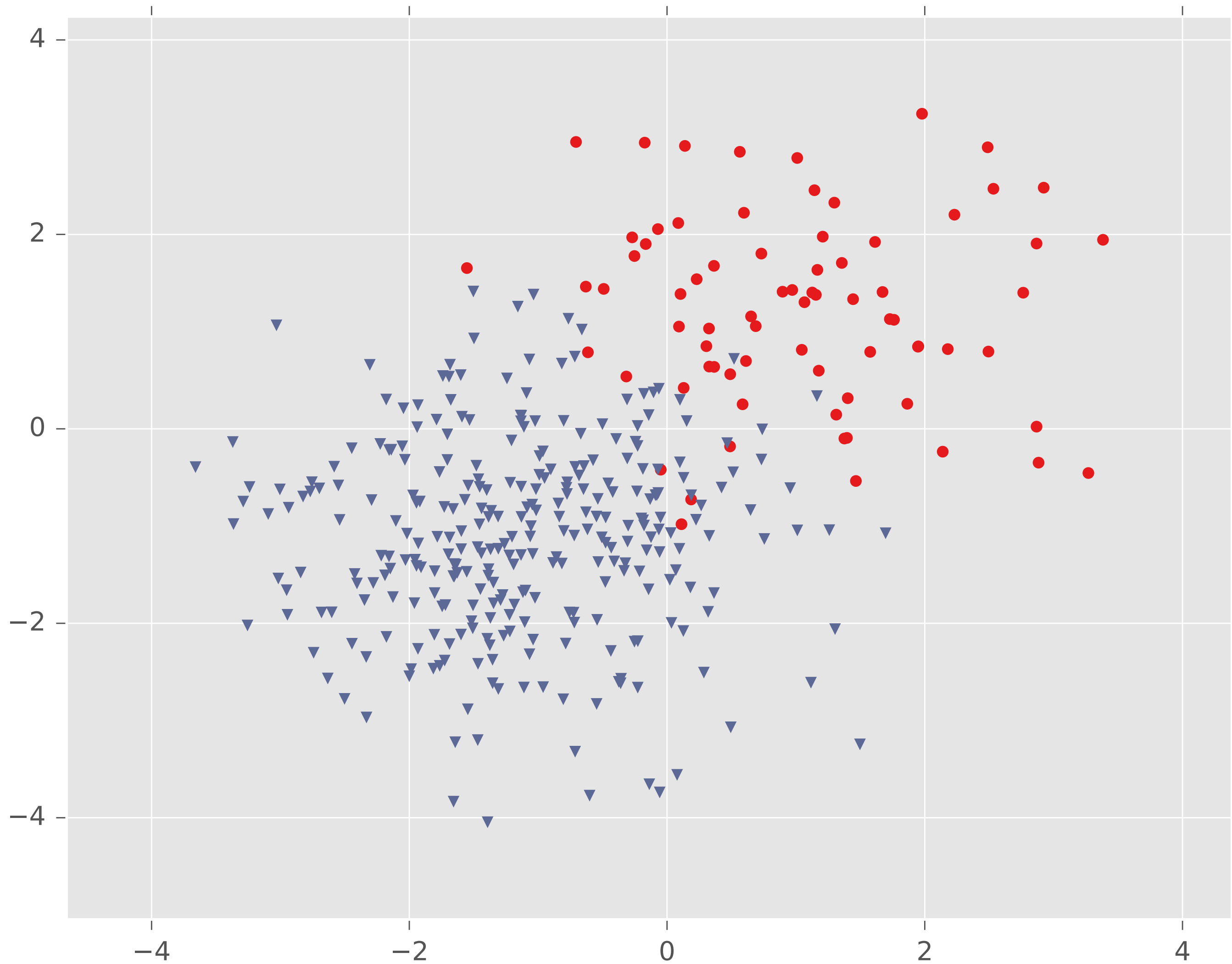
Why is it not "Logistic Classification"?

Whiteboard

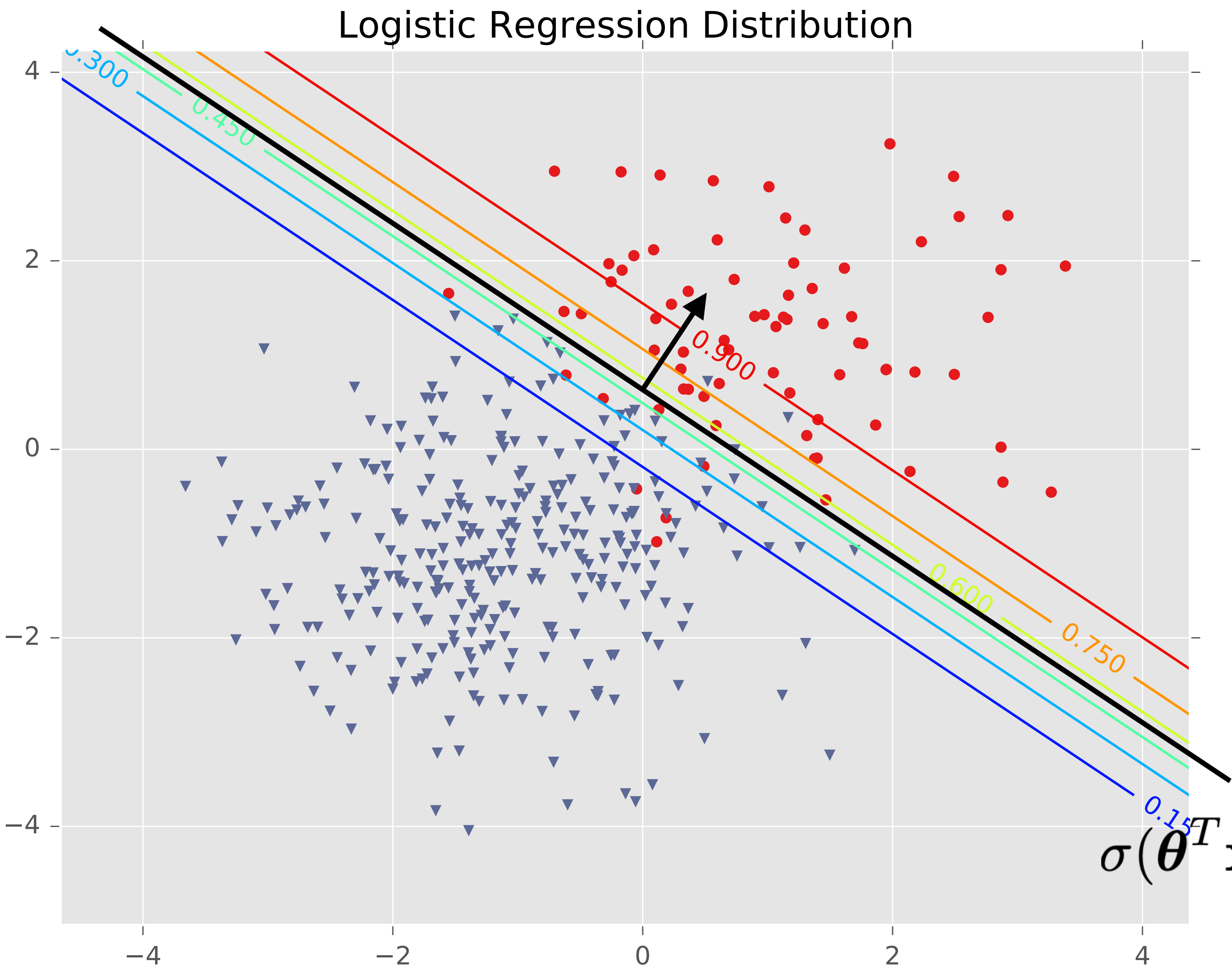
- Conceptual Change: 2D classification in 3D
- Why is it called Logistic *Regression* and not Logistic *Classification*?

LOGISTIC REGRESSION ON GAUSSIAN DATA

Logistic Regression

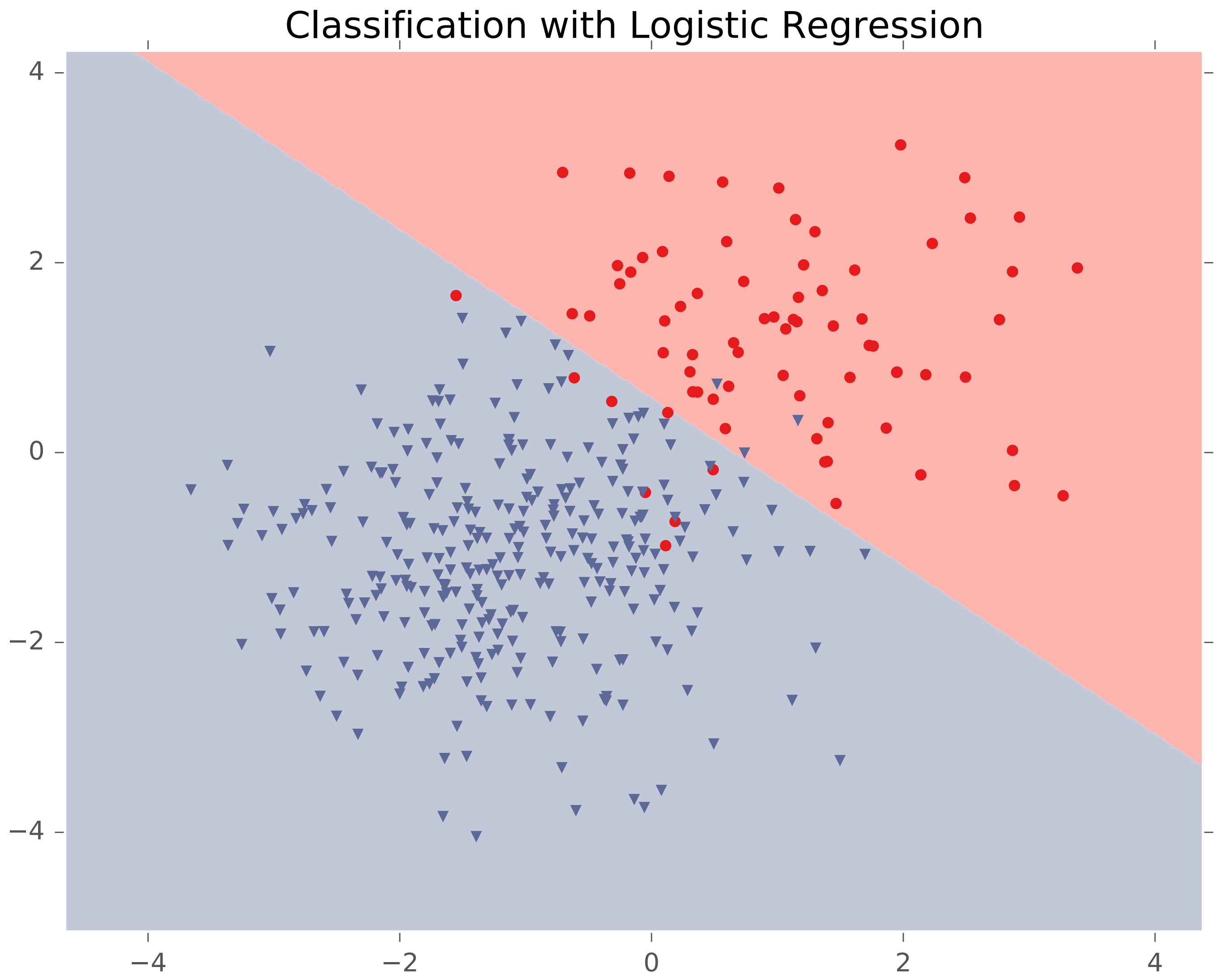


Logistic Regression



$$\sigma(\boldsymbol{\theta}^T \mathbf{x}) = 0.5$$

Logistic Regression



LEARNING LOGISTIC REGRESSION

Maximum **Conditional** Likelihood Estimation

Learning: finds the parameters that minimize some objective function.

$$\boldsymbol{\theta}^* = \operatorname*{argmin}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$$

We minimize the *negative* log conditional likelihood:

$$J(\boldsymbol{\theta}) = -\log \prod_{i=1}^{N} p_{\boldsymbol{\theta}}(y^{(i)}|\mathbf{x}^{(i)})$$

Why?

1. We can't maximize likelihood (as in Naïve Bayes) because we don't have a joint model p(x,y)
2. It worked well for Linear Regression (least squares is actually MCLE! more on this later…)

Maximum **Conditional** Likelihood Estimation

Learning: Four approaches to solving $\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}}{\text{argmin}}\, J(\boldsymbol{\theta})$

Approach 1: Gradient Descent
(take larger – more certain – steps opposite the gradient)

Approach 2: Stochastic Gradient Descent (SGD)
(take many small steps opposite the gradient)

Approach 3: Newton's Method
(use second derivatives to better follow curvature)

Approach 4: Closed Form???
(set derivatives equal to zero and solve for parameters)

Maximum **Conditional** Likelihood Estimation

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~~**Approach 4:** Closed Form???~~
~~(set derivatives equal to zero and solve for parameters)~~

Logistic Regression does not have a closed form solution for MLE parameters.

SGD for Logistic Regression

Question:

Which of the following is a correct description of SGD for Logistic Regression?

Answer:

At each step (i.e. iteration) of SGD for Logistic Regression we…

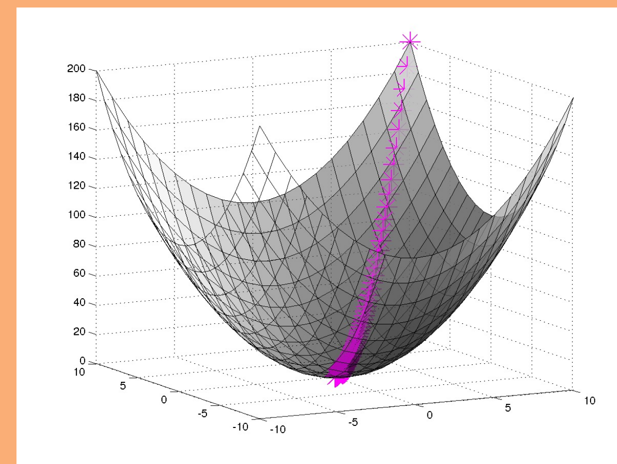
A. (1) compute the gradient of the log-likelihood for all examples (2) update all the parameters using the gradient
B. (1) ask Matt for a description of SGD for Logistic Regression, (2) write it down, (3) report that answer
C. (1) compute the gradient of the log-likelihood for all examples (2) randomly pick an example (3) update only the parameters for that example
D. (1) randomly pick a parameter, (2) compute the partial derivative of the log-likelihood with respect to that parameter, (3) update that parameter for all examples
E. (1) randomly pick an example, (2) compute the gradient of the log-likelihood for that example, (3) update all the parameters using that gradient
F. (1) randomly pick a parameter and an example, (2) compute the gradient of the log-likelihood for that example with respect to that parameter, (3) update that parameter using that gradient

Recall...

Gradient Descent

Algorithm 1 Gradient Descent

$$
\begin{array}{l}
\textbf{procedure } \mathrm{GD}(\mathcal{D}, \boldsymbol{\theta}^{(0)}) \\
\quad \boldsymbol{\theta} \leftarrow \boldsymbol{\theta}^{(0)} \\
\quad \textbf{while } \text{not converged } \textbf{do} \\
\quad\quad \boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \gamma \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \\
\quad \textbf{return } \boldsymbol{\theta}
\end{array}
$$



In order to apply GD to Logistic Regression all we need is the **gradient** of the objective function (i.e. vector of partial derivatives).

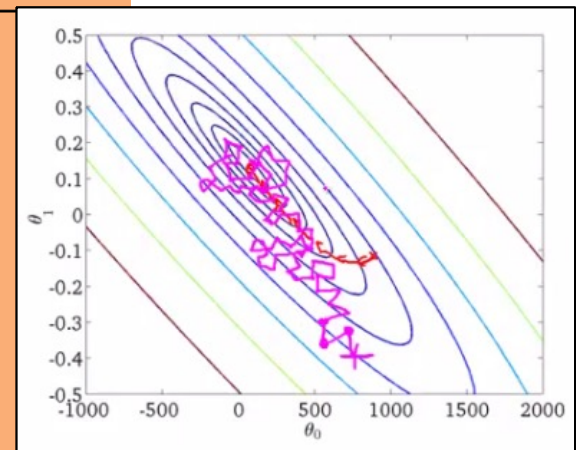
$$
\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \begin{bmatrix} \frac{d}{d\theta_1} J(\boldsymbol{\theta}) \\ \frac{d}{d\theta_2} J(\boldsymbol{\theta}) \\ \vdots \\ \frac{d}{d\theta_M} J(\boldsymbol{\theta}) \end{bmatrix}
$$

Recall...

Stochastic Gradient Descent (SGD)

Algorithm 1 Stochastic Gradient Descent (SGD)

procedure SGD($\mathcal{D}$, $\boldsymbol{\theta}^{(0)}$)
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta}^{(0)}$
    while not converged **do**
        for $i \in$ shuffle($\{1, 2, \ldots, N\}$) **do**
            $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \gamma \nabla_{\boldsymbol{\theta}} J^{(i)}(\boldsymbol{\theta})$
    return θ



We can also apply SGD to solve the MCLE problem for Logistic Regression.

We need a per-example objective:

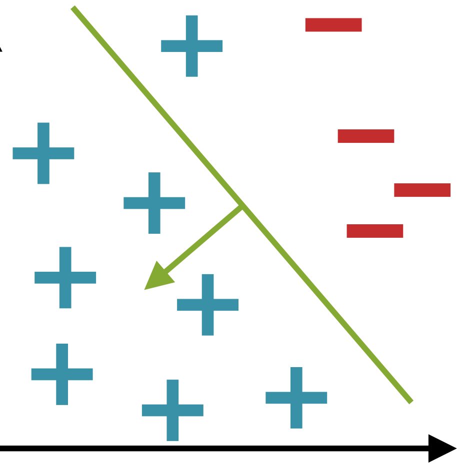
$$\text{Let } J(\boldsymbol{\theta}) = \sum_{i=1}^{N} J^{(i)}(\boldsymbol{\theta})$$
$$\text{where } J^{(i)}(\boldsymbol{\theta}) = -\log p_{\boldsymbol{\theta}}(y^i | \mathbf{x}^i).$$

Logistic Regression vs. Perceptron

Question:

True or False: Just like Perceptron, **one step** (i.e. iteration) **of SGD for Logistic Regression** will result in a change to the parameters **only** if the current example is **incorrectly** classified.

Answer:



BAYES OPTIMAL CLASSIFIER

Bayes Optimal Classifier

Suppose you knew the distribution $p^*(y \mid \mathbf{x})$ or had a good approximation to it.

Question:

How would you design a function $y = h(\mathbf{x})$ to predict a single label?

Answer:

You'd use the *Bayes optimal classifier*!

ur

approximates $c^*(\mathbf{x})$

Probabilistic Learning

Today, we assume that our output is **sampled** from a conditional **probability distribution:**

$$\mathbf{x}^{(i)} \sim p^*(\cdot)$$

$$y^{(i)} \sim p^*(\cdot|\mathbf{x}^{(i)})$$

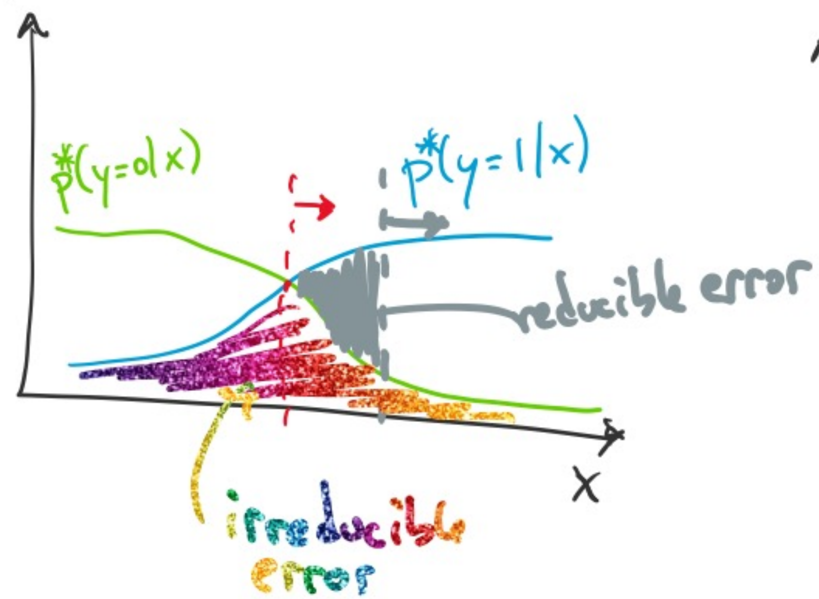
Our goal is to learn a probability distribution $p(y|\mathbf{x})$ that best approximates $p^*(y|\mathbf{x})$

Bayes Optimal Classifier

Def: an oracle knows everything (e.g. $p^*(y|x)$)

usually unknown

$y \in \{0,1\}$



Q: What is the optimal classifier in this setting?

A: $\hat{y} = h(\vec{x}) = \begin{cases} 1 & \text{if } p(y=1|x) \geq p(y=0|x) \\ 0 & \text{otherwise} \end{cases}$

$= \underset{y \in \{0,1\}}{\operatorname{argmax}}\ p(y|x)$

Bayes Optimal Classifier for 0/1 loss function

OPTIMIZATION METHOD #4: MINI-BATCH SGD

Mini-Batch SGD

- **Gradient Descent:**
  Compute true gradient exactly from all N examples
- **Stochastic Gradient Descent (SGD):**
  Approximate true gradient by the gradient of one randomly chosen example
- **Mini-Batch SGD:**
  Approximate true gradient by the average gradient of K randomly chosen examples

Mini-Batch SGD

while not converged: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \gamma \mathbf{g}$

Three variants of first-order optimization:

Gradient Descent: $\mathbf{g} = \nabla J(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^{N} \nabla J^{(i)}(\boldsymbol{\theta})$

SGD: $\mathbf{g} = \nabla J^{(i)}(\boldsymbol{\theta})$ where i sampled uniformly

Mini-batch SGD: $\mathbf{g} = \frac{1}{S} \sum_{s=1}^{S} \nabla J^{(i_s)}(\boldsymbol{\theta})$ where i_s sampled uniformly $\forall s$

Logistic Regression Objectives

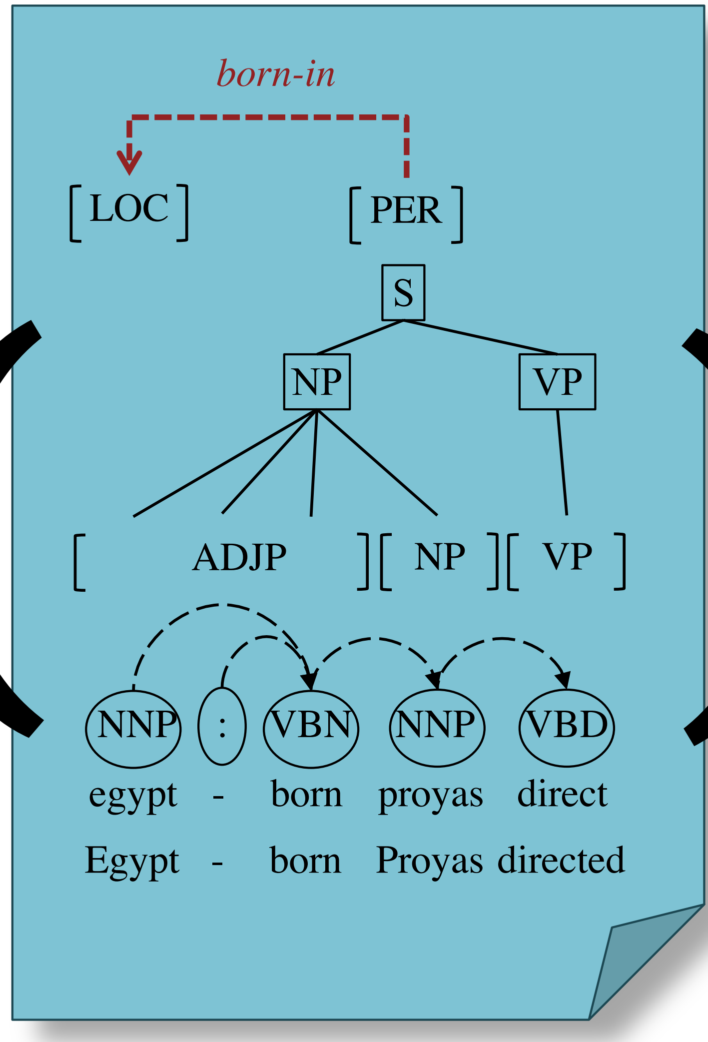
You should be able to…

- Apply the principle of maximum likelihood estimation (MLE) to learn the parameters of a probabilistic model
- Given a discriminative probabilistic model, derive the conditional log-likelihood, its gradient, and the corresponding Bayes Classifier
- Explain the practical reasons why we work with the **log** of the likelihood
- Implement logistic regression for binary or multiclass classification
- Prove that the decision boundary of binary logistic regression is linear
- For linear regression, show that the parameters which minimize squared error are equivalent to those that maximize conditional likelihood

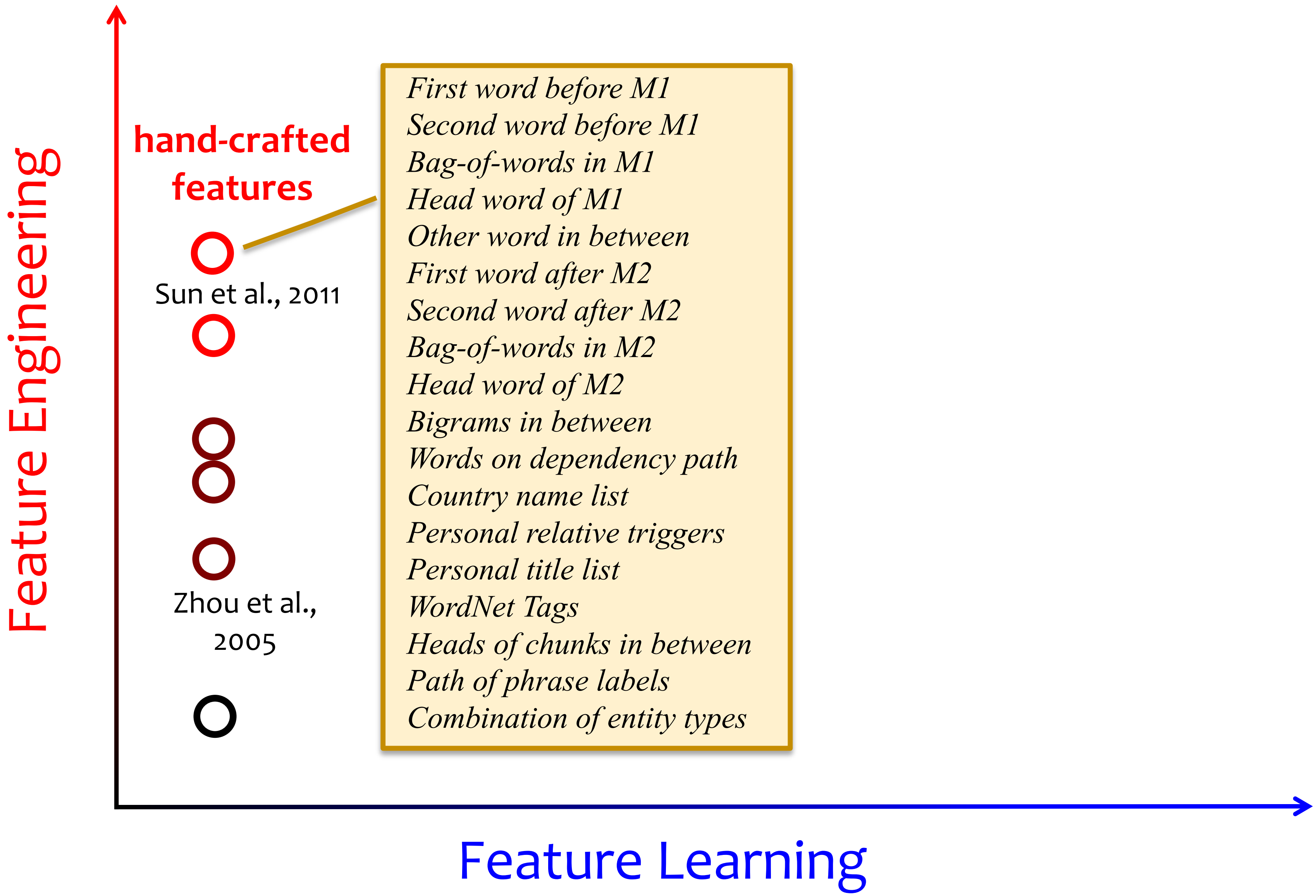
FEATURE ENGINEERING

Handcrafted Features

$$p(\textcolor{darkred}{y}|x) \propto \exp(\Theta_{\textcolor{darkred}{y}} \bullet f(\ \ \ ))$$



Where do features come from?



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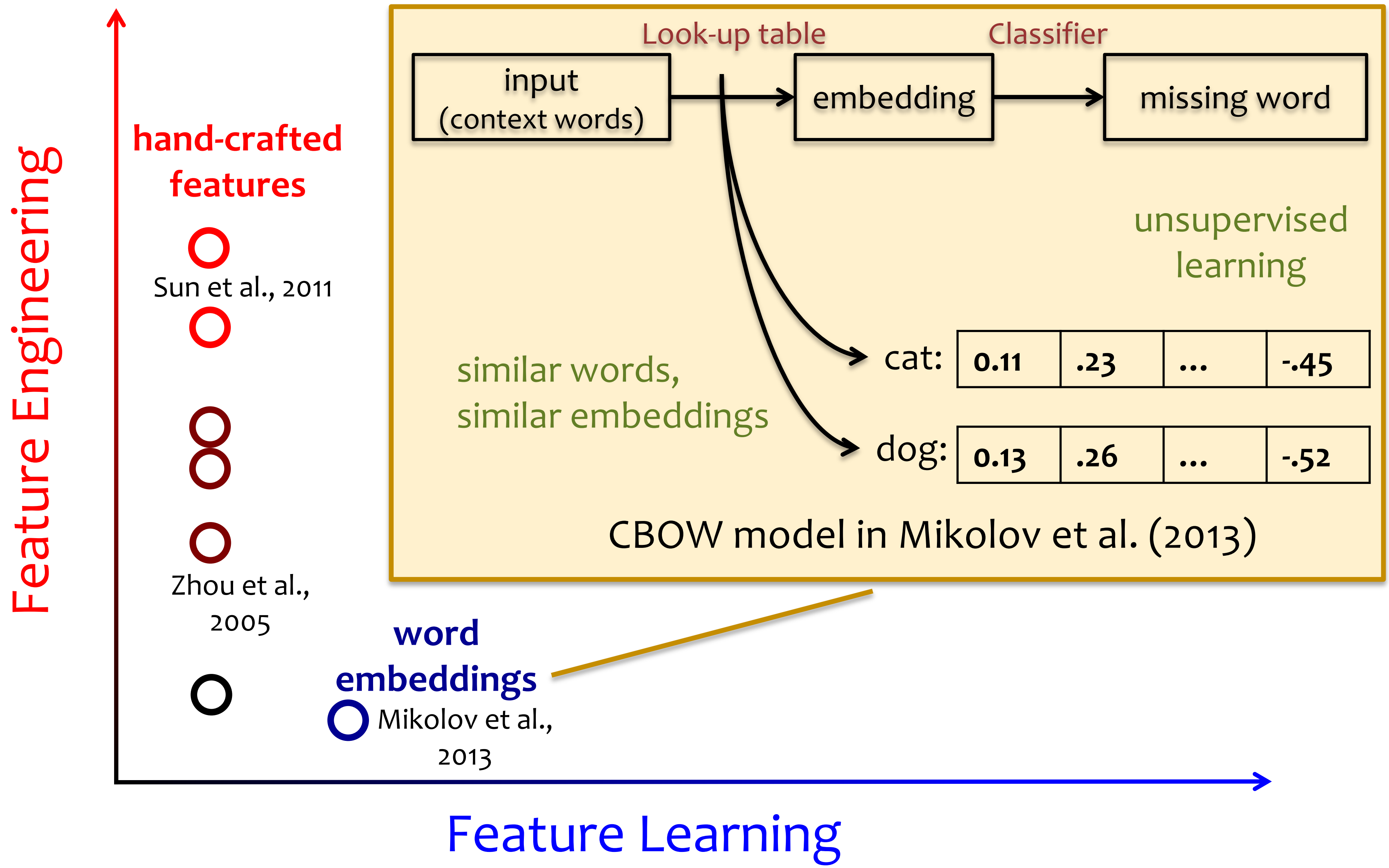

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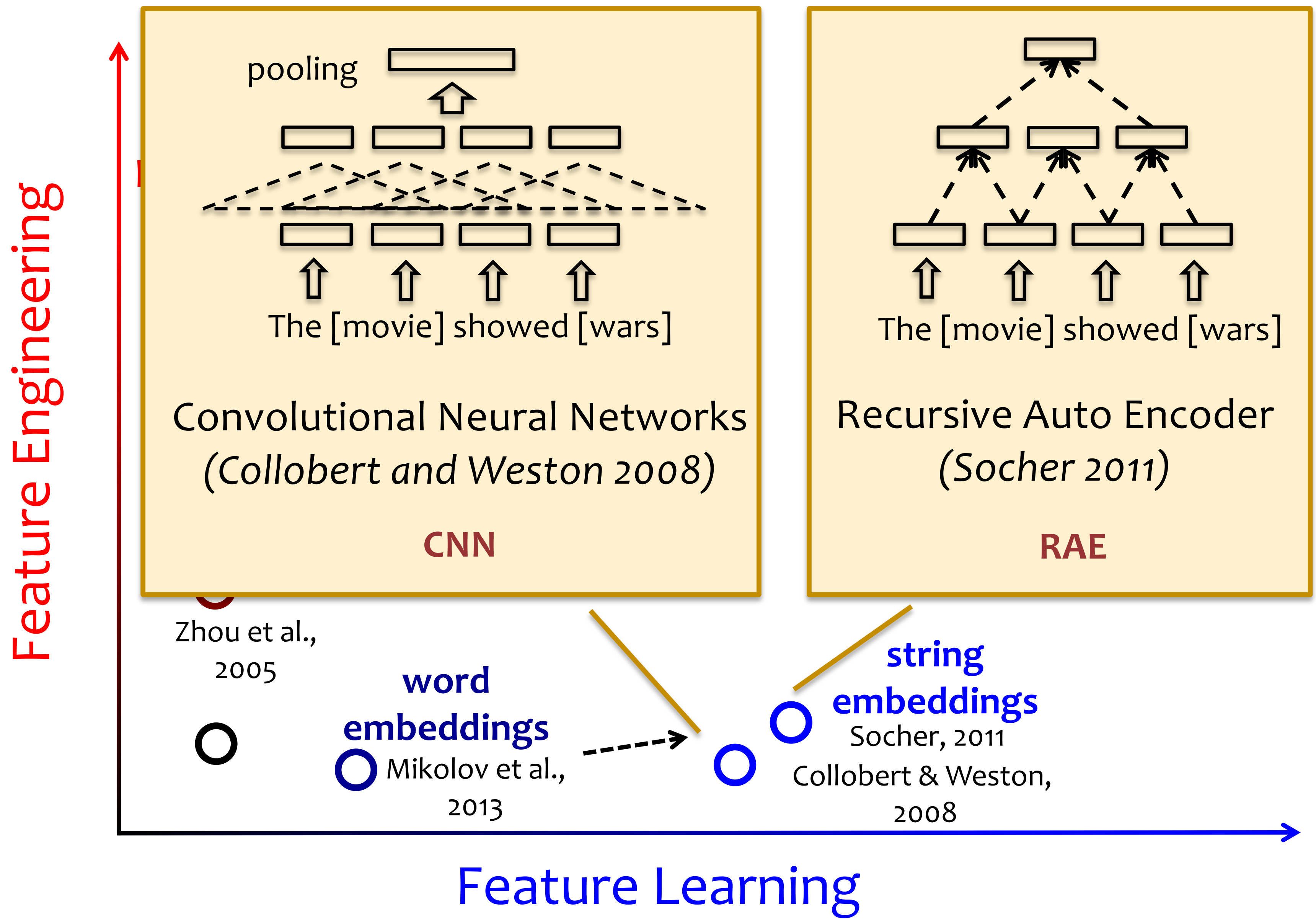

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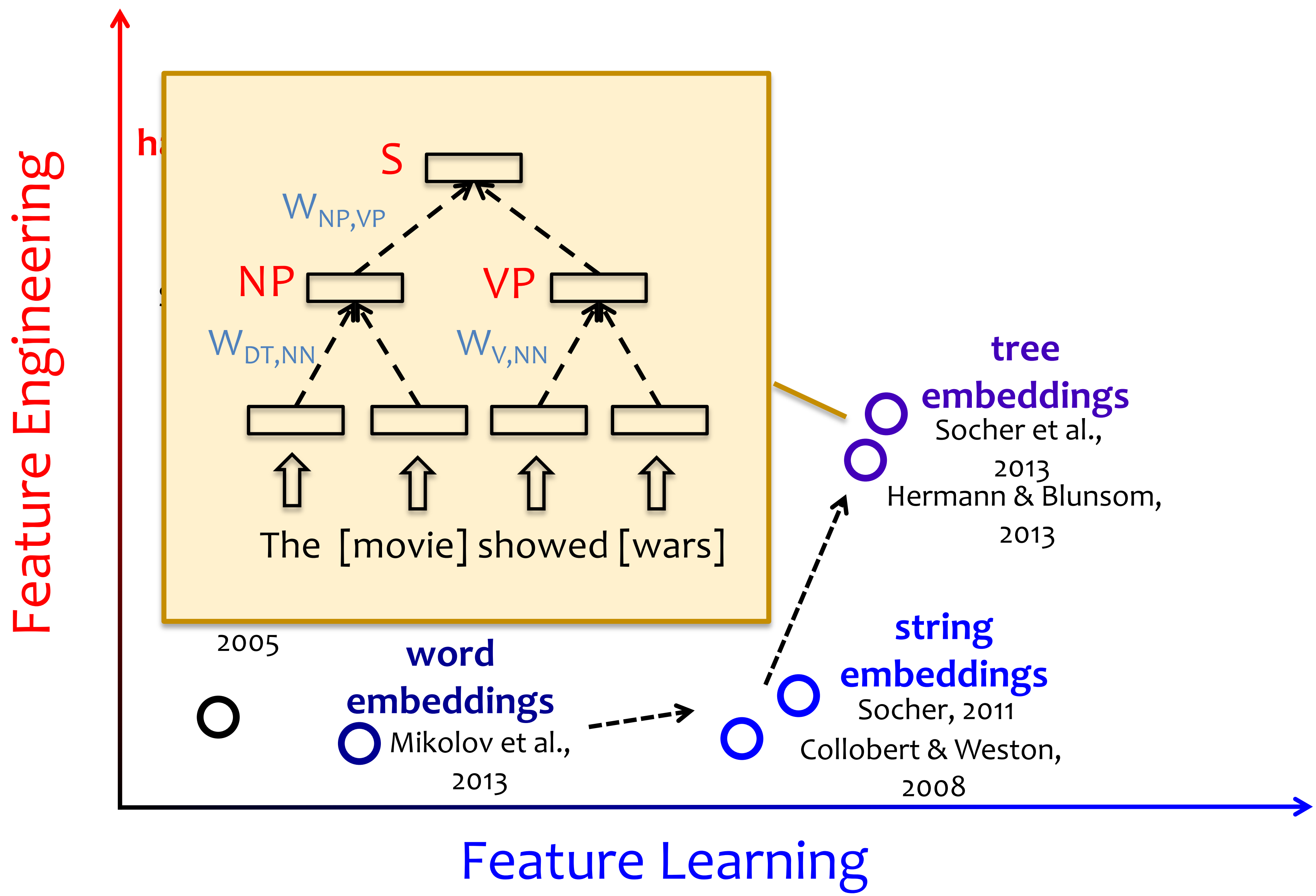

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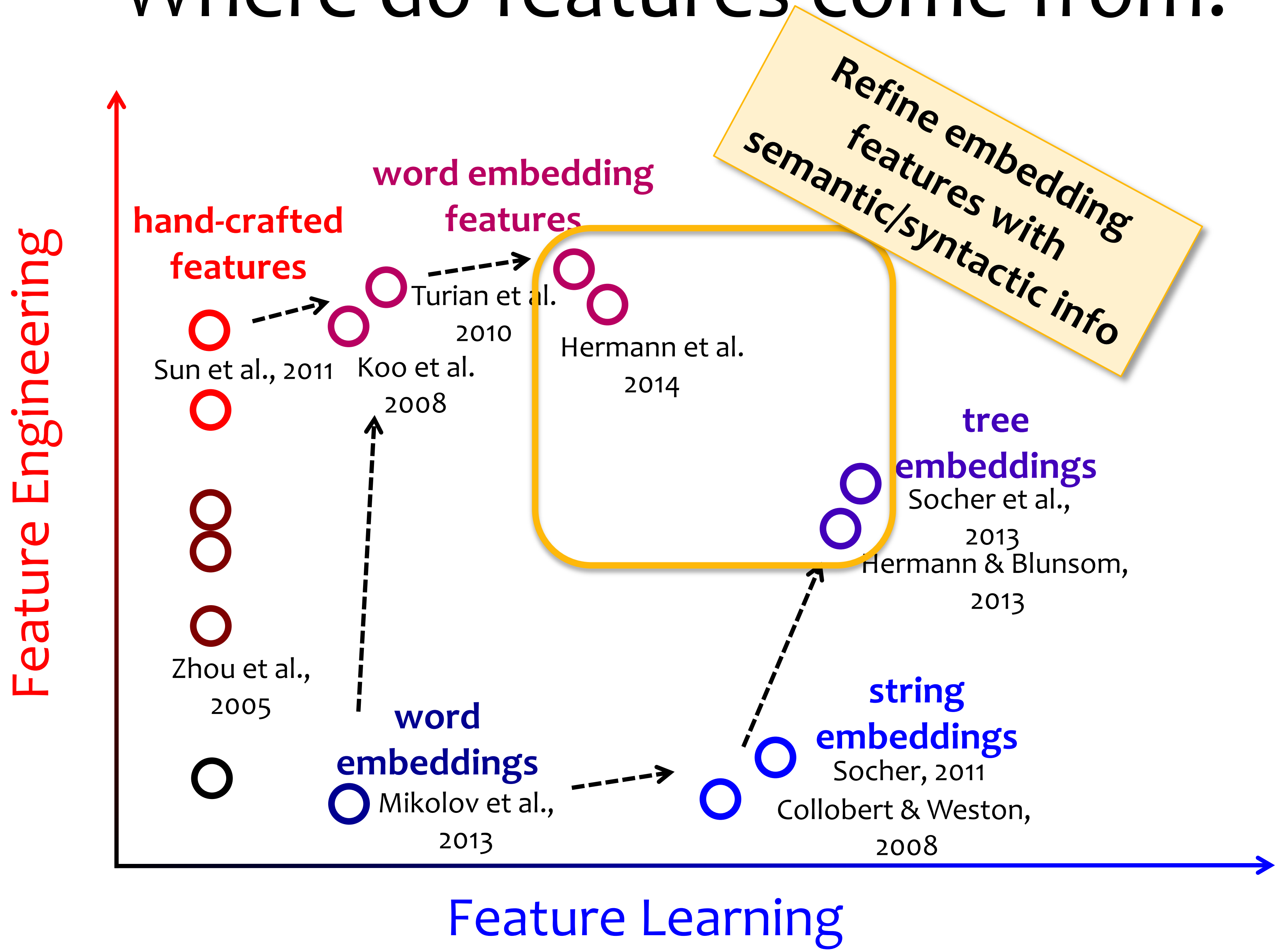

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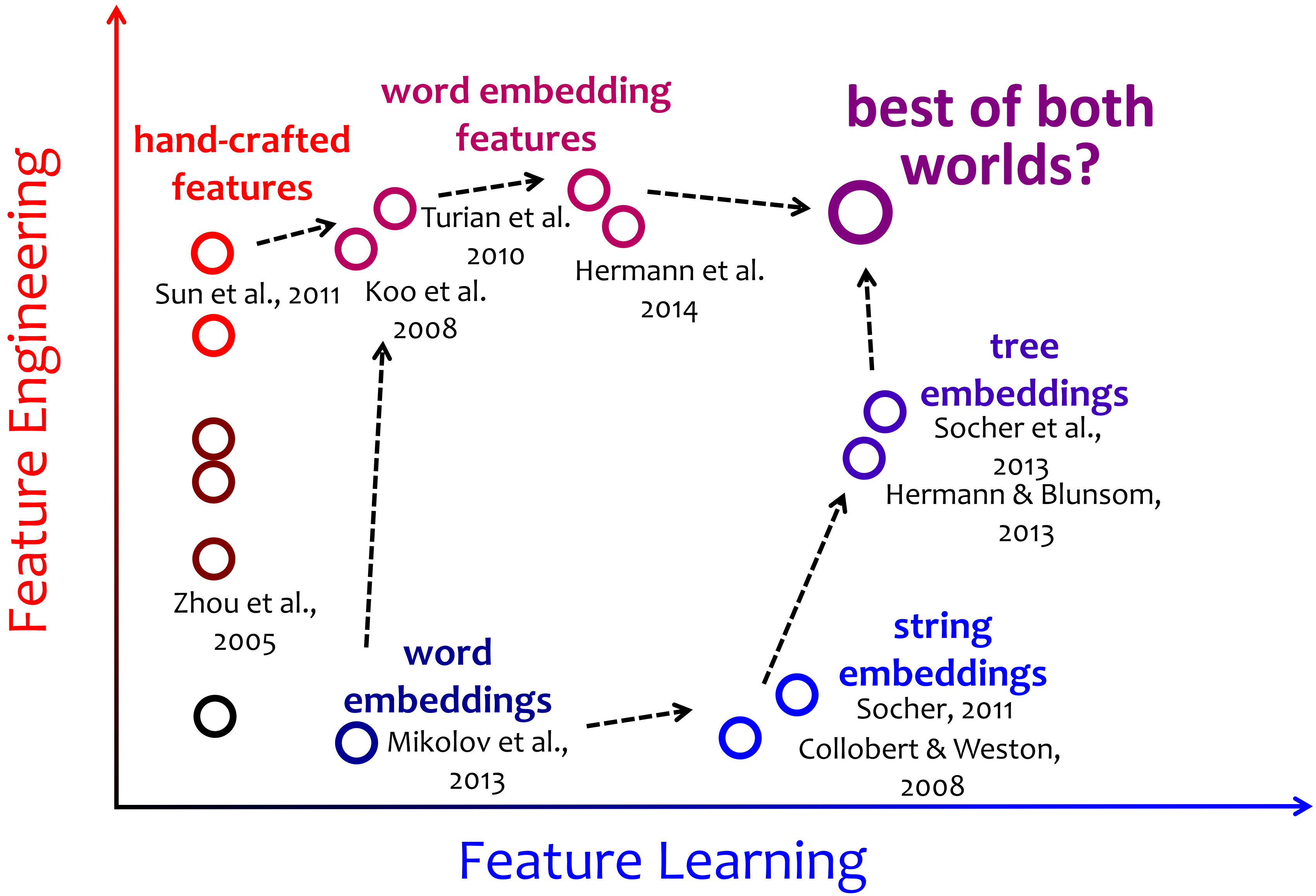
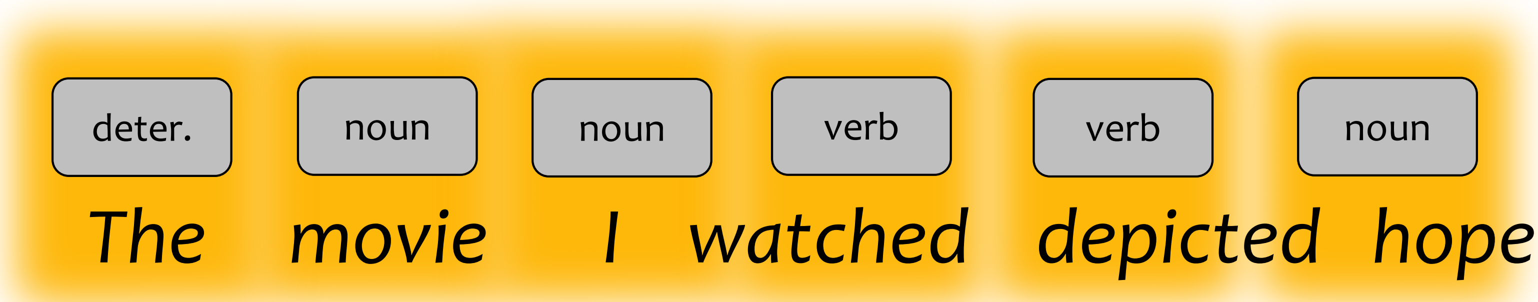


Feature Engineering for NLP

Suppose you build a logistic regression model to predict a part-of-speech (POS) tag for each word in a sentence.

What features should you use?



Feature Engineering for NLP

Per-word Features:

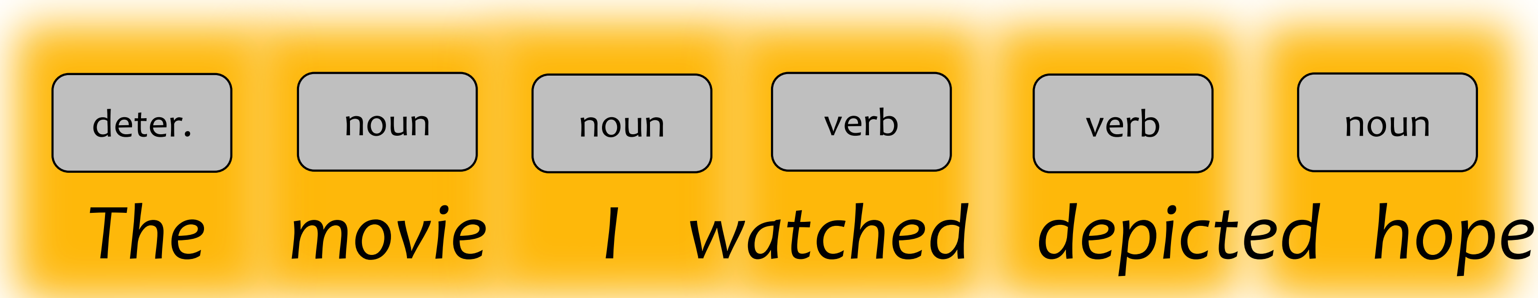
	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(4)}$	$x^{(5)}$	$x^{(6)}$
`is-capital(w_i)`	1	0	1	0	0	0
`endswith(w_i,"e")`	1	1	0	0	0	1
`endswith(w_i,"d")`	0	0	0	1	1	0
`endswith(w_i,"ed")`	0	0	0	1	1	0
`w_i == "aardvark"`	0	0	0	0	0	0
`w_i == "hope"`	0	0	0	0	0	1
…	…	…	…	…	…	…

deter.	noun	noun	verb	verb	noun
The	*movie*	*I*	*watched*	*depicted*	*hope*

Feature Engineering for NLP

Context Features:

	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(4)}$	$x^{(5)}$	$x^{(6)}$
…	…	…	…	…	…	…
`w_i == "watched"`	0	0	0	1	0	0
`w_{i+1} == "watched"`	0	0	1	0	0	0
`w_{i-1} == "watched"`	0	0	0	0	1	0
`w_{i+2} == "watched"`	0	1	0	0	0	0
`w_{i-2} == "watched"`	0	0	0	0	0	1
…	…	…	…	…	…	…



Feature Engineering for NLP

Context Features:

	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(4)}$	$x^{(5)}$	$x^{(6)}$
…	…	…	…	…	…	…
`w_i == "I"`	0	0	1	0	0	0
`w_{i+1} == "I"`	0	1	0	0	0	0
`w_{i-1} == "I"`	0	0	0	1	0	0
`w_{i+2} == "I"`	1	0	0	0	0	0
`w_{i-2} == "I"`	0	0	0	0	1	0
…	…	…	…	…	…	…

deter.	noun	noun	verb	verb	noun
The	*movie*	*I*	*watched*	*depicted*	*hope*

Table from Manning (2011)

Feature Engineering for NLP

Table 3. Tagging accuracies with different feature templates and other changes on the *WSJ* 19-21 development set.

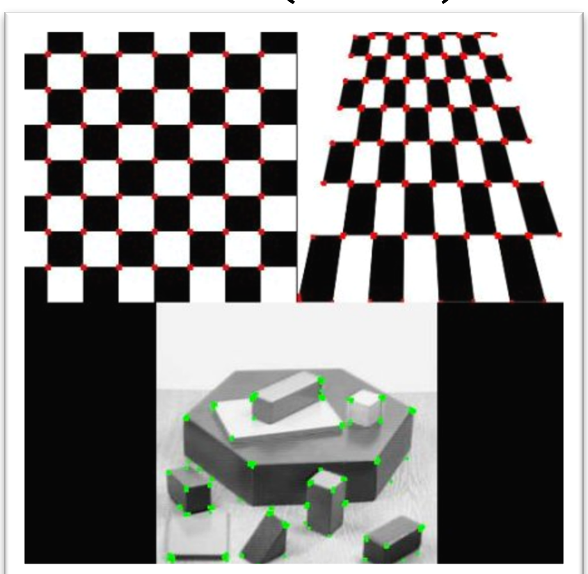
Model	Feature Templates	# Feats	Sent. Acc.	Token Acc.	Unk. Acc.
3GRAMMEMM	See text	248,798	52.07%	96.92%	88.99%
NAACL 2003	See text and [1]	460,552	55.31%	97.15%	88.61%
Replication	See text and [1]	460,551	55.62%	97.18%	88.92%
Replication′	+rareFeatureThresh = 5	482,364	55.67%	97.19%	88.96%
5W	$+\langle t_0, w_{-2}\rangle, \langle t_0, w_2\rangle$	730,178	56.23%	97.20%	89.03%
5WSHAPES	$+\langle t_0, s_{-1}\rangle, \langle t_0, s_0\rangle, \langle t_0, s_{+1}\rangle$	731,661	56.52%	97.25%	89.81%
5WSHAPESDS	+ distributional similarity	737,955	56.79%	97.28%	90.46%

deter.	noun	noun	verb	verb	noun
The	*movie*	*I*	*watched*	*depicted*	*hope*

Feature Engineering for CV

Edge detection (Canny)

Corner Detection (Harris)



Figures from http://opencv.org

Feature Engineering for CV

Scale Invariant Feature Transform (SIFT)

Figure 3: Model images of planar objects are shown in the top row. Recognition results below show model outlines and image keys used for matching.

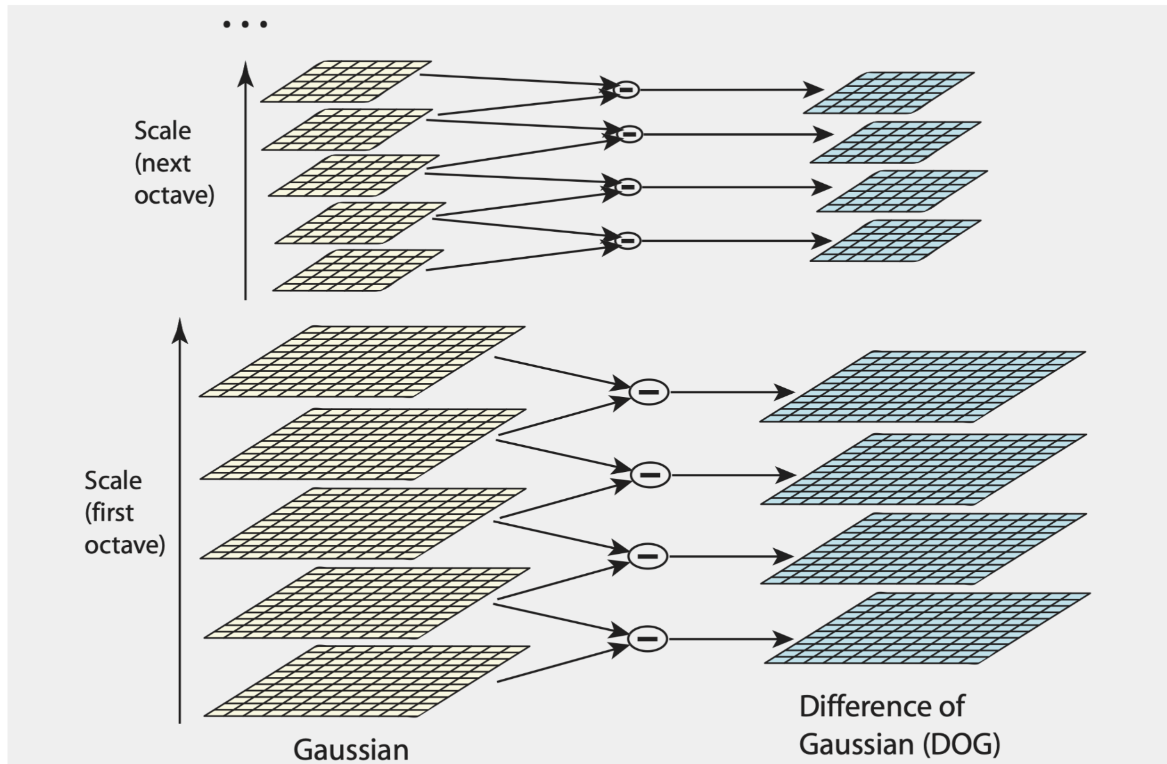


Figure 1: For each octave of scale space, the initial image is repeatedly convolved with Gaussians to produce the set of scale space images shown on the left. Adjacent Gaussian images are subtracted to produce the difference-of-Gaussian images on the right. After each octave, the Gaussian image is down-sampled by a factor of 2, and the process repeated.

Figure from Lowe (1999) and Lowe (2004)

NON-LINEAR FEATURES

Nonlinear Features

- aka. "nonlinear basis functions"
- So far, input was always $\mathbf{x} = [x_1, \ldots, x_M]$
- **Key Idea:** let input be some function of **x**
  - original input: $\mathbf{x} \in \mathbb{R}^M$
  - new input: $\mathbf{x}' \in \mathbb{R}^{M'}$

    where $M' > M$ (usually)
  - define $\mathbf{x}' = b(\mathbf{x}) = [b_1(\mathbf{x}), b_2(\mathbf{x}), \ldots, b_{M'}(\mathbf{x})]$

    where $b_i : \mathbb{R}^M \to \mathbb{R}$ is any function
- **Examples:** (M = 1)

polynomial	$b_j(x) = x^j \quad \forall j \in \{1, \ldots, J\}$
radial basis function	$b_j(x) = \exp\left(\frac{-(x - \mu_j)^2}{2\sigma_j^2}\right)$
sigmoid	$b_j(x) = \frac{1}{1 + \exp(-\omega_j x)}$
log	$b_j(x) = \log(x)$

For a linear model: still a linear function of b(**x**) even though a nonlinear function of **x**

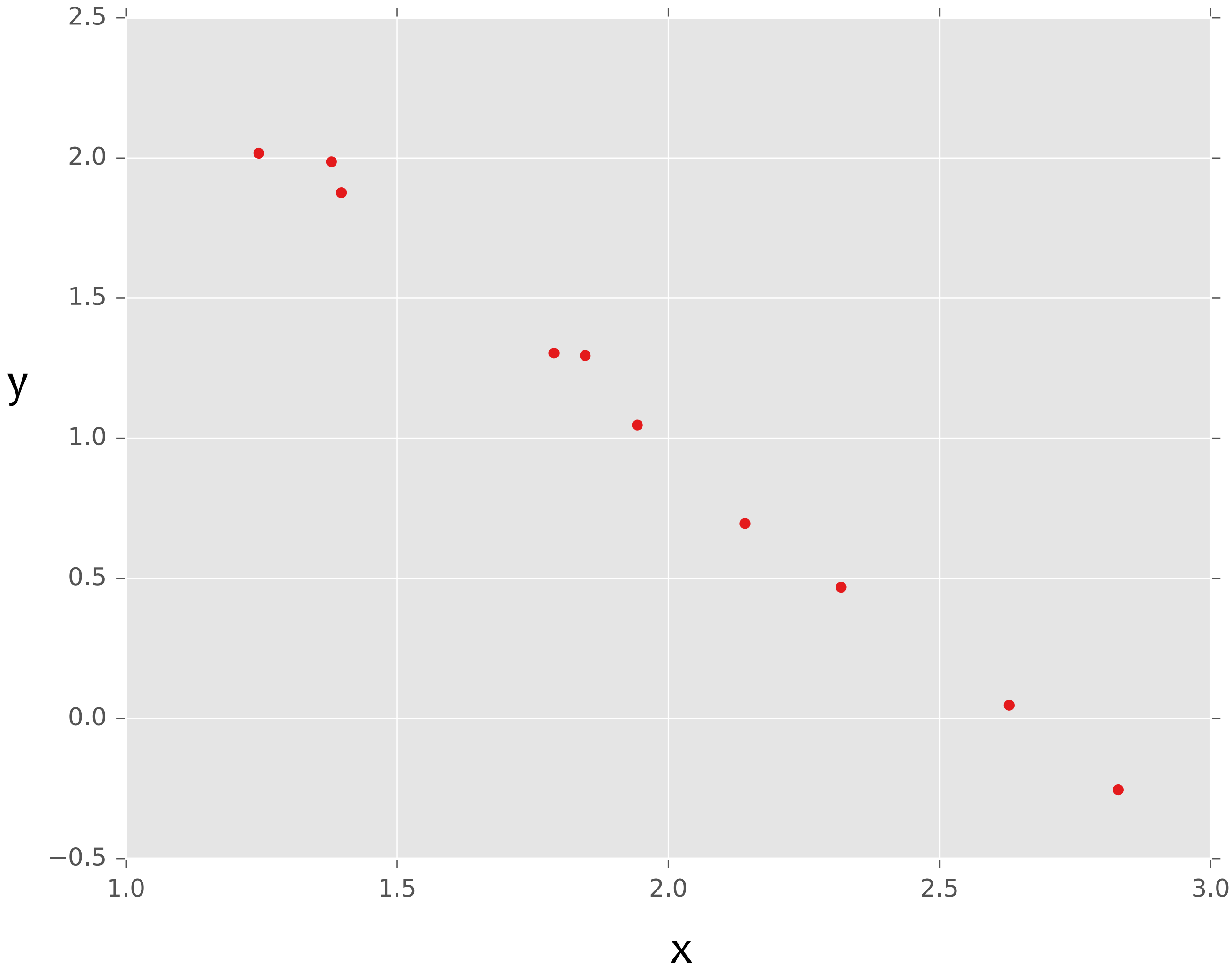
Examples:
- Perceptron
- Linear regression
- Logistic regression

Example: Linear Regression

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function

i	y	x
1	2.0	1.2
2	1.3	1.7
…	…	…
10	1.1	1.9

true "unknown" target function is linear with negative slope and gaussian noise



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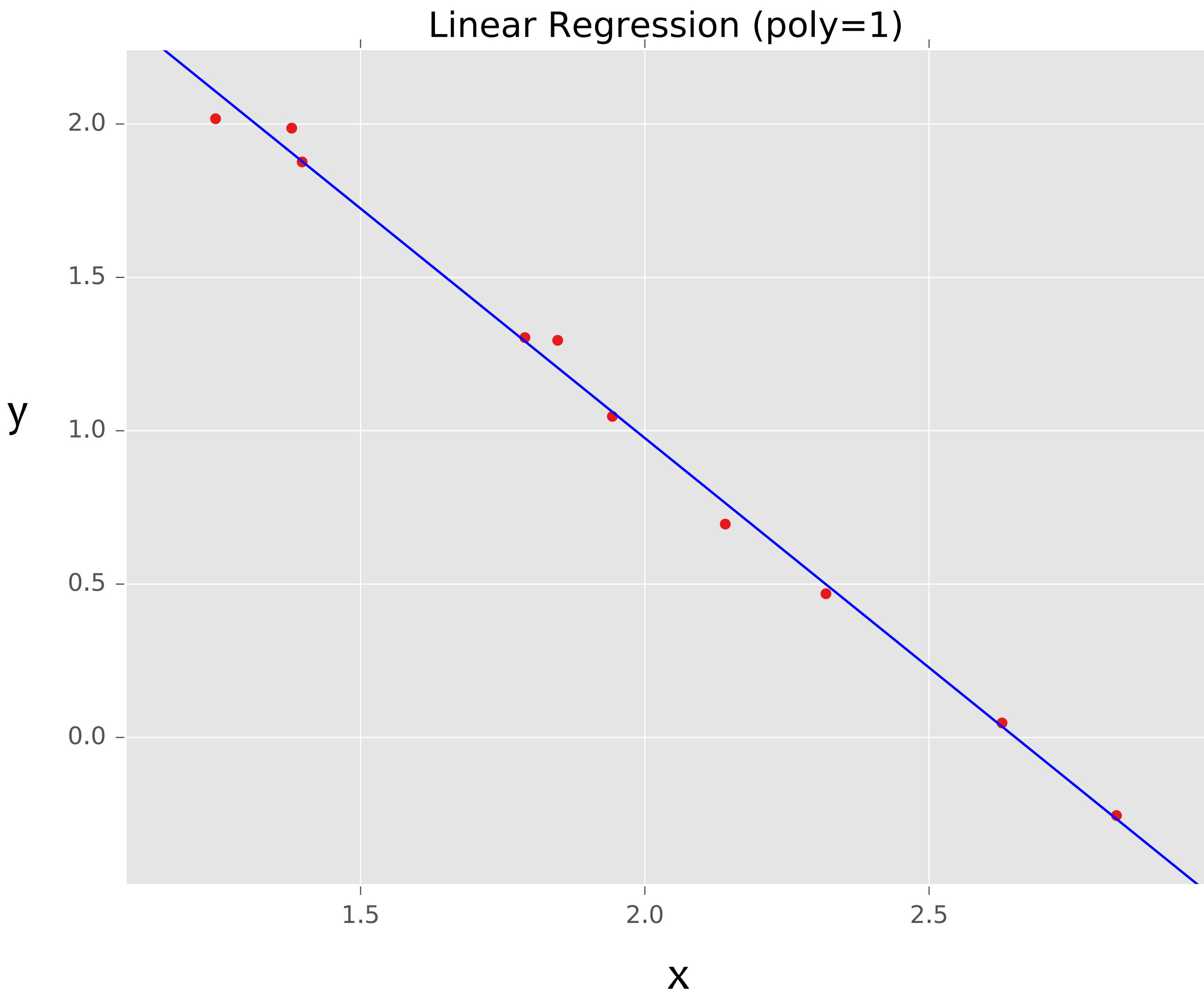
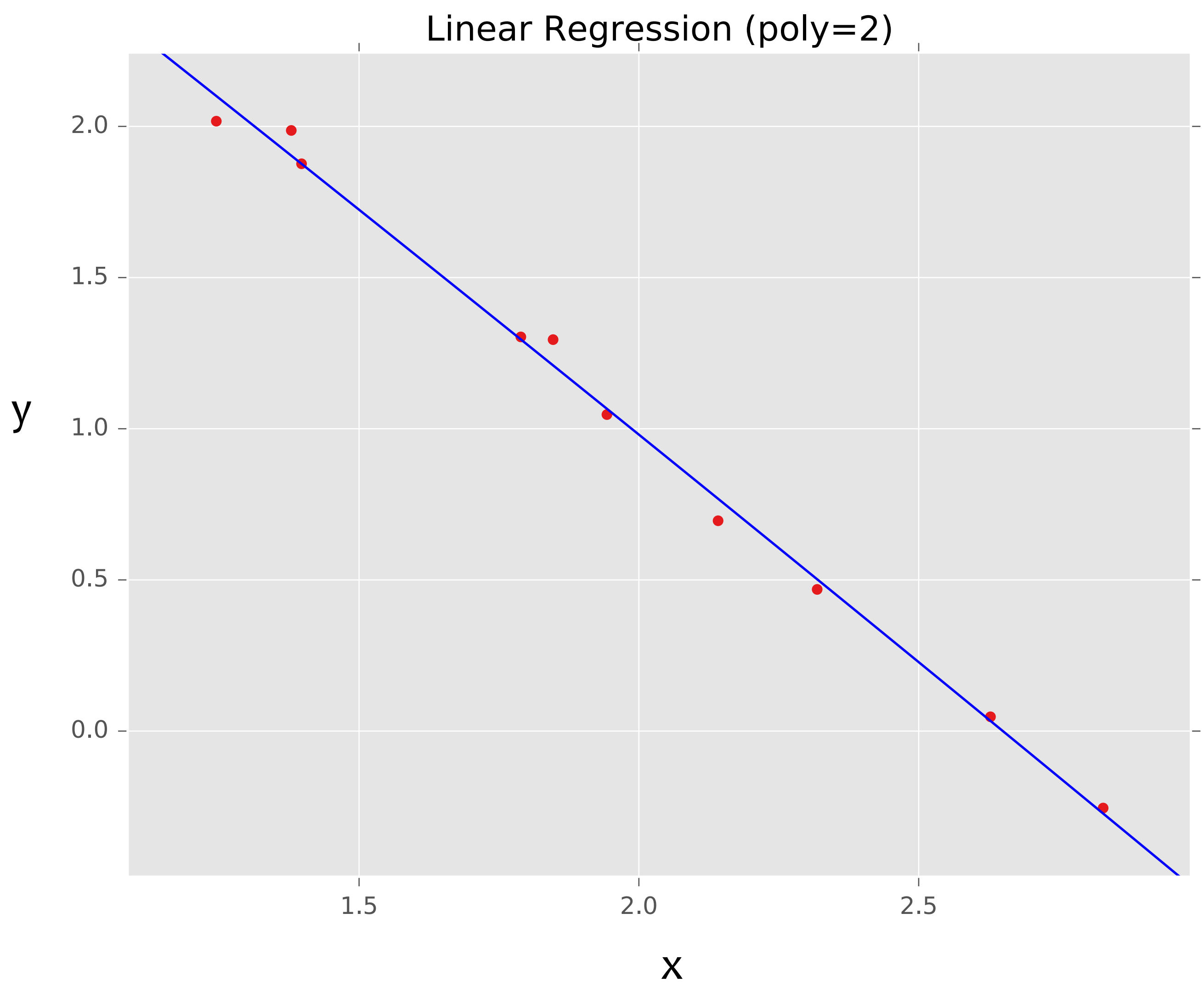


Example: Linear Regression

Goal: Learn y = $\mathbf{w}^T$ f($\mathbf{x}$) + b
where f(.) is a polynomial basis function

i	y	x	x^2
1	2.0	1.2	$(1.2)^2$
2	1.3	1.7	$(1.7)^2$
…	…	…	…
10	1.1	1.9	$(1.9)^2$

true "unknown" target function is linear with negative slope and gaussian noise

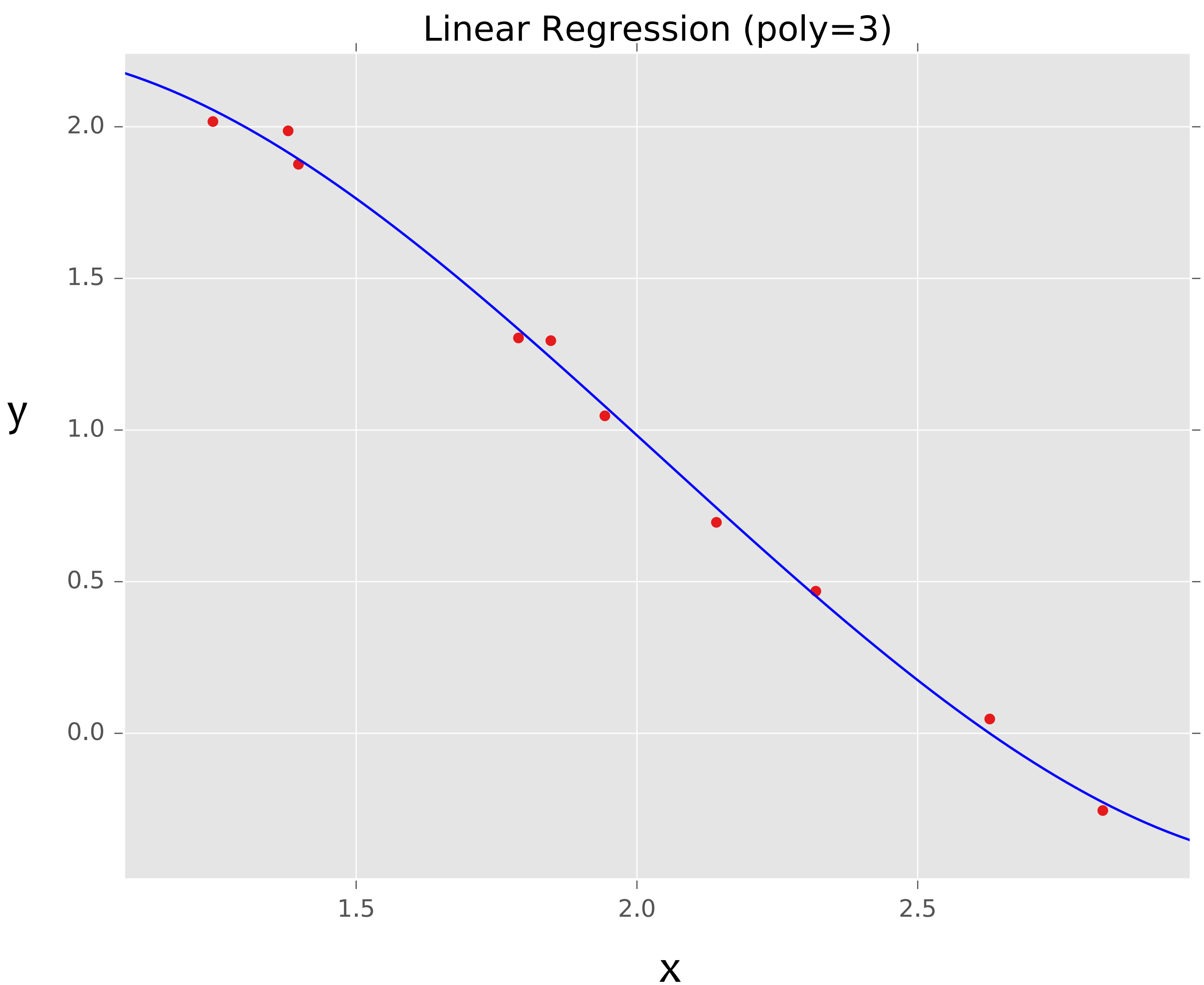


Example: Linear Regression

Goal: Learn y = $\mathbf{w}^T$ f($\mathbf{x}$) + b
where f(.) is a polynomial
basis function

i	y	x	x^2	x^3
1	2.0	1.2	$(1.2)^2$	$(1.2)^3$
2	1.3	1.7	$(1.7)^2$	$(1.7)^3$
…	…	…	…	…
10	1.1	1.9	$(1.9)^2$	$(1.9)^3$

true "unknown" target function is linear with negative slope and gaussian noise

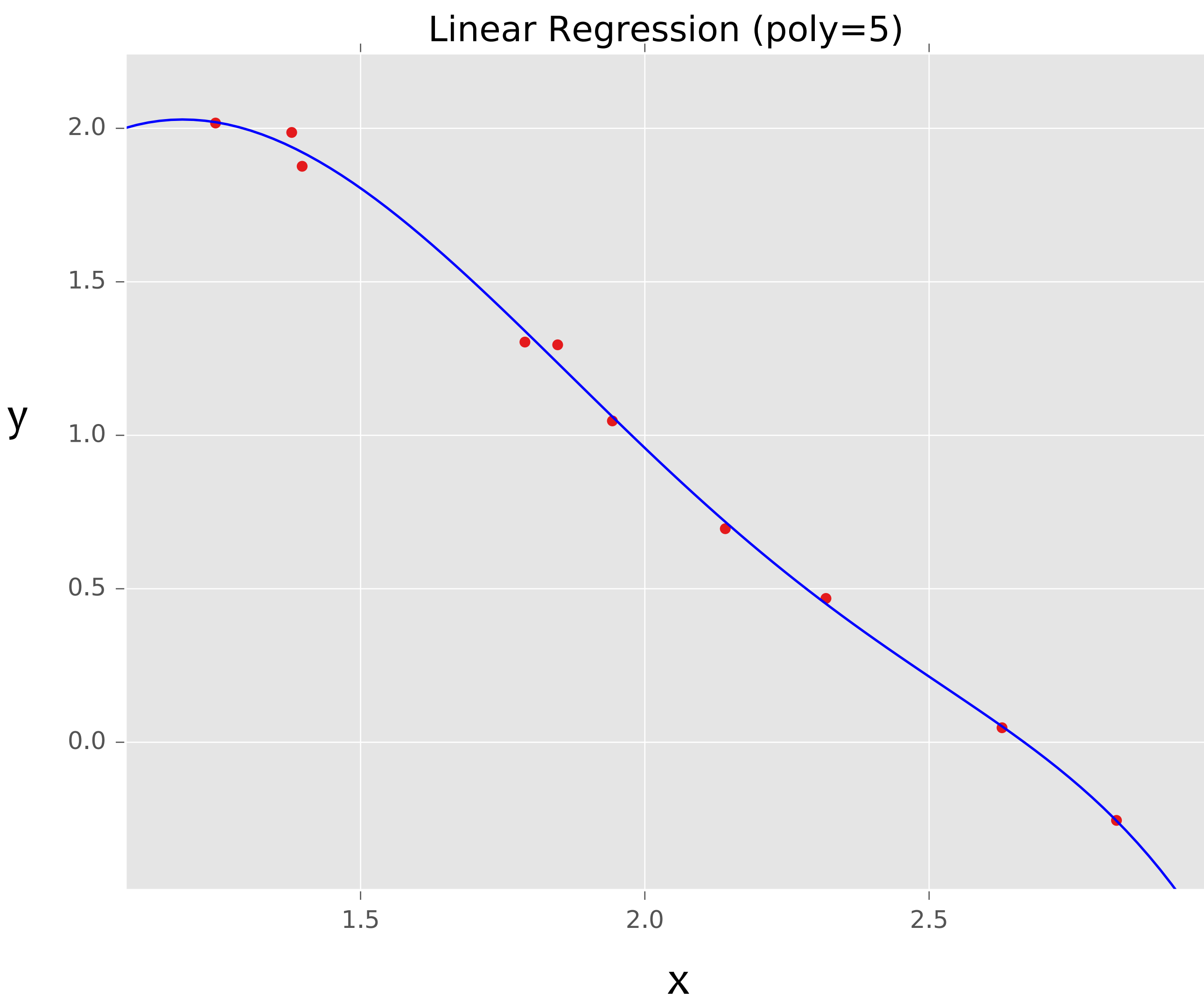


Example: Linear Regression

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function

i	y	x	…	x^5
1	2.0	1.2	…	$(1.2)^5$
2	1.3	1.7	…	$(1.7)^5$
…	…	…	…	…
10	1.1	1.9	…	$(1.9)^5$

true "unknown" target function is linear with negative slope and gaussian noise

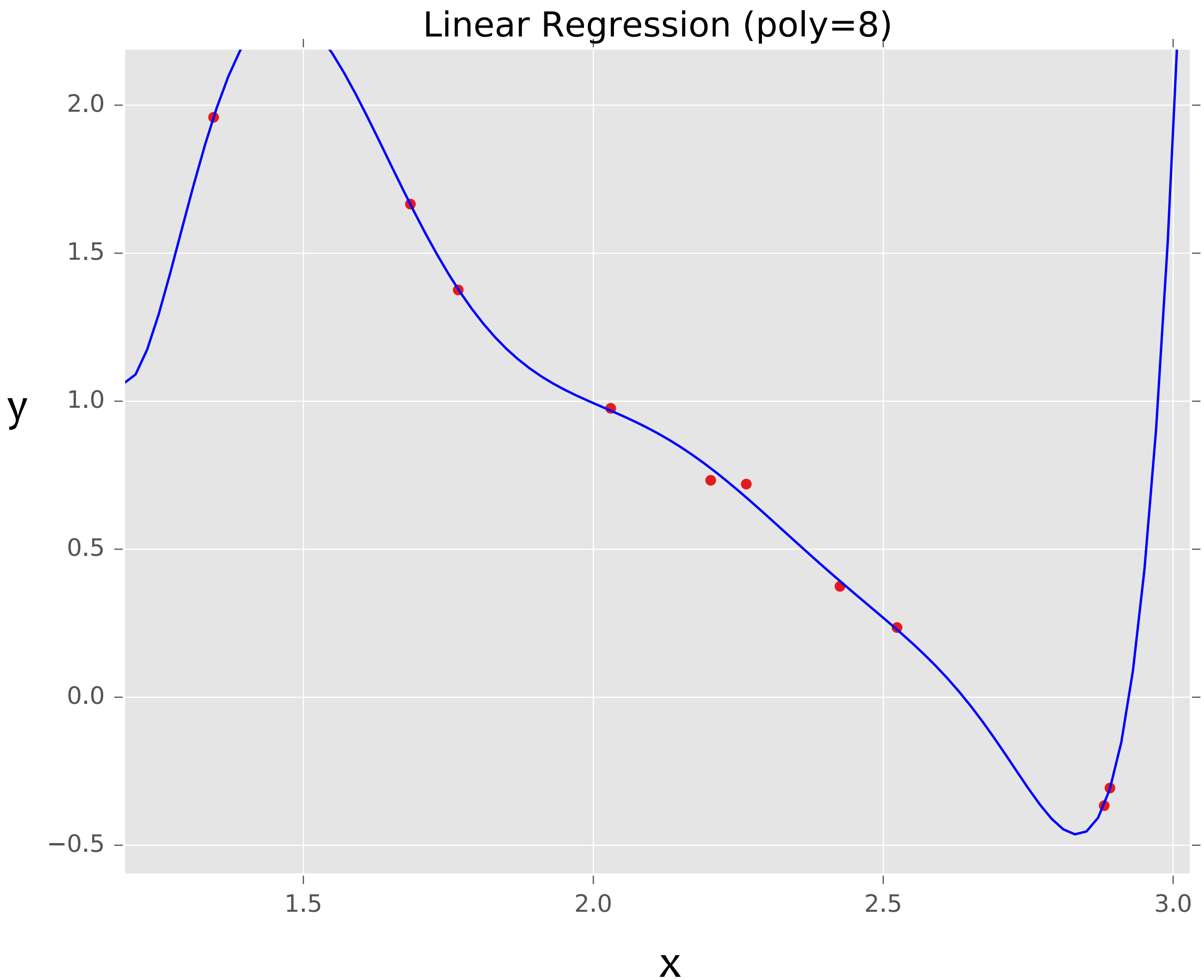


Example: Linear Regression

Goal: Learn y = $\mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function

i	y	x	…	x^8
1	2.0	1.2	…	$(1.2)^8$
2	1.3	1.7	…	$(1.7)^8$
…	…	…	…	…
10	1.1	1.9	…	$(1.9)^8$

true “unknown” target function is linear with negative slope and gaussian noise

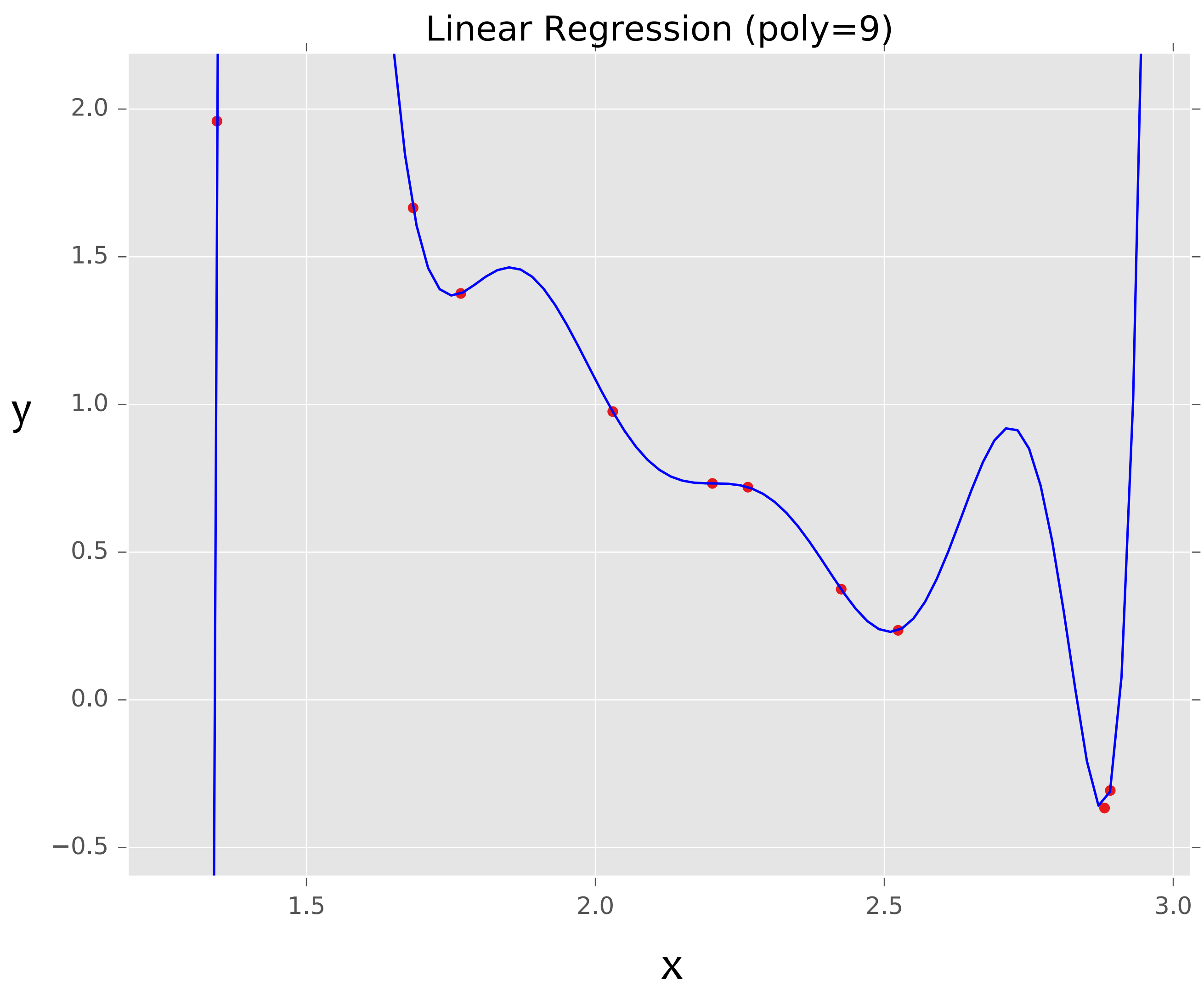


Example: Linear Regression

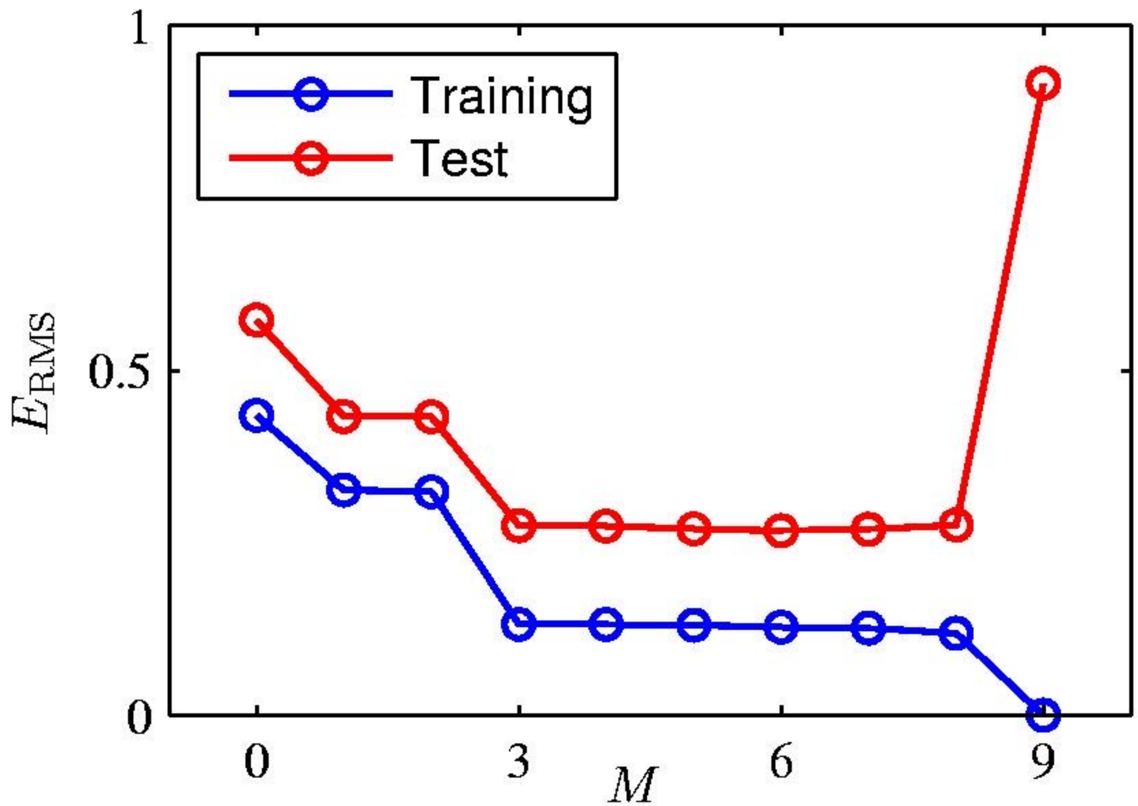
Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$
where f(.) is a polynomial
basis function

i	y	x	…	x^9
1	2.0	1.2	…	$(1.2)^9$
2	1.3	1.7	…	$(1.7)^9$
…	…	…	…	…
10	1.1	1.9	…	$(1.9)^9$

true "unknown" target function is linear with negative slope and gaussian noise



Over-fitting



Root-Mean-Square (RMS) Error: $E_{\mathrm{RMS}} = \sqrt{2E(\mathbf{w}^\star)/N}$

Slide courtesy of William Cohen

Polynomial Coefficients

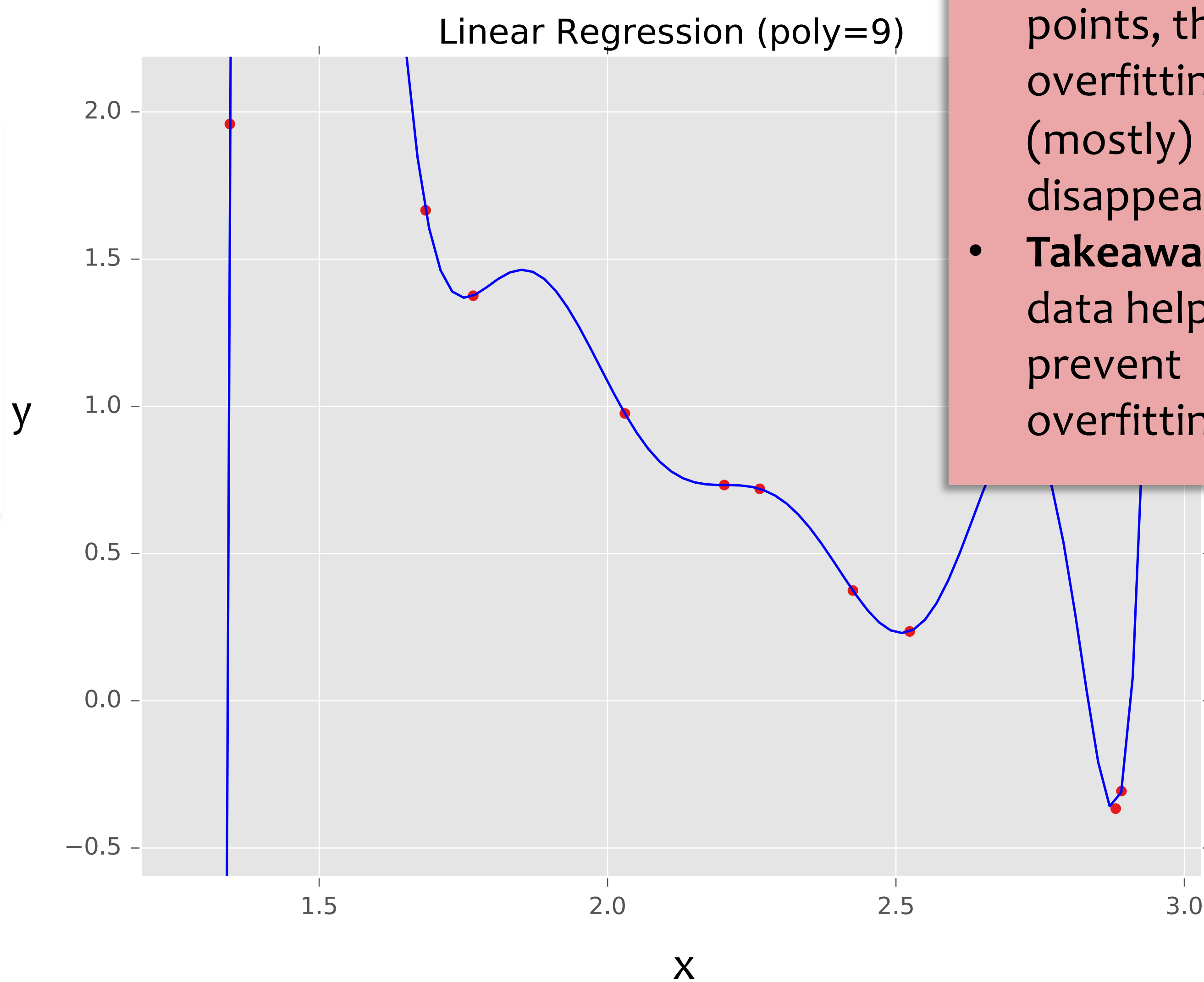
	$M=0$	$M=1$	$M=3$	$M=9$
θ_0	0.19	0.82	0.31	0.35
θ_1		-1.27	7.99	232.37
θ_2			-25.43	-5321.83
θ_3			17.37	48568.31
θ_4				-231639.30
θ_5				640042.26
θ_6				-1061800.52
θ_7				1042400.18
θ_8				-557682.99
θ_9				125201.43

Slide courtesy of William Cohen

Example: Linear Regression

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function

i	y	x	…	x^9
1	2.0	1.2	…	$(1.2)^9$
2	1.3	1.7	…	$(1.7)^9$
…	…	…	…	…
10	1.1	1.9	…	$(1.9)^9$



- With just N = 10 points we overfit!
- But with N = 100 points, the overfitting (mostly) disappears
- **Takeaway:** more data helps prevent overfitting

Example: Linear Regression

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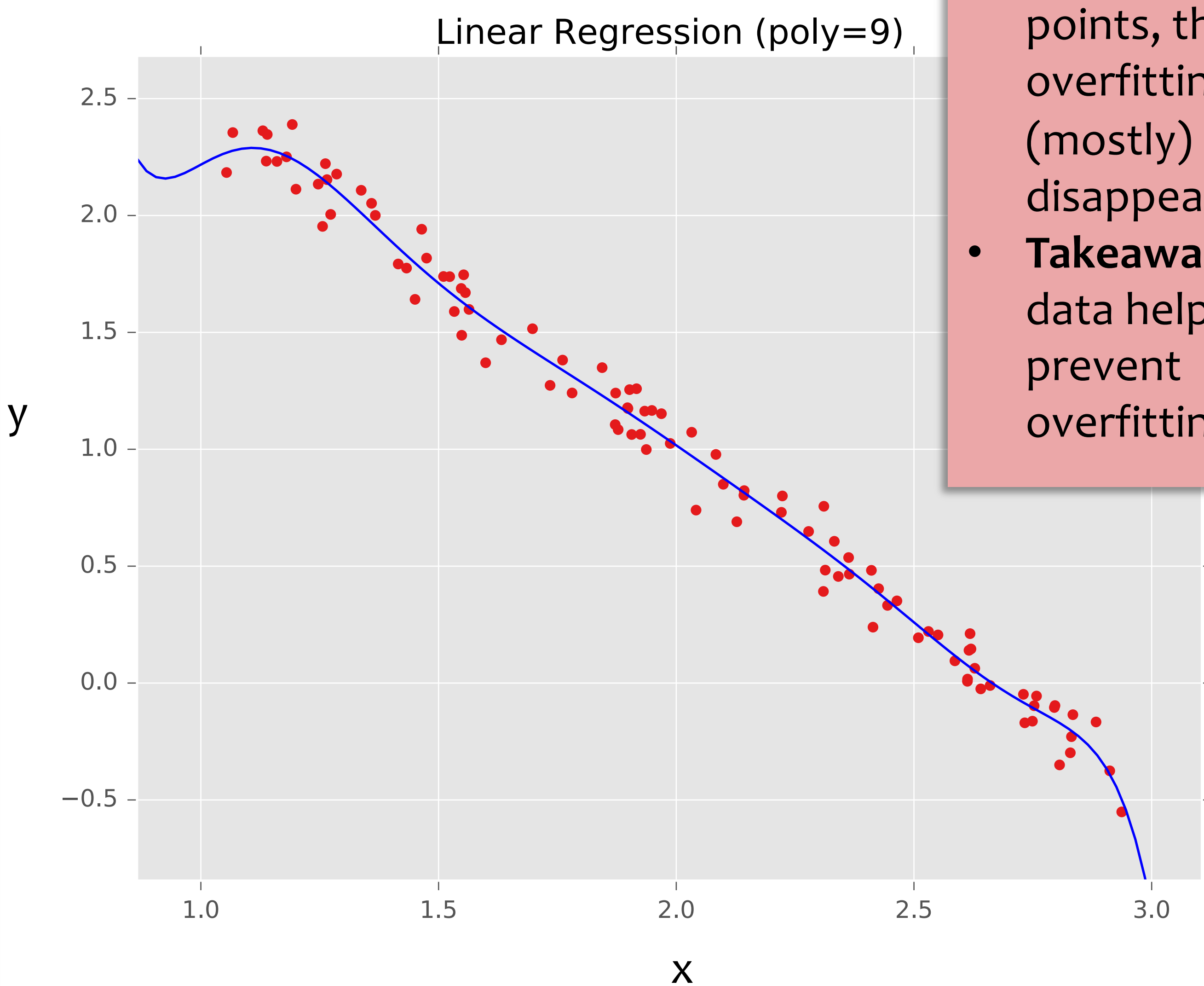
i	y	x	…	x^9
1	2.0	1.2	…	$(1.2)^9$
2	1.3	1.7	…	$(1.7)^9$
3	0.1	2.7	…	$(2.7)^9$
4	1.1	1.9	…	$(1.9)^9$
…	…	…	…	…
…	…	…	…	…
…	…	…	…	…
98	…	…	…	…
99	…	…	…	…
100	0.9	1.5	…	$(1.5)^9$



- With just N = 10 points we overfit!
- But with N = 100 points, the overfitting (mostly) disappears
- **Takeaway:** more data helps prevent overfitting

REGULARIZATION

Overfitting

Definition: The problem of **overfitting** is when the model captures the noise in the training data instead of the underlying structure

Overfitting can occur in all the models we've seen so far:

- Decision Trees (e.g. when tree is too deep)
- KNN (e.g. when k is small)
- Perceptron (e.g. when sample isn't representative)
- Linear Regression (e.g. with nonlinear features)
- Logistic Regression (e.g. with many rare features)

Motivation: Regularization

- **Occam's Razor:** prefer the simplest hypothesis

- What does it mean for a hypothesis (or model) to be **simple**?
  1. small number of features (**model selection**)
  2. small number of "important" features (**shrinkage**)

Regularization

- **Given** objective function: J(θ)
- **Goal** is to find: $\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\text{argmin}}\, J(\boldsymbol{\theta}) + \lambda r(\boldsymbol{\theta})$
- **Key idea:** Define regularizer r(θ) s.t. we tradeoff between fitting the data and keeping the model simple
- **Choose form of r(θ):**
  - Example: q-norm (usually p-norm): $\|\boldsymbol{\theta}\|_q = \left(\sum_{m=1}^{M} |\theta_m|^q\right)^{\frac{1}{q}}$

q	$r(\boldsymbol{\theta})$	**yields parameters that are...**	**name**	**optimization notes**
0	$\|\|\boldsymbol{\theta}\|\|_0 = \sum \mathbb{1}(\theta_m \neq 0)$	zero values	L0 reg.	no good computational solutions
1	$\|\|\boldsymbol{\theta}\|\|_1 = \sum \|\theta_m\|$	zero values	L1 reg.	subdifferentiable
2	$(\|\|\boldsymbol{\theta}\|\|_2)^2 = \sum \theta_m^2$	small values	L2 reg.	differentiable

Regularization Examples

Add an **L2 regularizer** to Linear Regression (aka. Ridge Regression)

$$J_{\text{RR}}(\boldsymbol{\theta}) = J(\boldsymbol{\theta}) + \boxed{\lambda ||\boldsymbol{\theta}||_2^2}$$

$$= \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} (\boldsymbol{\theta}^T \mathbf{x}^{(i)} - y^{(i)})^2 + \boxed{\lambda \sum_{m=1}^{M} \theta_m^2}$$

Add an **L1 regularizer** to Linear Regression (aka. LASSO)

$$J_{\text{LASSO}}(\boldsymbol{\theta}) = J(\boldsymbol{\theta}) + \boxed{\lambda ||\boldsymbol{\theta}||_1}$$

$$= \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2} (\boldsymbol{\theta}^T \mathbf{x}^{(i)} - y^{(i)})^2 + \boxed{\lambda \sum_{m=1}^{M} |\theta_m|}$$

Regularization Examples

Add an **L2 regularizer** to Logistic Regression

$$J'(\boldsymbol{\theta}) = J(\boldsymbol{\theta}) + \boxed{\lambda ||\boldsymbol{\theta}||_2^2}$$
$$= \frac{1}{N} \sum_{i=1}^{N} - \log p(y^{(i)} \mid \mathbf{x}^{(i)}, \boldsymbol{\theta}) + \boxed{\lambda \sum_{m=1}^{M} \theta_m^2}$$

Add an **L1 regularizer** to Logistic Regression

$$J'(\boldsymbol{\theta}) = J(\boldsymbol{\theta}) + \boxed{\lambda ||\boldsymbol{\theta}||_1}$$
$$= \frac{1}{N} \sum_{i=1}^{N} - \log p(y^{(i)} \mid \mathbf{x}^{(i)}, \boldsymbol{\theta}) + \boxed{\lambda \sum_{m=1}^{M} |\theta_m|}$$

Regularization

Question:

Suppose we are minimizing J'(θ) where

$$J'(\boldsymbol{\theta}) = J(\boldsymbol{\theta}) + \lambda r(\boldsymbol{\theta})$$

As λ increases, the minimum of J'(θ) will…

A. … move towards the midpoint between J(θ) and r(θ)

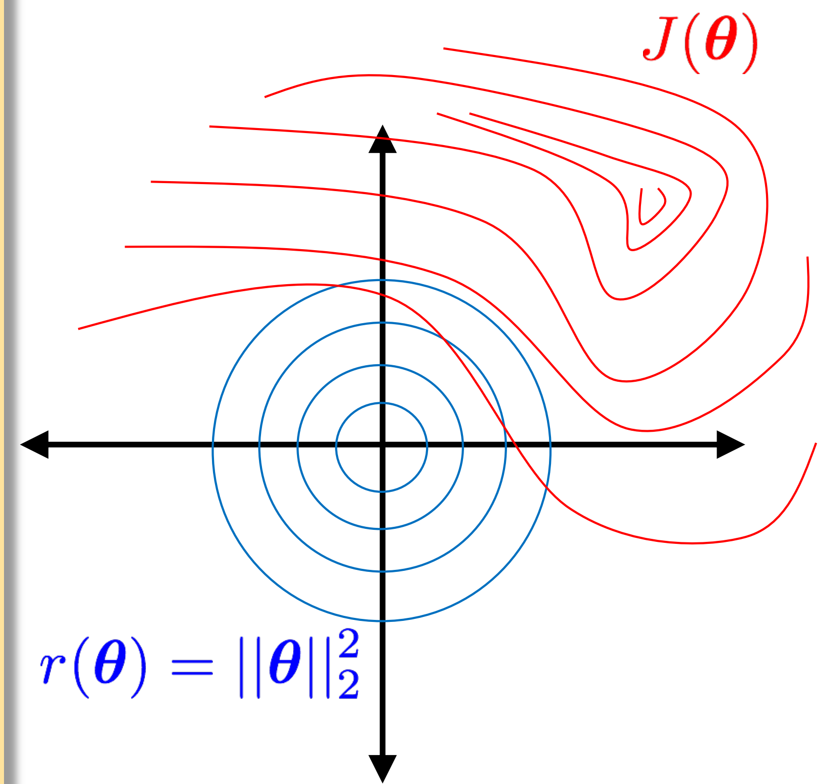
B. … move towards the minimum of J(θ)

C. … move towards the minimum of r(θ)

D. … move towards a theta vector of positive infinities

E. … move towards a theta vector of negative infinities

F. … stay the same



Regularization

Whiteboard

- Why does L2 regularization lead to small numbers and L1 regularization lead to zeros?

Regularization

Don't Regularize the Bias (Intercept) Parameter!

- In our models so far, the bias / intercept parameter is usually denoted by θ_0 -- that is, the parameter for which we fixed $x_0 = 1$
- Regularizers always avoid penalizing this bias / intercept parameter
- Why? Because otherwise the learning algorithms wouldn't be invariant to a shift in the y-values

Whitening Data

- It's common to *whiten* each feature by subtracting its mean and dividing by its variance
- For regularization, this helps all the features be penalized in the same units
(e.g. convert both centimeters and kilometers to z-scores)

Regularization Exercise

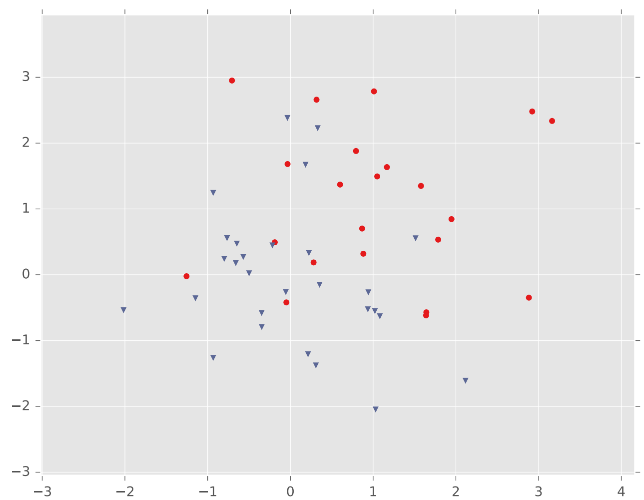
In-class Exercise

1. Plot train error vs. regularization weight (cartoon)
2. Plot validation error vs . regularization weight (cartoon)

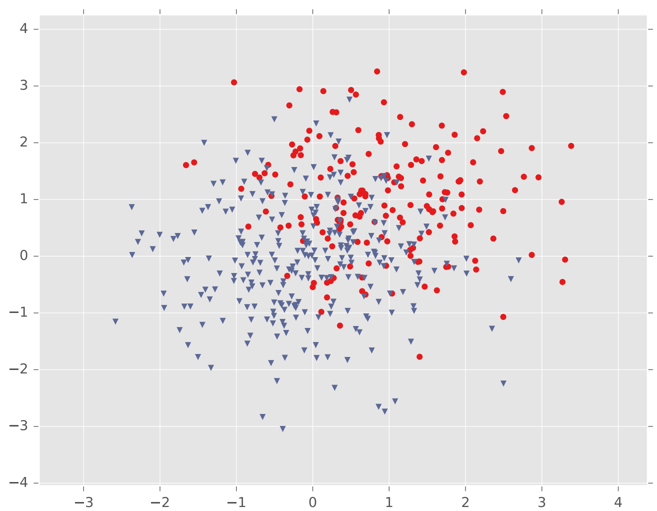
REGULARIZATION EXAMPLE: LOGISTIC REGRESSION

Example: Logistic Regression

Training Data

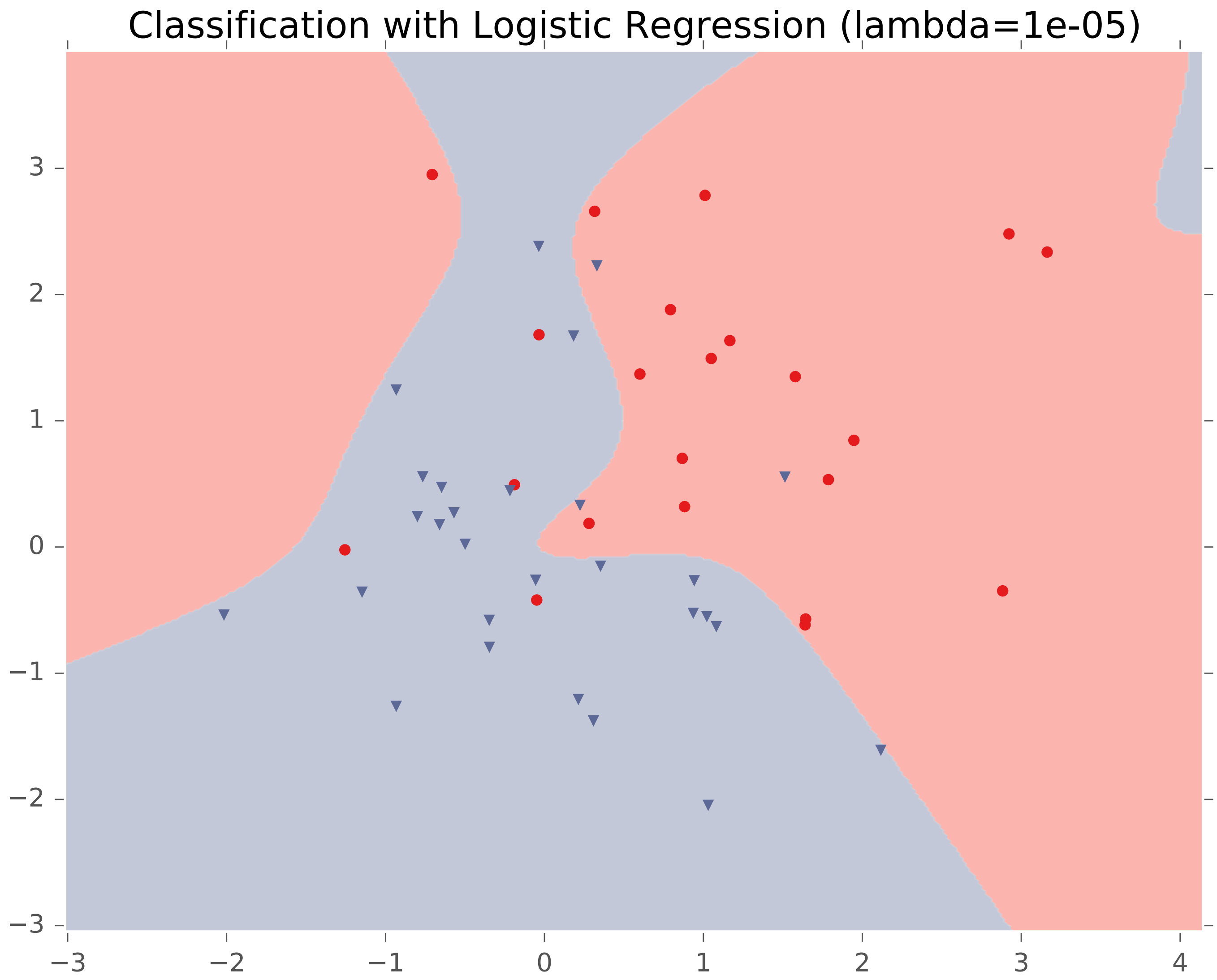


Test Data

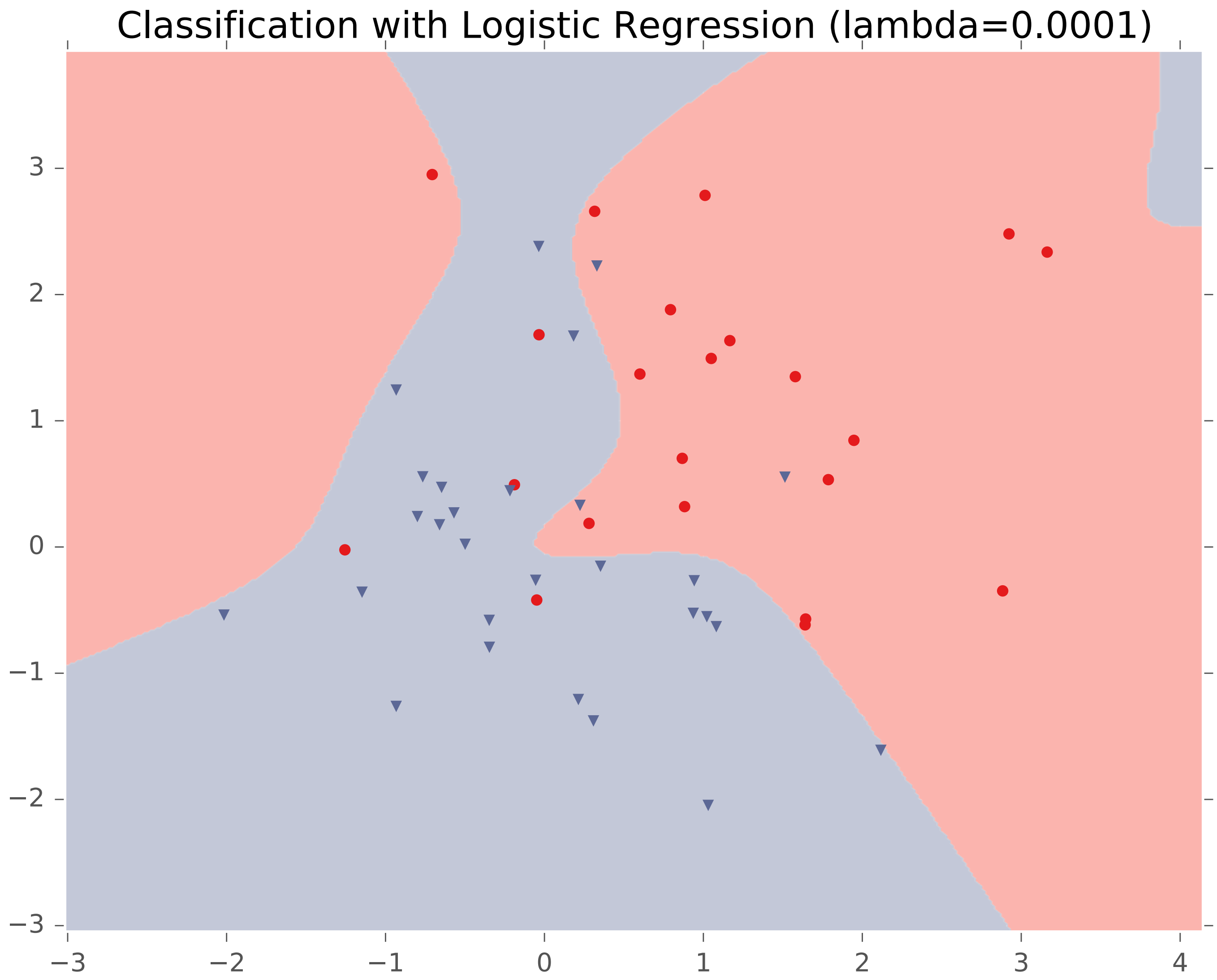


- For this example, we construct **nonlinear features** (i.e. feature engineering)
- Specifically, we add **polynomials up to order 9** of the two original features x_1 and x_2
- Thus our classifier is **linear** in the **high-dimensional feature space**, but the decision boundary is **nonlinear** when visualized in **low-dimensions** (i.e. the original two dimensions)

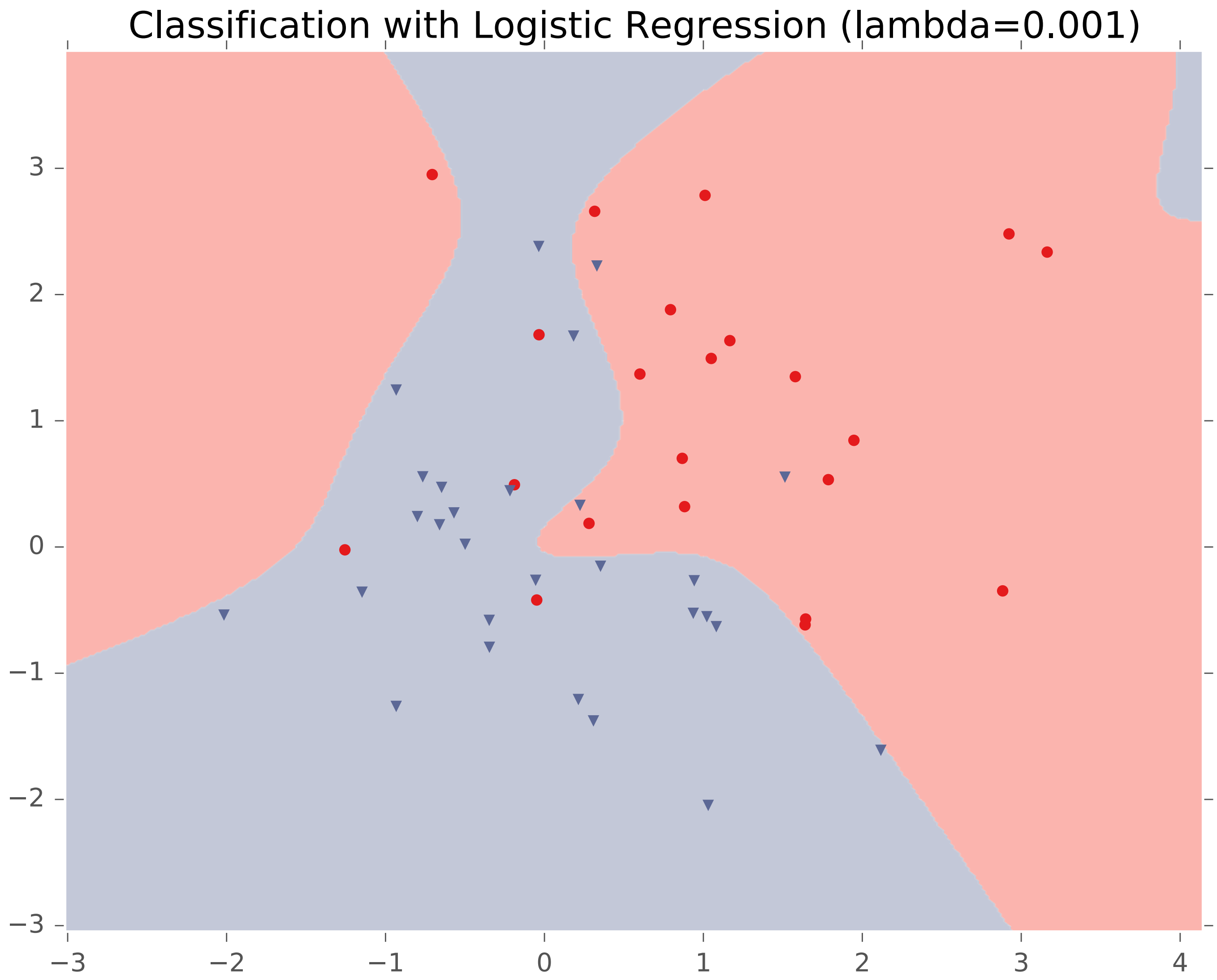
Example: Logistic Regression



Example: Logistic Regression



Example: Logistic Regression



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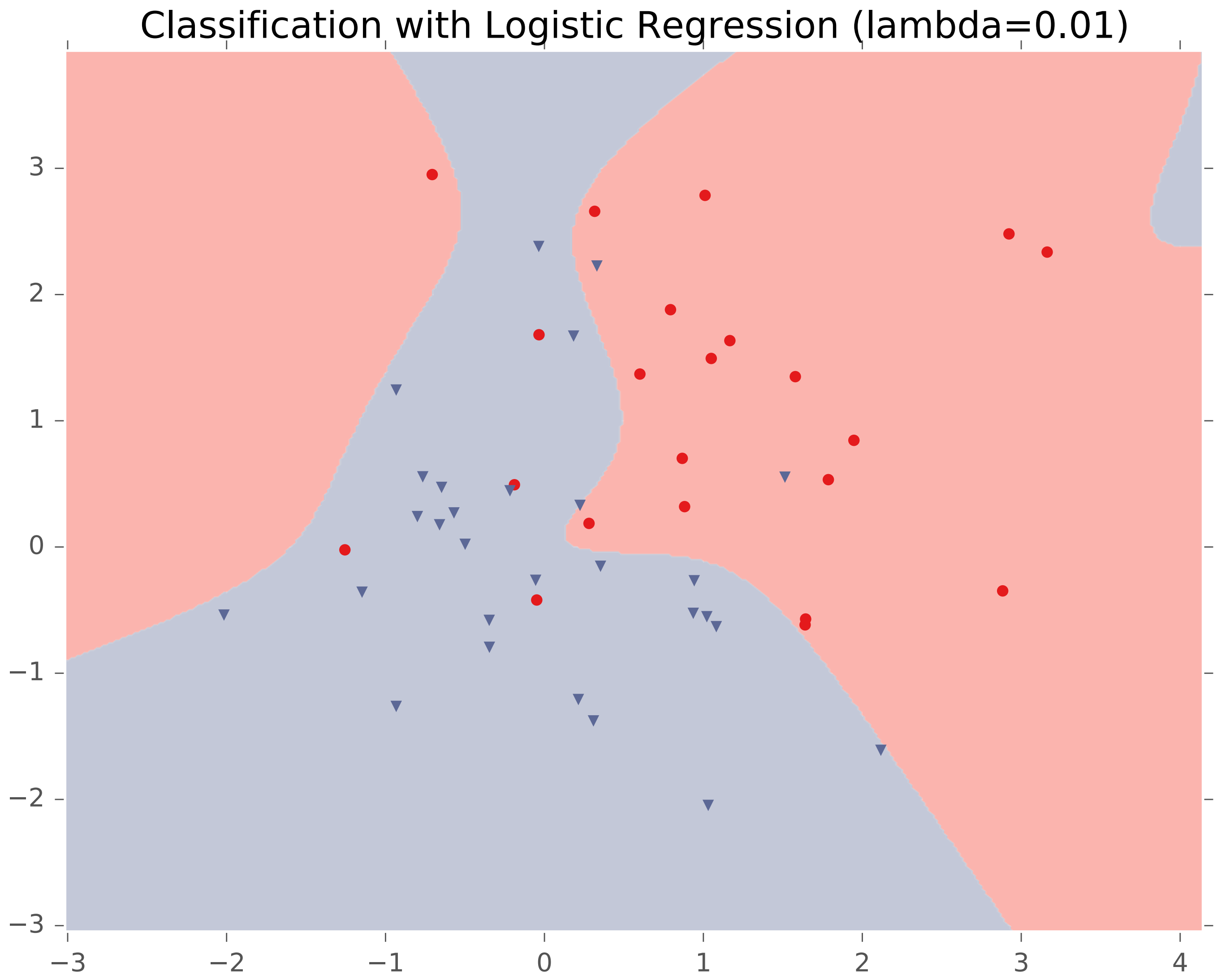

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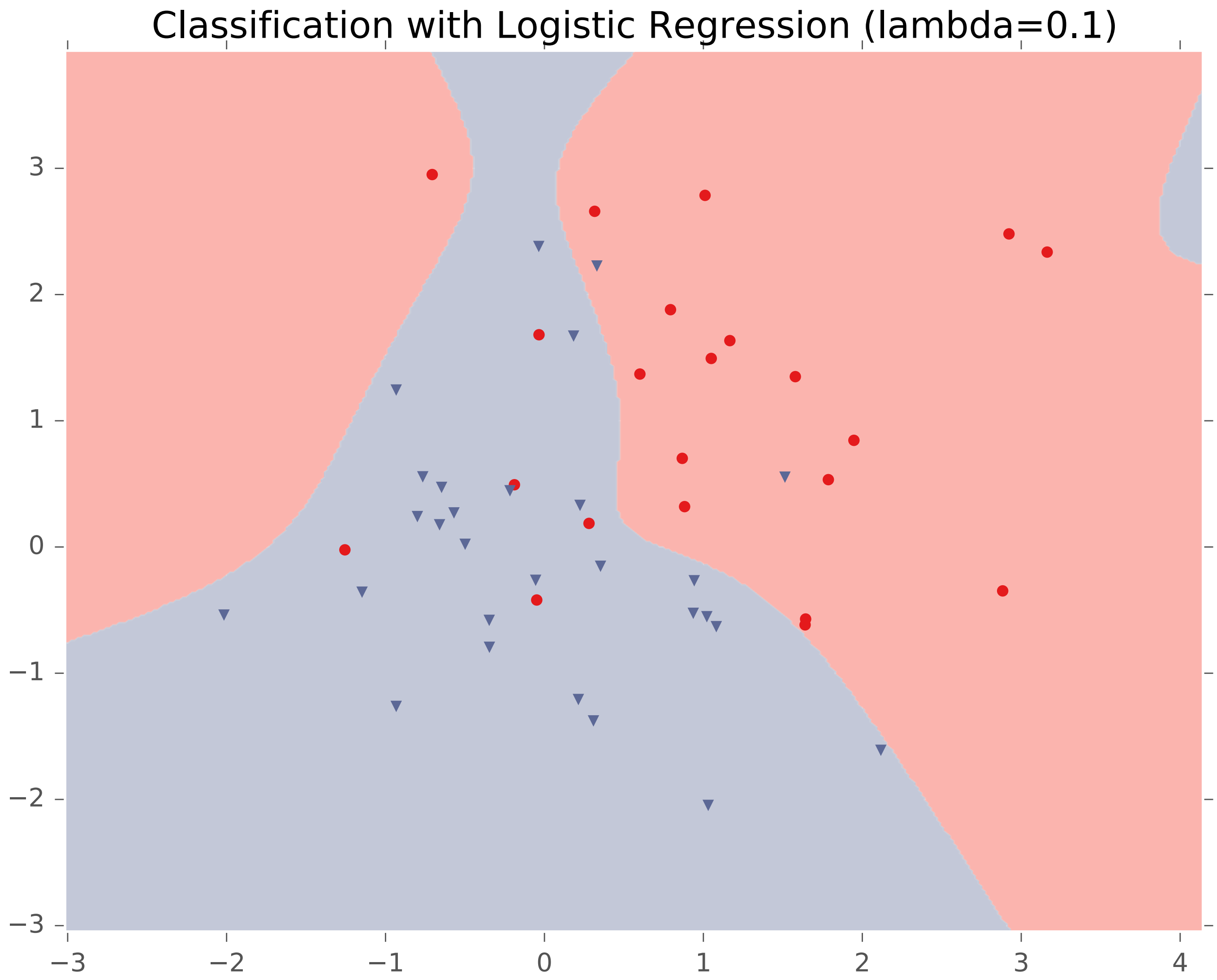

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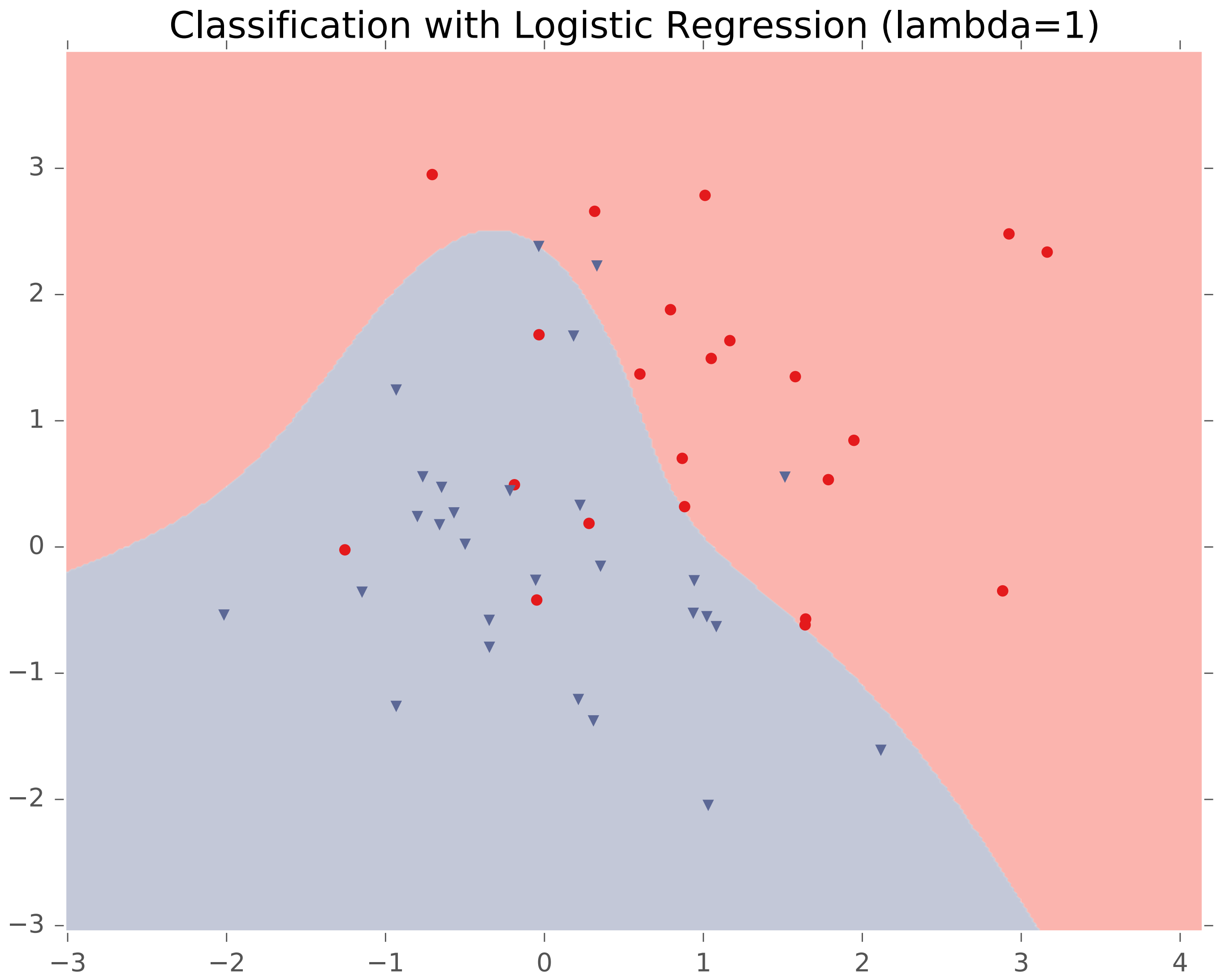

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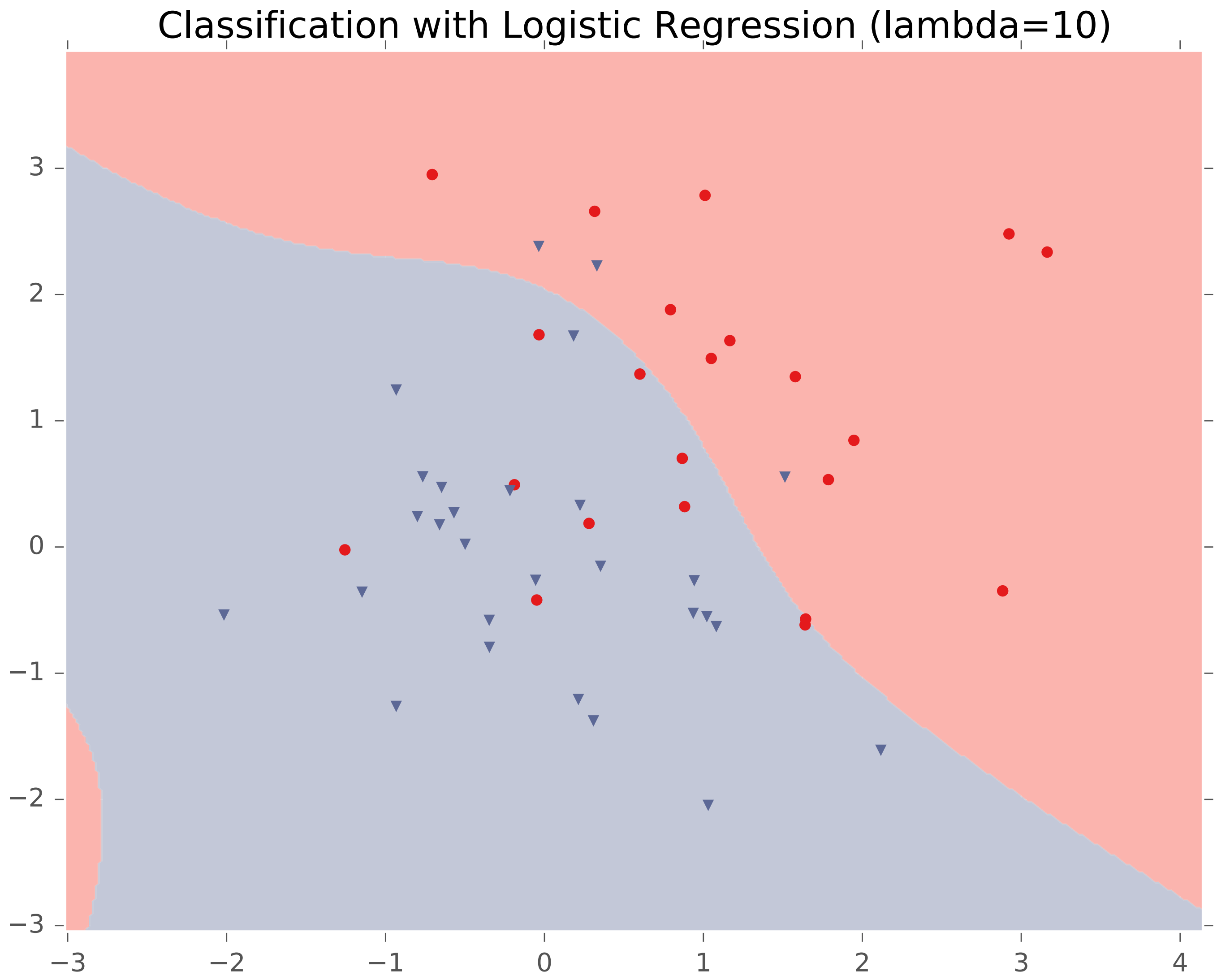

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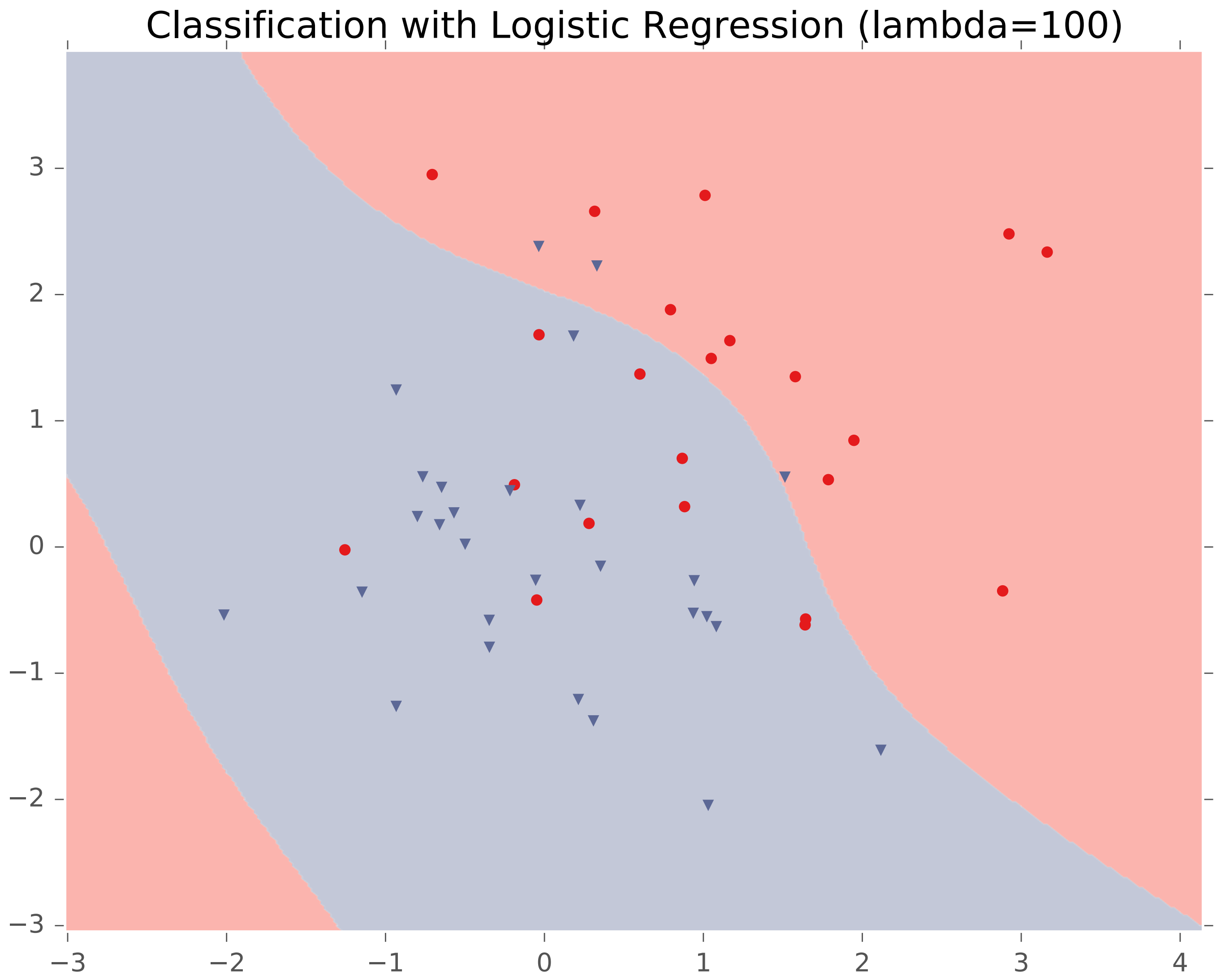
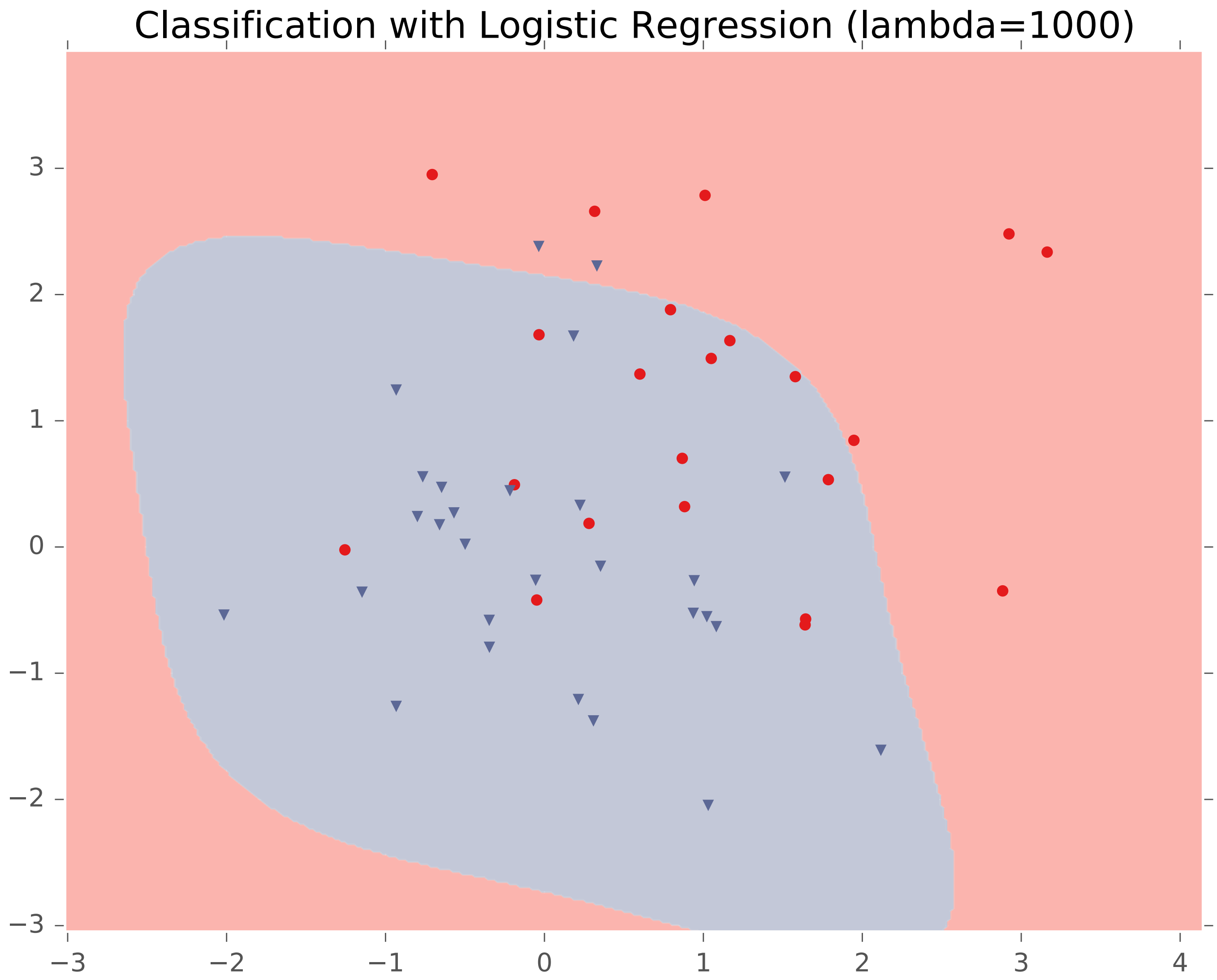


Example: Logistic Regression



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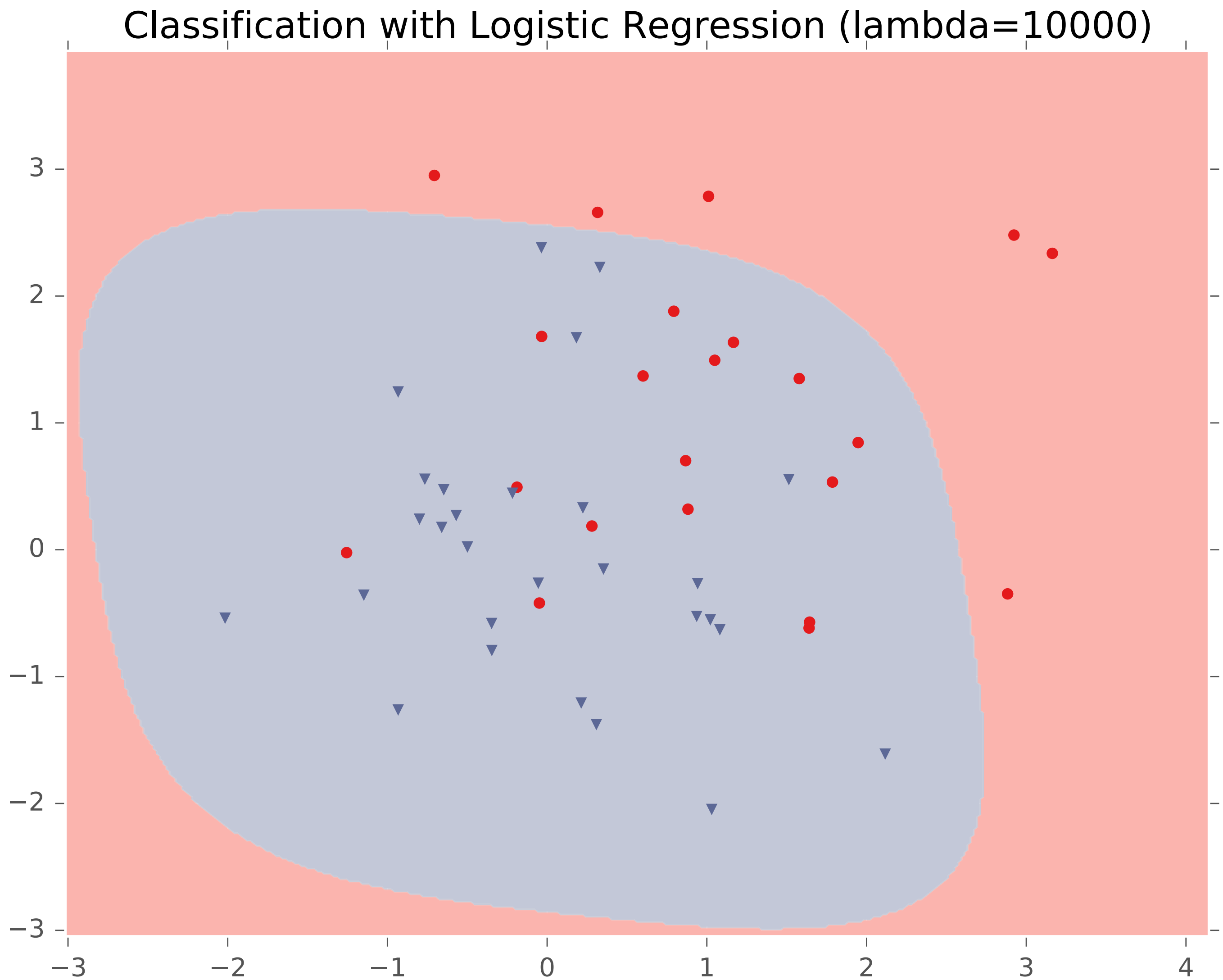

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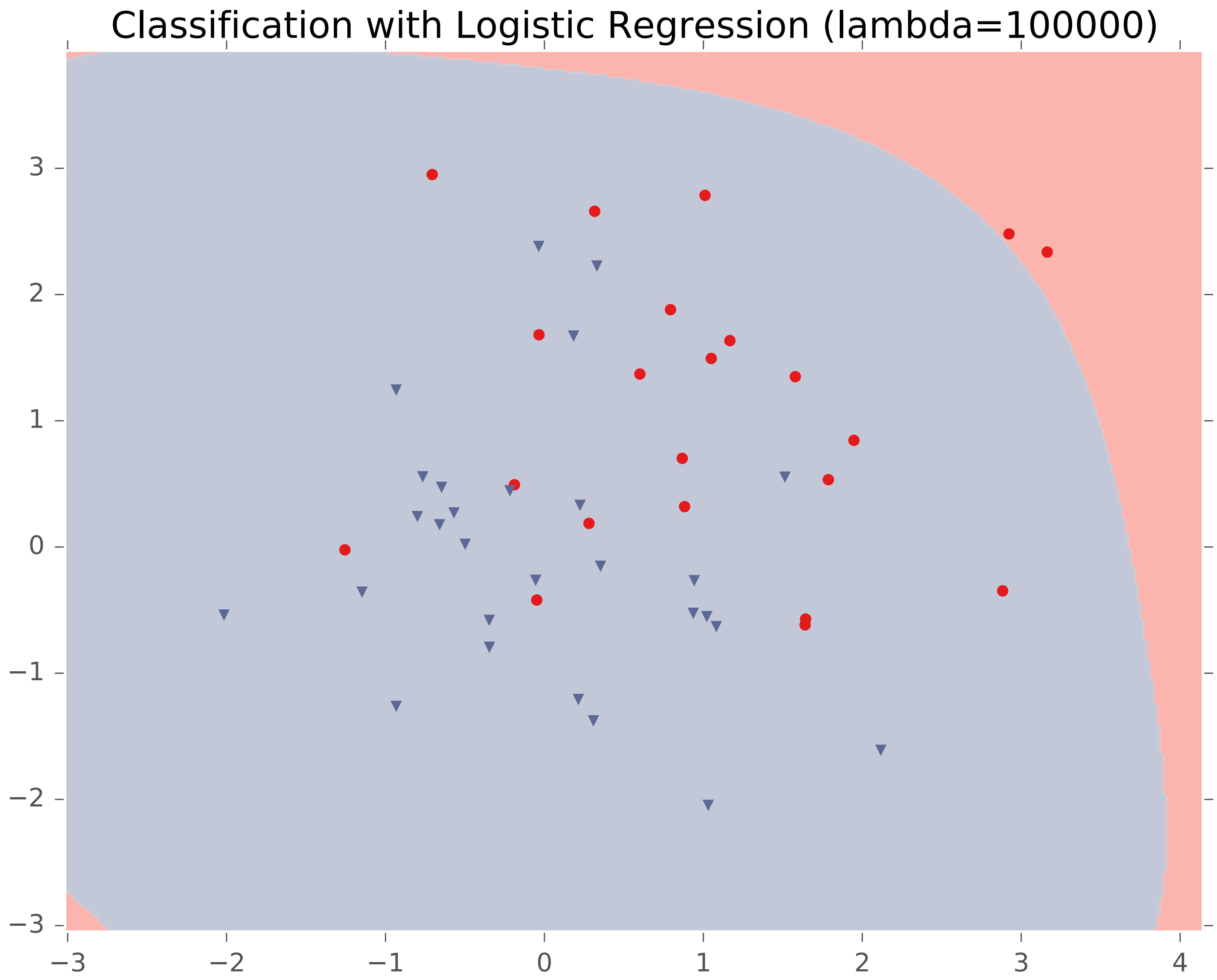
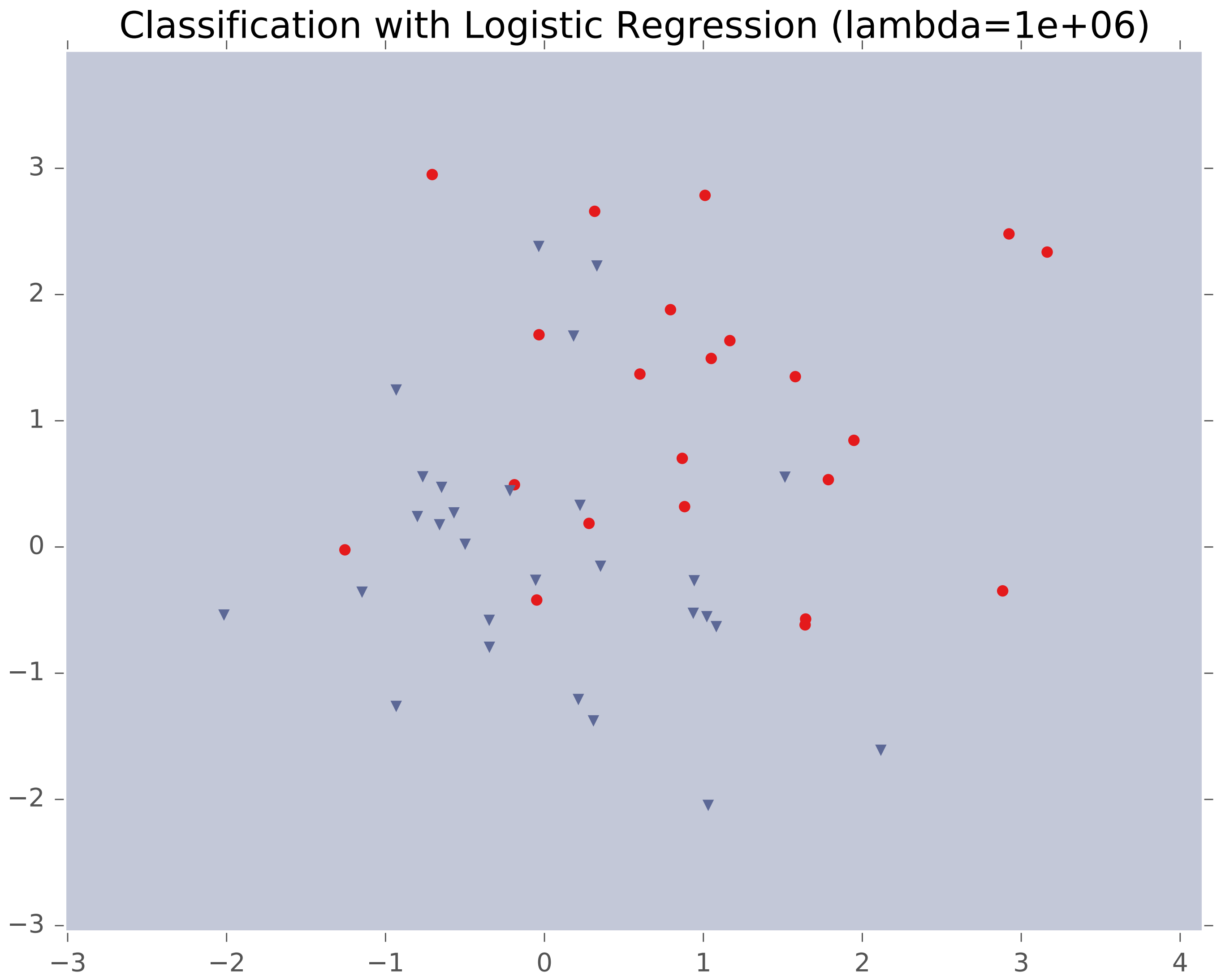


Example: Logistic Regression



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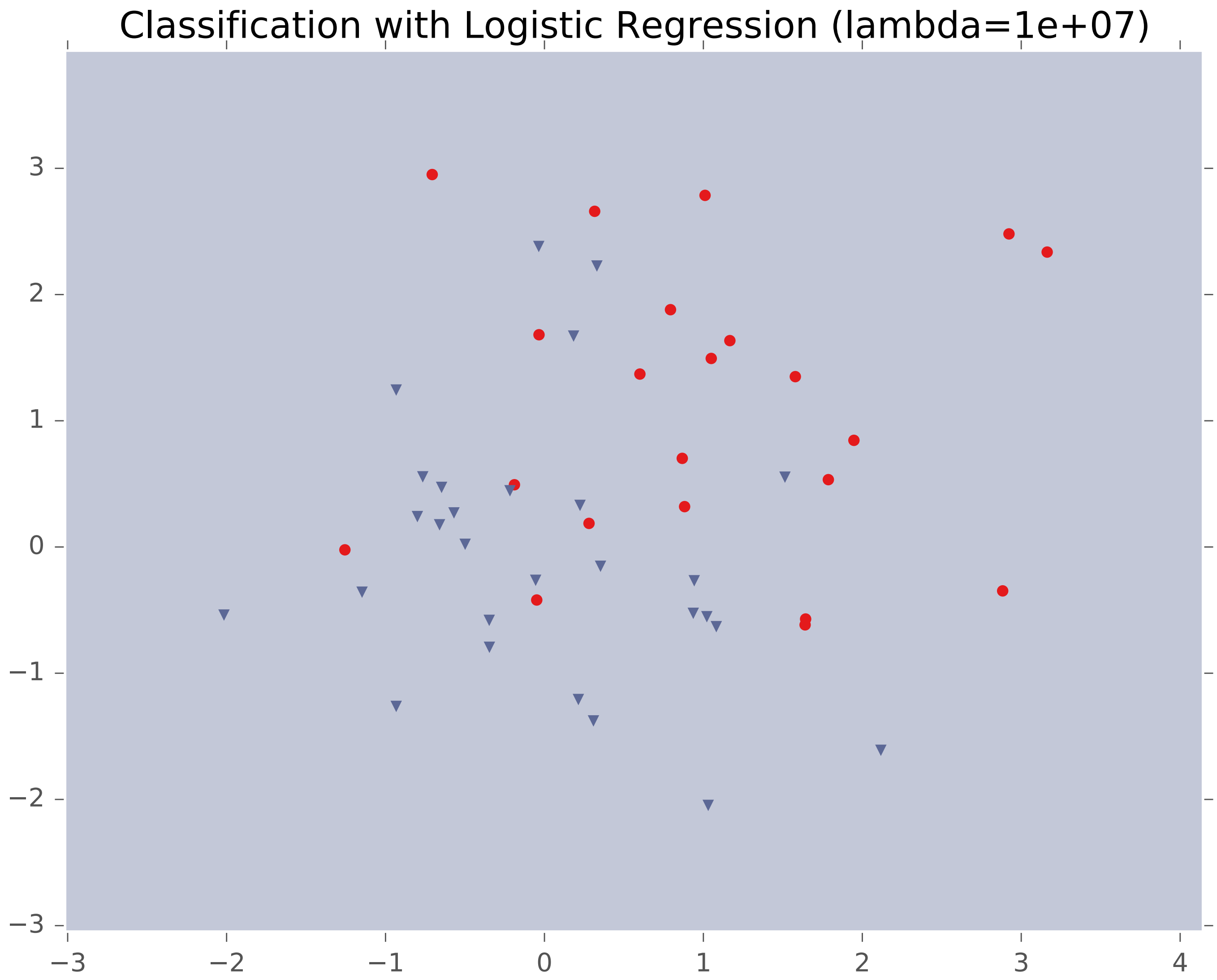

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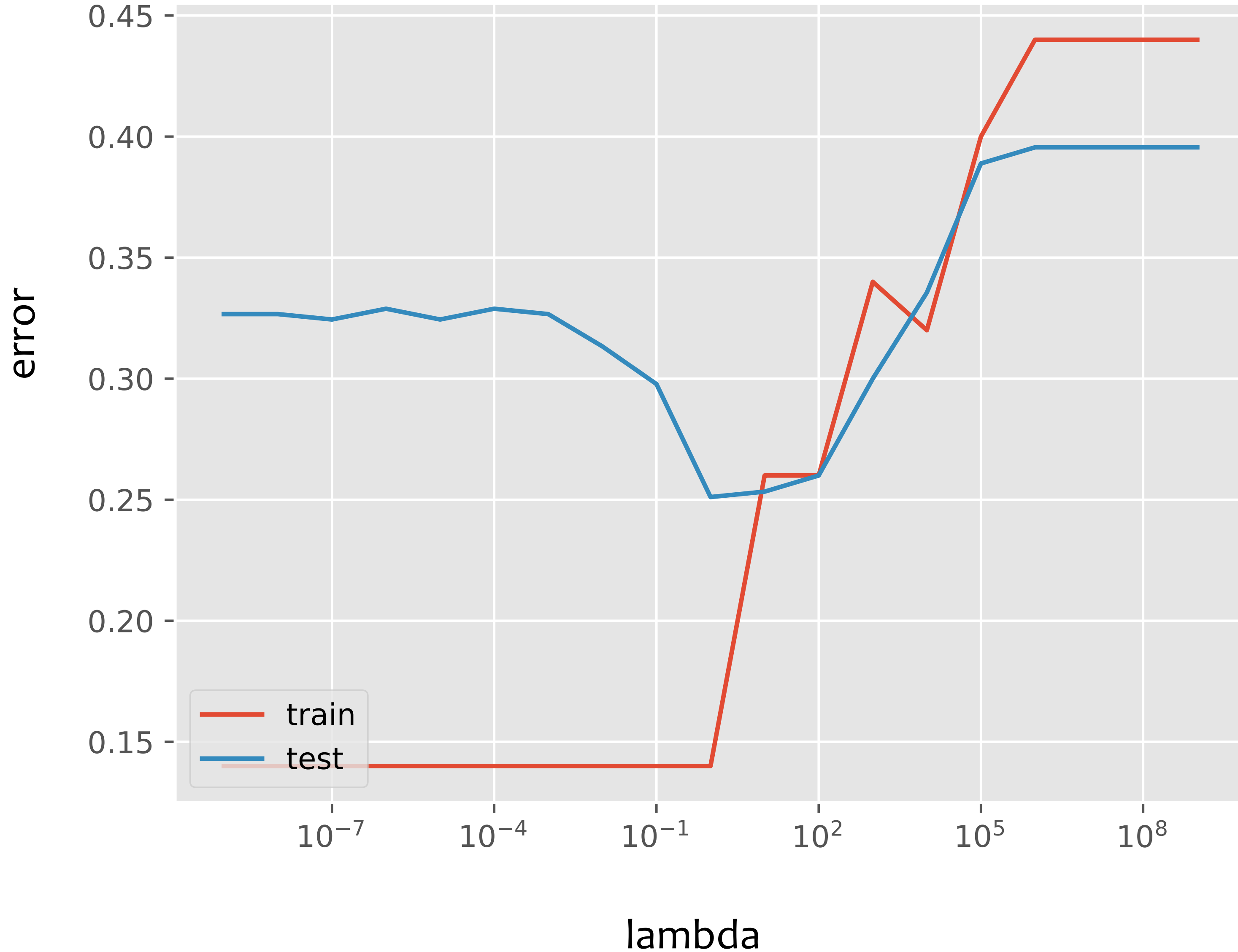

Regularization as MAP

- L1 and L2 regularization can be interpreted as **maximum a-posteriori (MAP) estimation** of the parameters
- To be discussed later in the course…

Takeaways

1. **Nonlinear basis functions** allow **linear models** (e.g. Linear Regression, Logistic Regression) to capture **nonlinear** aspects of the original input
2. Nonlinear features are **require no changes to the model** (i.e. just preprocessing)
3. **Regularization** helps to avoid **overfitting**
4. **Regularization** and **MAP estimation** are equivalent for appropriately chosen priors

Feature Engineering / Regularization Objectives

You should be able to…

- Engineer appropriate features for a new task
- Use feature selection techniques to identify and remove irrelevant features
- Identify when a model is overfitting
- Add a regularizer to an existing objective in order to combat overfitting
- Explain why we should **not** regularize the bias term
- Convert linearly inseparable dataset to a linearly separable dataset in higher dimensions
- Describe feature engineering in common application areas