10-301/601 Introduction to Machine Learning

Machine Learning Department
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Decision Trees

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Lecture 3
Jan. 26, 2022

Q&A

Q: In our medical diagnosis example, suppose two of our doctors (i.e. experts) disagree about whether (+) or not (-) the patient is sick. How would the decision tree represent this situation?

A: Today we will define decision trees that predict a single class by a majority vote at the leaf. More generally, the leaf could provide a probability distribution over output classes $p(y|\mathbf{x})$

Q&A

Q: How do these In-Class Polls work?

A:
- Sign into **Google Form (c**lick [Poll] link on Schedule page http://mlcourse.org/schedule.html) using **Andrew Email**
- Answer **during lecture for full credit,** or within 24 hours for half credit
- Avoid the **toxic option** which gives negative points!
- 8 "free poll points" but can't use more than 3 free polls consecutively. All the questions for one lecture are worth 1 point total.

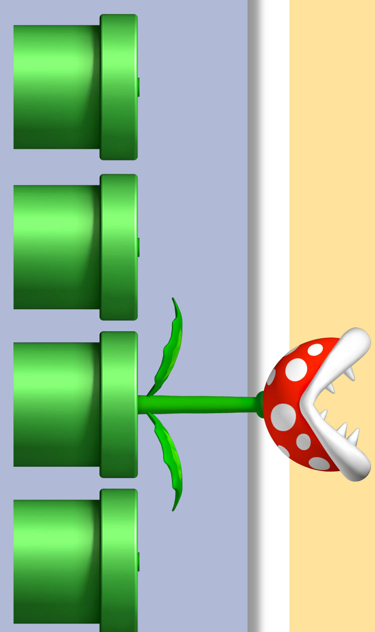
Latest Poll link: http://poll.mlcourse.org

First In-Class Poll

Question:

Which of the following did you bring to class today?

A. Smartphone
B. Flip phone
C. Pay phone
D. No phone



Answer:

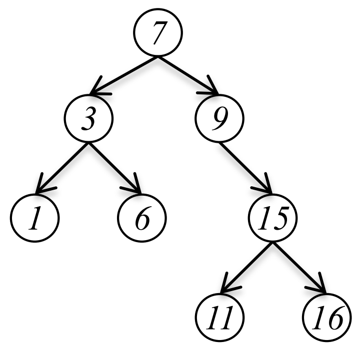
Reminders

- **Homework 1: Background**
  - **Out: Wed, Jan 19 (1st lecture)**
  - **Due: Wed, Jan 26 at 11:59pm**
  - unique policy for this assignment: we will grant (essentially) any and all extension requests
- **Homework 2: Decision Trees**
  - **Out: Wed, Jan. 26**
  - **Due: Fri, Feb. 4 at 11:59pm**

MAKING PREDICTIONS WITH A DECISION TREES

Background: Recursion

- *Def*: a **binary search tree** (BST) consists of nodes, where each node:
  - has a value, v
  - up to 2 children
  - all its left descendants have values less than v, and its right descendants have values greater than v
- We like BSTs because they permit search in O(log(n)) time, assuming n nodes in the tree



Recursive Search

```
def contains(node, key):
        if key < node.value & node.left != null:
                return contains(node.left, key)
        else if node.value < key & node.right != null:
                return contains(node.right, key)
        else:
                return key == node.value
```

Node Data Structure

```
class Node:
        int value
        Node left
        Node right
```

Iterative Search

```
def contains(node, key):
        cur = node
        while true:
                if key < cur.value & cur.left != null:
                        cur = cur.left
                else if cur.value < key & cur.right != null:
                        cur = cur.right
                else:
                        break
        return key == cur.value
```

Decision Trees

Whiteboard

- Example Decision Tree as a hypothesis
- Defining h(x) for a decision tree

Tree to Predict C-Section Risk

Learned from medical records of 1000 women (Sims et al., 2000)

Negative examples are C-sections

```
[833+,167-] .83+ .17-
Fetal_Presentation = 1: [822+,116-] .88+ .12-
| Previous_Csection = 0: [767+,81-] .90+ .10-
| | Primiparous = 0: [399+,13-] .97+ .03-
| | Primiparous = 1: [368+,68-] .84+ .16-
| | | Fetal_Distress = 0: [334+,47-] .88+ .12-
| | | | Birth_Weight < 3349: [201+,10.6-] .95+ .
| | | | Birth_Weight >= 3349: [133+,36.4-] .78+
| | | Fetal_Distress = 1: [34+,21-] .62+ .38-
| Previous_Csection = 1: [55+,35-] .61+ .39-
Fetal_Presentation = 2: [3+,29-] .11+ .89-
Fetal_Presentation = 3: [8+,22-] .27+ .73-
```


Figure from Tom Mitchell

LEARNING A DECISION TREE

Decision Trees

Whiteboard

– Decision Tree Learning

Recursive Training for Decision Trees

def train(dataset D'):

- Let p = new Node()
- ***Base Case:*** If (1) all labels $y^{(i)}$ in D' are identical (2) D' is empty (3) for each attribute, all values are identical
  - p.type = Leaf // The node p is a leaf node
  - p.label = majority_vote(D') // Store the label
  - return p
- ***Recursive Step:*** Otherwise
  - Make an internal node
    - p.type = Internal // The node p is an internal node
  - Pick the *best* attribute X_m according to splitting criterion
    - p.attr = argmax_m splitting_criterion(D', X_m) // Store the attribute on which to split
  - For each value v of attribute X_m:
    - $D_{X_m = v}$ = {($\mathbf{x}$,y) in D' : x_m = v} // Select a partition of the data
    - child_v = train($D_{X_m = v}$) // Recursively build the child
    - p.branches[v] = child_v // Create a branch with label v
    - return p

Decision Tree Learning Example

Dataset:

Output Y, Attributes A, B, C

Y	A	B	C
-	1	0	0
-	1	0	1
-	1	0	0
+	0	0	1
+	1	1	0
+	1	1	1
+	1	1	0
+	1	1	1

In-Class Exercise

Using **error rate** as the splitting criterion, what decision tree would be learned?

Decision Trees

Whiteboard

- Example of Decision Tree Learning with Error Rate as splitting criterion

SPLITTING CRITERION: ERROR RATE

Decision Tree Learning

- *Definition*: a **splitting criterion** is a function that measures the effectiveness of splitting on a particular attribute
- Our decision tree learner **selects the "best" attribute** as the one that maximizes the splitting criterion
- Lots of options for a splitting criterion:
  - error rate (or *accuracy* if we want to pick the tree that *maximizes* the criterion)
  - Gini gain
  - Mutual information
  - random
  - …

Decision Tree Learning Example

Dataset:

Output Y, Attributes A and B

Y	A	B
-	1	0
-	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1

In-Class Exercise

Which attribute would **error rate** select for the next split?

1. A
2. B
3. A or B (tie)
4. Neither

Decision Tree Learning Example

Dataset:

Output Y, Attributes A and B

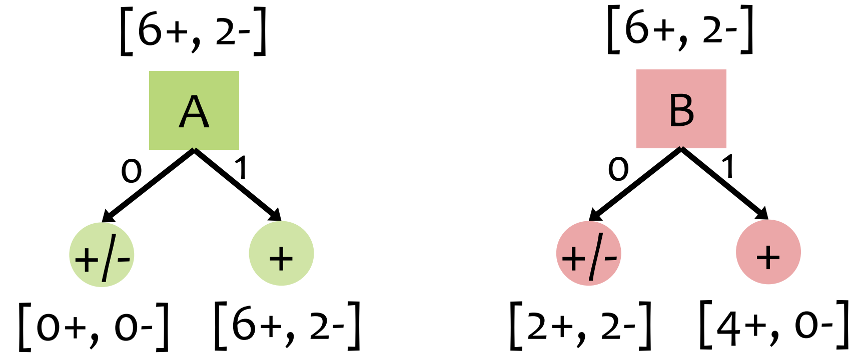
Y	A	B
-	1	0
-	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1

Decision Tree Learning Example

Dataset:

Output Y, Attributes A and B

Y	A	B
-	1	0
-	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1



Error Rate

$$\text{error}(h_A, D) = 2/8$$
$$\text{error}(h_B, D) = 2/8$$

error rate treats attributes A and B as equally good

SPLITTING CRITERION: MUTUAL INFORMATION

Information Theory & DTs

Whiteboard

- Information Theory primer
  - Entropy
  - (Specific) Conditional Entropy
  - Conditional Entropy
  - Information Gain / Mutual Information
- Information Gain as DT splitting criterion

Mutual Information

Let X be a random variable with $X \in \mathcal{X}$.
Let Y be a random variable with $Y \in \mathcal{Y}$.

Entropy: $H(Y) = -\sum_{y \in \mathcal{Y}} P(Y = y) \log_2 P(Y = y)$

Specific Conditional Entropy: $H(Y \mid X = x) = -\sum_{y \in \mathcal{Y}} P(Y = y \mid X = x) \log_2 P(Y = y \mid X = x)$

Conditional Entropy: $H(Y \mid X) = \sum_{x \in \mathcal{X}} P(X = x) H(Y \mid X = x)$

Mutual Information: $I(Y; X) = H(Y) - H(Y|X) = H(X) - H(X|Y)$

- For a decision tree, we can use **mutual information** of the output class *Y* and some attribute *X* on which to split **as a splitting criterion**
- Given a dataset *D* of training examples, we can estimate the required probabilities as…

$$P(Y = y) = N_{Y=y}/N$$
$$P(X = x) = N_{X=x}/N$$
$$P(Y = y|X = x) = N_{Y=y,X=x}/N_{X=x}$$

where $N_{Y=y}$ is the number of examples for which $Y = y$ and so on.

Mutual Information

Let X be a random variable with $X \in \mathcal{X}$.
Let Y be a random variable with $Y \in \mathcal{Y}$.

Entropy: $H(Y) = -\sum_{y \in \mathcal{Y}} P(Y=y) \log_2 P(Y=y)$

Specific Conditional Entropy: $H(Y \mid X=x) = -\sum_{y \in \mathcal{Y}} P(Y=y \mid X=x) \log_2 P(Y=y \mid X=x)$

Conditional Entropy: $H(Y \mid X) = \sum_{x \in \mathcal{X}} P(X=x) H(Y \mid X=x)$

Mutual Information: $I(Y;X) = H(Y) - H(Y|X) = H(X) - H(X|Y)$

- **Entropy** measures the **expected # of bits** to code one random draw from X.
- For a decision tree, we want to **reduce the entropy of the random variable we are trying to predict**!

Conditional entropy is the expected value of specific conditional entropy $E_{P(X=x)}[H(Y \mid X = x)]$

Informally, we say that **mutual information** is a measure of the following: *If we know X, how much does this reduce our uncertainty about Y?*

Decision Tree Learning Example

Dataset:

Output Y, Attributes A and B

Y	A	B
-	1	0
-	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1

In-Class Exercise

Which attribute would **mutual information** select for the next split?

1. A
2. B
3. A or B (tie)
4. Neither

Decision Tree Learning Example

Dataset:

Output Y, Attributes A and B

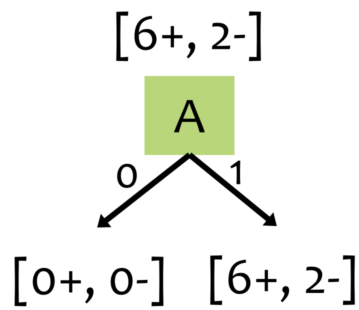
Y	A	B
-	1	0
-	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1

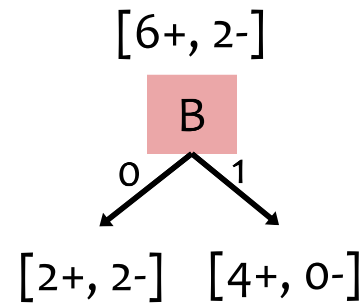
Decision Tree Learning Example

Dataset:

Output Y, Attributes A and B

Y	A	B
-	1	0
-	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1





Mutual Information

$H(Y) = -2/8 \log(2/8) - 6/8 \log(6/8)$

$H(Y|A=0) =$ "undefined"

$H(Y|A=1) = -2/8 \log(2/8) - 6/8 \log(6/8)$
$= H(Y)$

$H(Y|A) = P(A=0)H(Y|A=0) + P(A=1)H(Y|A=1)$
$= 0 + H(Y|A=1) = H(Y)$

$I(Y; A) = H(Y) - H(Y|A=1) = 0$

$H(Y|B=0) = -2/4 \log(2/4) - 2/4 \log(2/4)$

$H(Y|B=1) = -0 \log(0) - 1 \log(1) = 0$

$H(Y|B) = 4/8(0) + 4/8(H(Y|B=0))$

$I(Y; B) = H(Y) - 4/8\, H(Y|B=0) > 0$

Tennis Example

Test your understanding

Dataset:

Day	Outlook	Temperature	Humidity	Wind	PlayTennis?
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Figure from Tom Mitchell

Tennis Example

Test your understanding

Which attribute yields the best classifier?

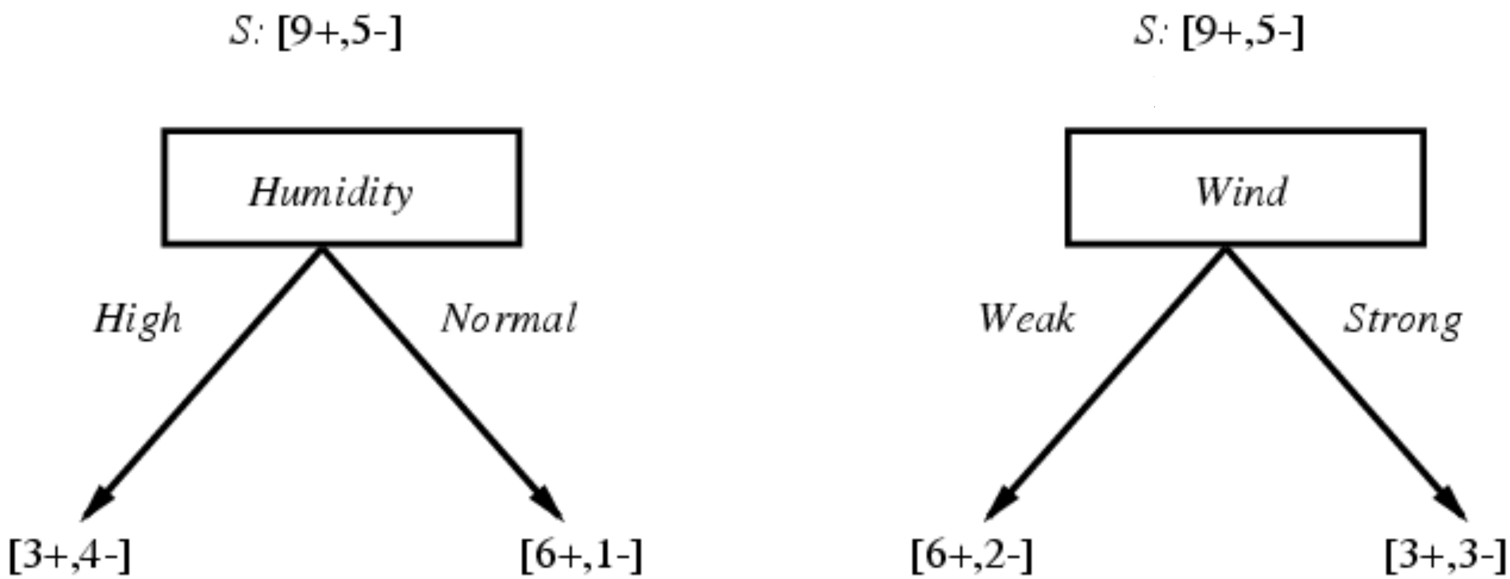


Figure from Tom Mitchell

Tennis Example

Test your understanding

Which attribute yields the best classifier?

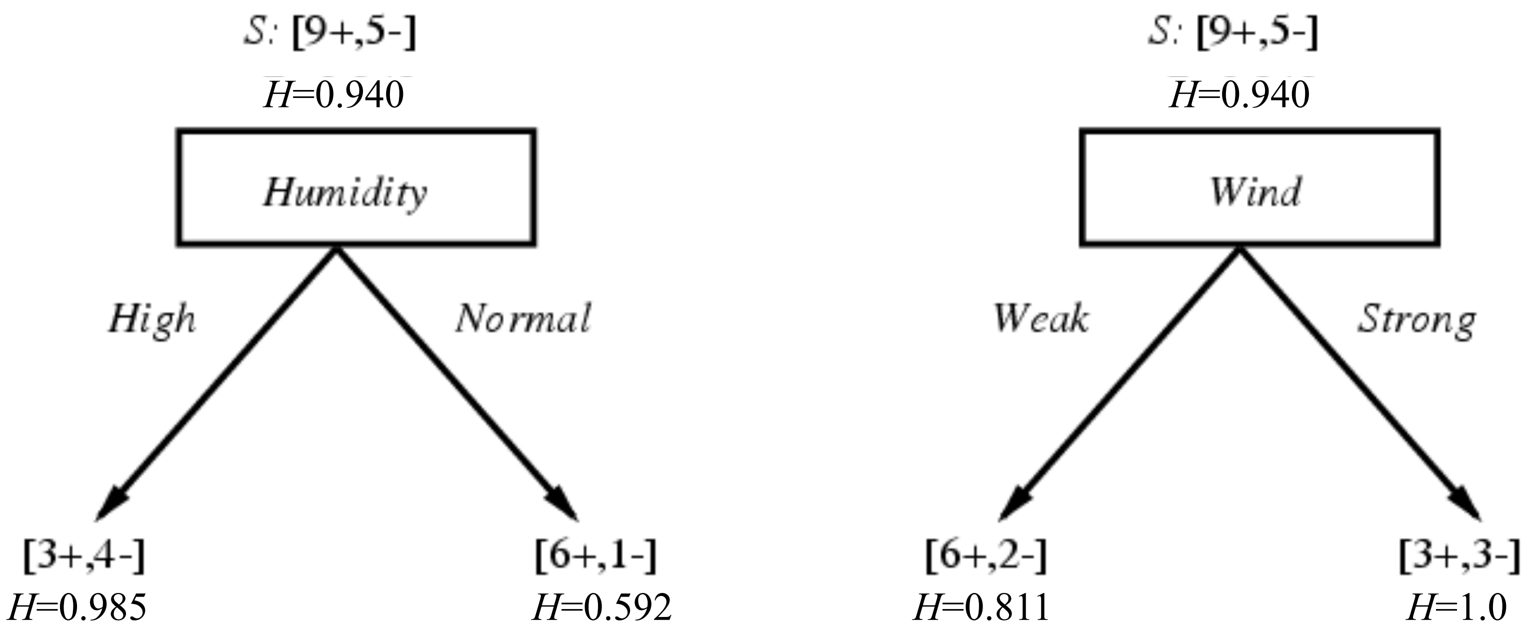


Figure from Tom Mitchell

Tennis Example

Test your understanding

Which attribute yields the best classifier?

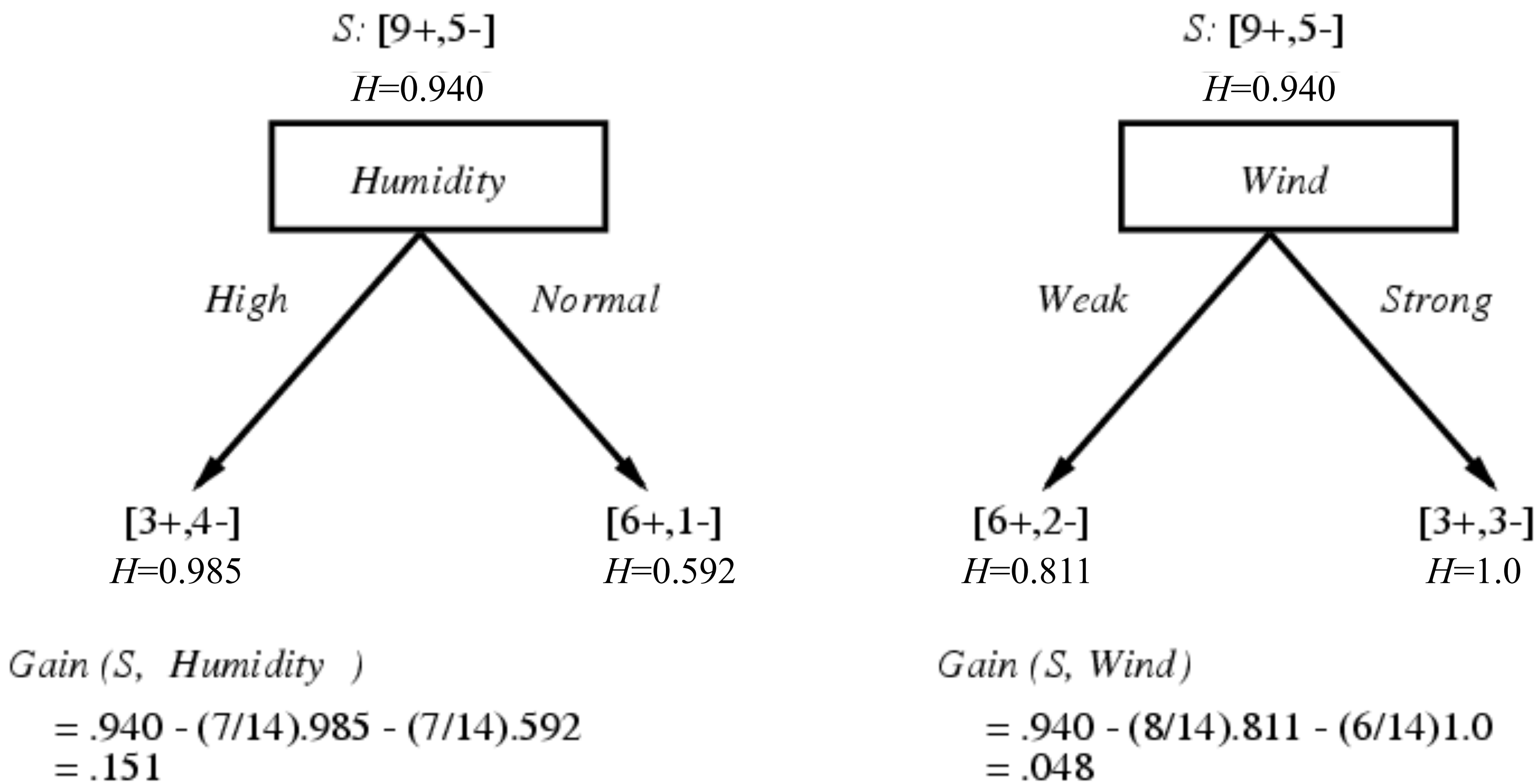
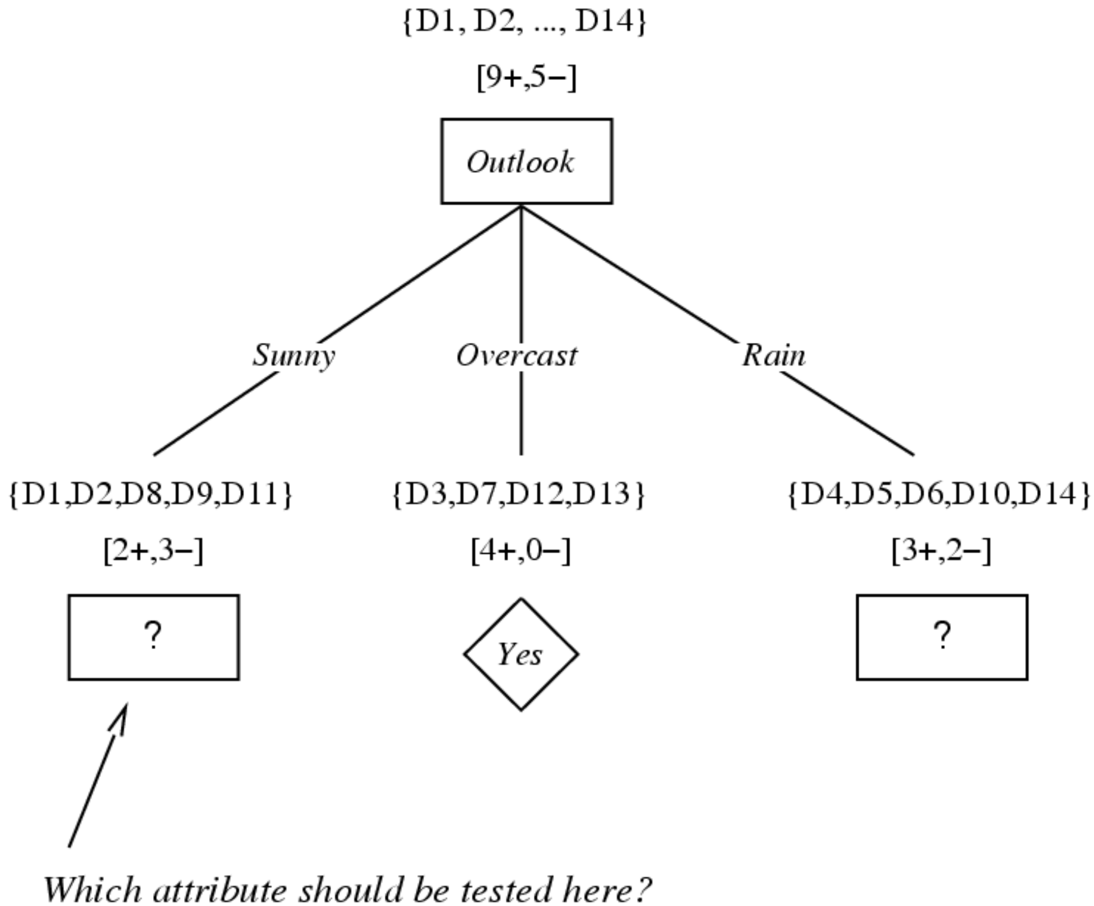


Figure from Tom Mitchell

Tennis Example



S_{sunny} = {D1,D2,D8,D9,D11}

$Gain\ (S_{sunny}, Humidity) = .970 - (3/5)\,0.0 - (2/5)\,0.0 = .970$

$Gain\ (S_{sunny}, Temperature) = .970 - (2/5)\,0.0 - (2/5)\,1.0 - (1/5)\,0.0 = .570$

$Gain\ (S_{sunny}, Wind) = .970 - (2/5)\,1.0 - (3/5)\,.918 = .019$

Figure from Tom Mitchell