10-301/10-601 Introduction to Machine Learning

Machine Learning Department
School of Computer Science
Carnegie Mellon University

Ensemble Methods: Boosting

+

Recommender Systems

Matt Gormley
Lecture 26
Apr. 19, 2023

Reminders

- **Homework 8: Reinforcement Learning**
  - Out: Mon, Apr. 10
  - Due: Fri, Apr. 21 at 11:59pm
- **Homework 9: Learning Paradigms**
  - Out: Fri, Apr. 21
  - Due: Fri, Dec. 9 at 11:59pm (only two grace/late days permitted)

Learning Paradigms

Paradigm	**Data**
Supervised	$\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^N$ $\quad \mathbf{x} \sim p^*(\cdot)$ and $y = c^*(\cdot)$
$\hookrightarrow$ Regression	$y^{(i)} \in \mathbb{R}$
$\hookrightarrow$ Classification	$y^{(i)} \in \{1, \ldots, K\}$
$\hookrightarrow$ Binary classification	$y^{(i)} \in \{+1, -1\}$
$\hookrightarrow$ Structured Prediction	$\mathbf{y}^{(i)}$ is a vector
Unsupervised	$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N$ $\quad \mathbf{x} \sim p^*(\cdot)$
$\hookrightarrow$ Clustering	predict $\{z^{(i)}\}_{i=1}^N$ where $z^{(i)} \in \{1, \ldots, K\}$
$\hookrightarrow$ Dimensionality Reduction	convert each $\mathbf{x}^{(i)} \in \mathbb{R}^M$ to $\mathbf{u}^{(i)} \in \mathbb{R}^K$ with $K << M$
Semi-supervised	$\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^{N_1} \cup \{\mathbf{x}^{(j)}\}_{j=1}^{N_2}$
Online	$\mathcal{D} = \{(\mathbf{x}^{(1)}, y^{(1)}), (\mathbf{x}^{(2)}, y^{(2)}), (\mathbf{x}^{(3)}, y^{(3)}), \ldots\}$
Active Learning	$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N$ and can query $y^{(i)} = c^*(\cdot)$ at a cost
Imitation Learning	$\mathcal{D} = \{(s^{(1)}, a^{(1)}), (s^{(2)}, a^{(2)}), \ldots\}$
Reinforcement Learning	$\mathcal{D} = \{(s^{(1)}, a^{(1)}, r^{(1)}), (s^{(2)}, a^{(2)}, r^{(2)}), \ldots\}$

ML Big Picture

Learning Paradigms:

What data is available and when? What form of prediction?

- supervised learning
- unsupervised learning
- semi-supervised learning
- reinforcement learning
- active learning
- imitation learning
- domain adaptation
- online learning
- density estimation
- recommender systems
- feature learning
- manifold learning
- dimensionality reduction
- ensemble learning
- distant supervision
- hyperparameter optimization

Problem Formulation:

What is the structure of our output prediction?

boolean	Binary Classification
categorical	Multiclass Classification
ordinal	Ordinal Classification
real	Regression
ordering	Ranking
multiple discrete	Structured Prediction
multiple continuous	(e.g. dynamical systems)
both discrete & cont.	(e.g. mixed graphical models)

Application Areas

Key challenges?

NLP, Speech, Computer Vision, Robotics, Medicine, Search

Theoretical Foundations:

What principles guide learning?

- ❑ probabilistic
- ❑ information theoretic
- ❑ evolutionary search
- ❑ ML as optimization

Facets of Building ML Systems:

How to build systems that are robust, efficient, adaptive, effective?

1. *Data prep*
2. *Model selection*
3. *Training (optimization / search)*
4. *Hyperparameter tuning on validation data*
5. *(Blind) Assessment on test data*

Big Ideas in ML:

Which are the ideas driving development of the field?

- *inductive bias*
- *generalization / overfitting*
- *bias-variance decomposition*
- *generative vs. discriminative*
- *deep nets, graphical models*
- *PAC learning*
- *distant rewards*

Outline for Today

We'll talk about two distinct topics:

1. **Ensemble Methods:** combine or learn multiple classifiers into one
   (i.e. a family of algorithms)
2. **Recommender Systems:** produce recommendations of what a user will like
   (i.e. the solution to a particular type of task)

We'll use a prominent example of a recommender systems (the Netflix Prize) to motivate both topics…

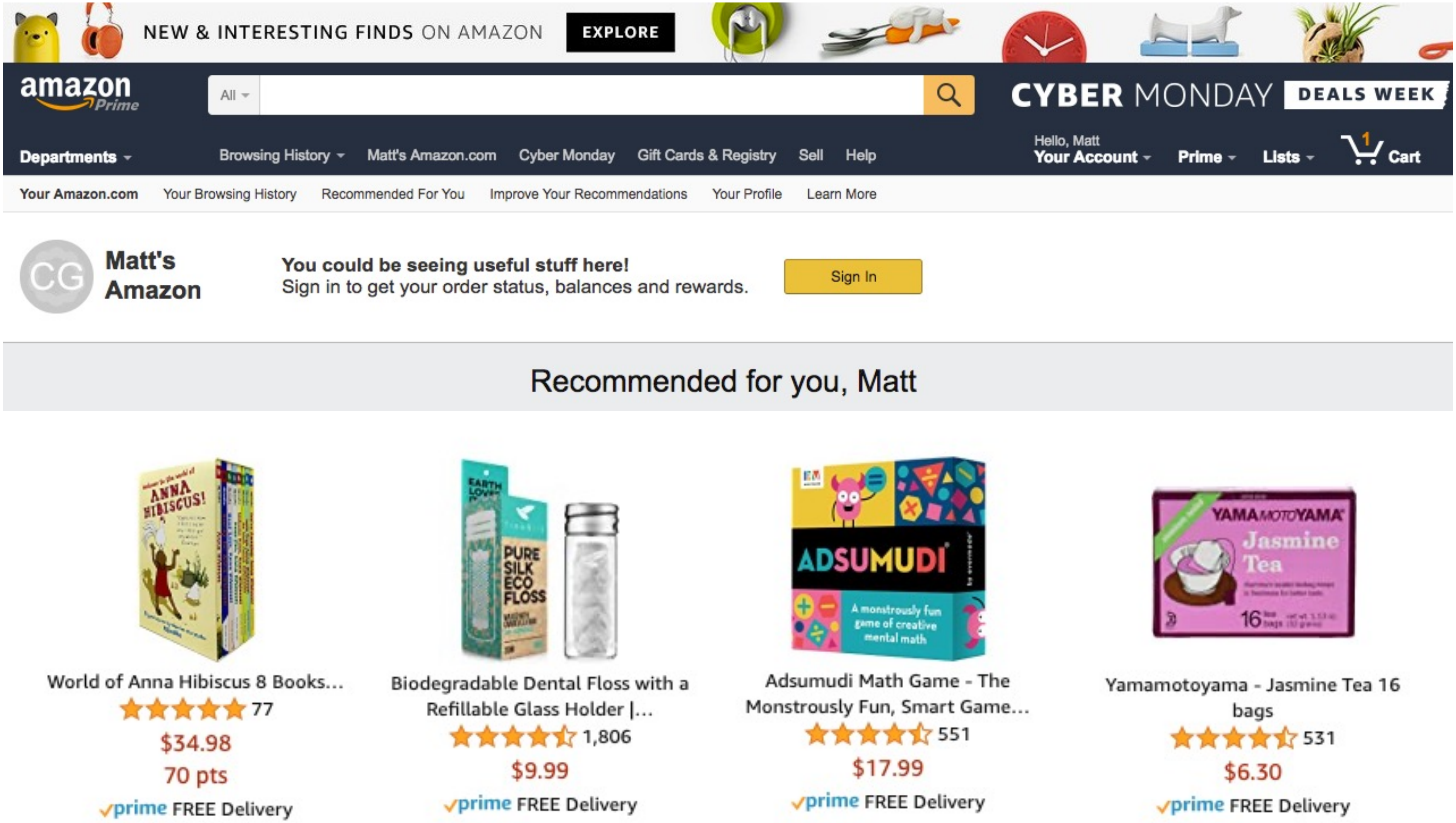
RECOMMENDER SYSTEMS

Recommender Systems

A Common Challenge:

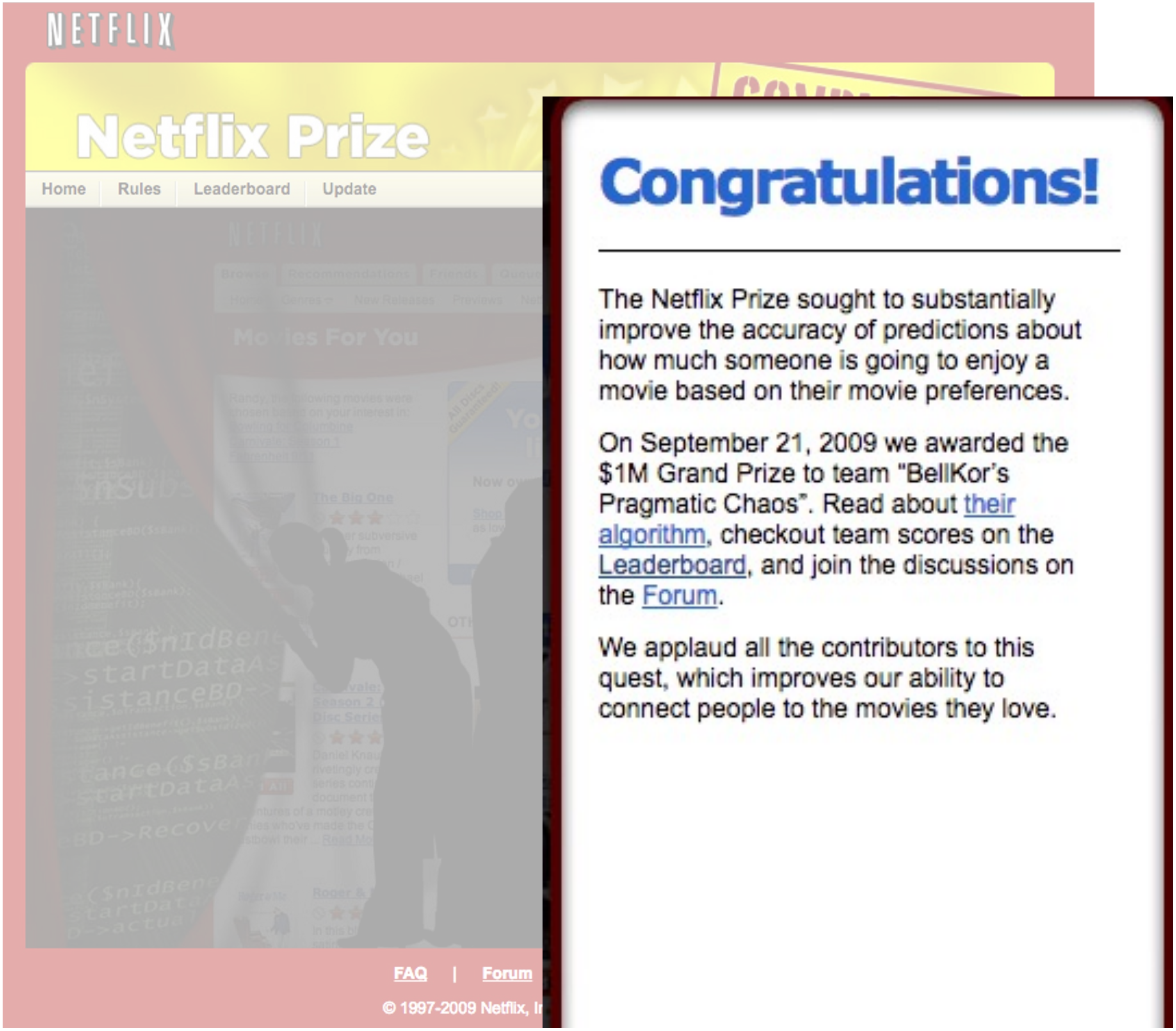
- Assume you're a company selling **items** of some sort: movies, songs, products, etc.
- Company collects millions of **ratings** from **users** of their **items**
- To maximize profit / user happiness, you want to **recommend** items that users are likely to want

Recommender Systems



Recommender Systems

Recommender Systems



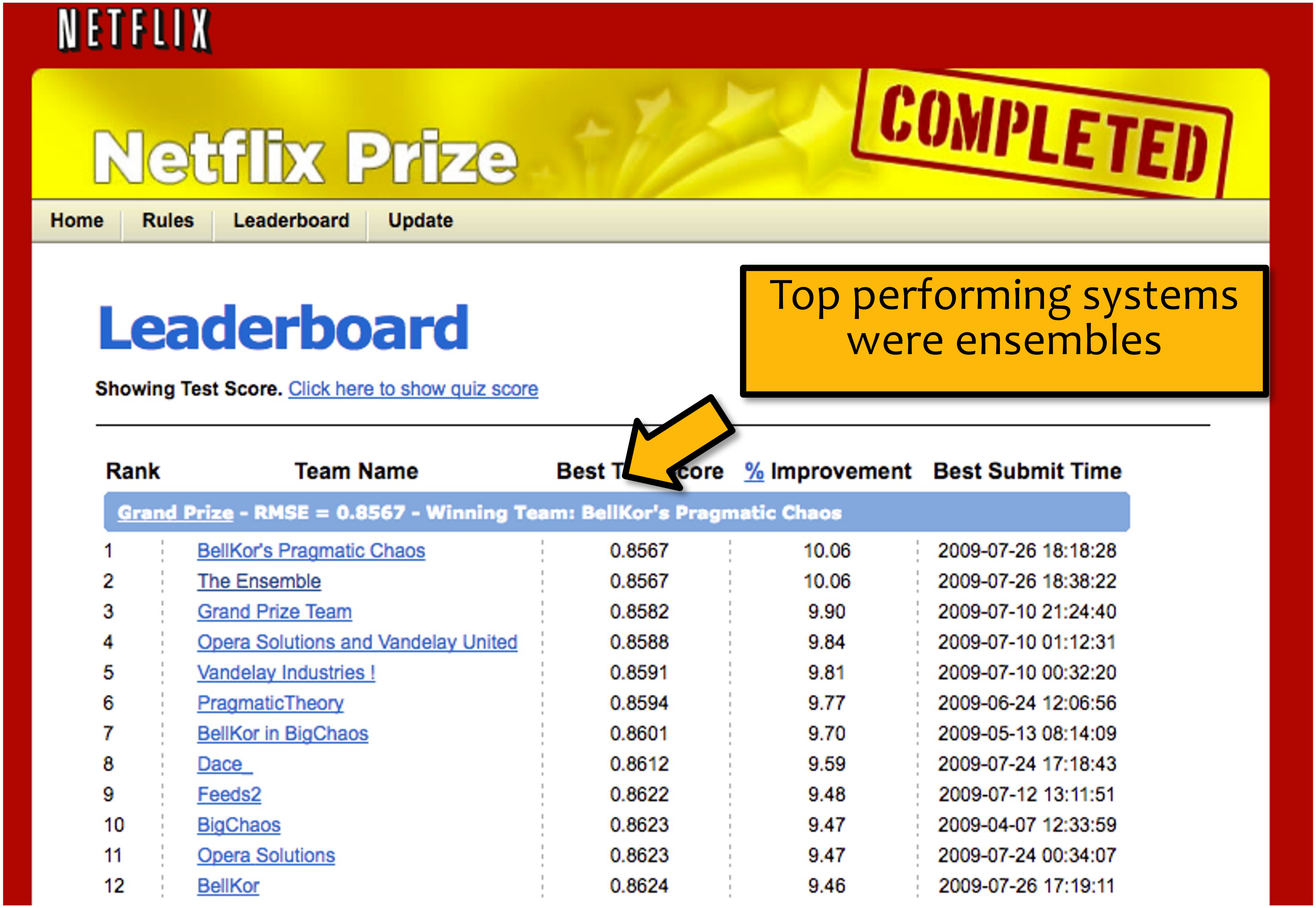
Recommender Systems

Problem Setup

- 500,000 users
- 20,000 movies
- 100 million ratings
- Goal: To obtain lower root mean squared error (RMSE) than Netflix's existing system on 3 million held out ratings

ENSEMBLE METHODS

Recommender Systems



Rank	Team Name	Best T… core	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
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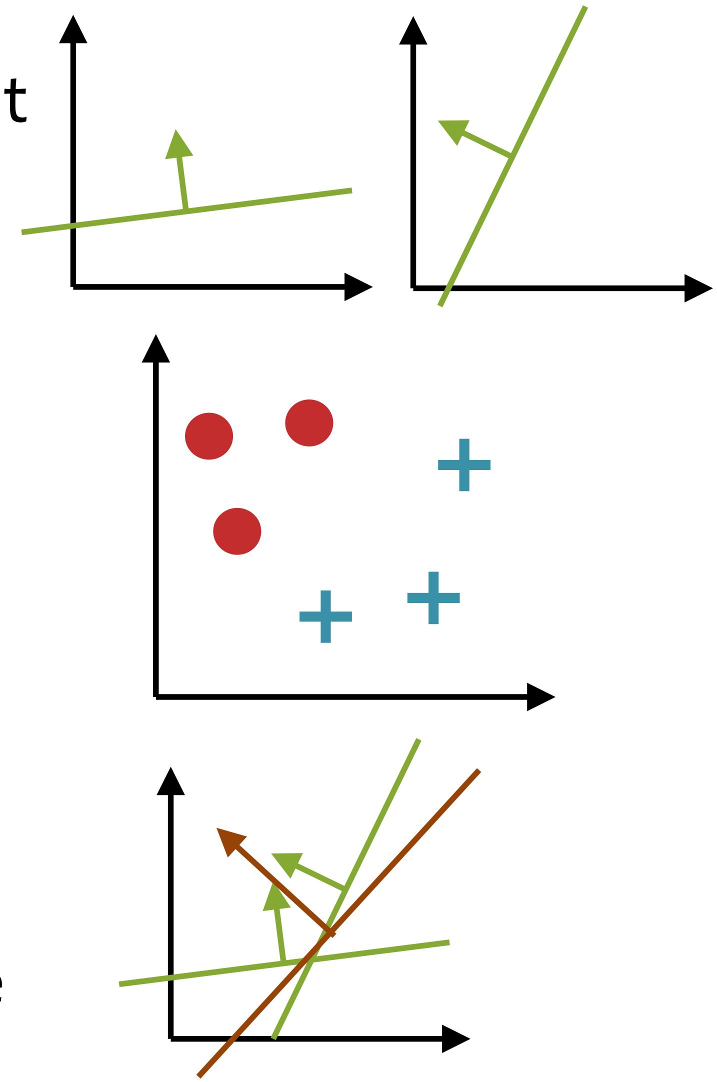
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11

Top performing systems were ensembles

Weighted Majority Algorithm

(Littlestone & Warmuth, 1994)

- **Given:** pool *A* of binary classifiers (that you know nothing about)
- **Data:** stream of examples (i.e. online learning setting)
- **Goal:** design a new learner that uses the predictions of the pool to make new predictions
- **Algorithm:**
  – Initially weight all classifiers equally
  – Receive a training example and predict the (weighted) majority vote of the classifiers in the pool
  – Down-weight classifiers that contribute to a mistake by a factor of β



Weighted Majority Algorithm

(Littlestone & Warmuth, 1994)

Suppose we have a pool of T binary classifiers $\mathcal{A} = \{h_1, \dots, h_T\}$ where $h_t : \mathbb{R}^M \rightarrow \{+1, -1\}$. Let α_t be the weight for classifier h_t.

Algorithm 1 Weighted Majority Algorithm

procedure WeightedMajority($\mathcal{A}$, β)
    Initialize classifier weights $\alpha_t = 1, \;\forall t \in \{1, \dots, T\}$
    for each training example $(\mathbf{x}, y)$ **do**
        Predict majority vote class (splitting ties randomly)

$$\hat{h}(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

        if a mistake is made $\hat{h}(x) \neq y$ **then**
            for each classifier $t \in \{1, \dots, T\}$ **do**
                If $h_t(x) \neq y$, then $\alpha_t \leftarrow \beta \alpha_t$

Weighted Majority Algorithm

Theorems (Littlestone & Warmuth, 1994)

For the general case where WM is applied to a pool $\mathcal{A}$ of algorithms we show the following upper bounds on the number of mistakes made in a given sequence of trials:

1. $O(\log|\mathcal{A}| + m)$, if one algorithm of $\mathcal{A}$ makes at most m mistakes.
2. $O(\log\frac{|\mathcal{A}|}{k} + m)$, if each of a subpool of k algorithms of $\mathcal{A}$ makes at most m mistakes.
3. $O(\log\frac{|\mathcal{A}|}{k} + \frac{m}{k})$, if the total number of mistakes of a subpool of k algorithms of $\mathcal{A}$ is at most m.

These are "mistake bounds" of the variety we saw for the Perceptron algorithm

ADABOOST

Comparison

Weighted Majority Algorithm

- an example of an ensemble method
- assumes the classifiers are learned ahead of time
- only learns (majority vote) weight for each classifiers

AdaBoost

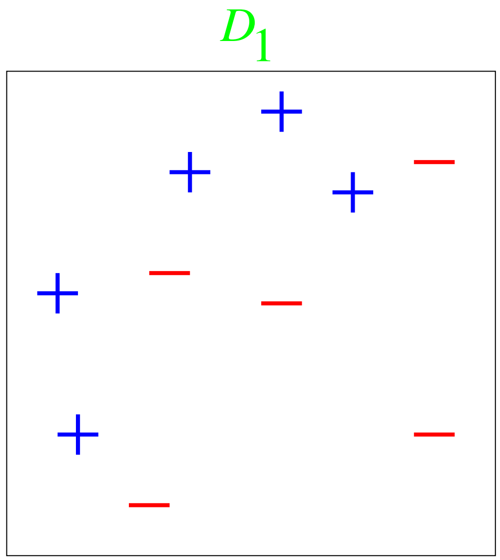
- an example of a boosting method
- simultaneously learns:
  - the classifiers themselves
  - (majority vote) weight for each classifiers

AdaBoost

- Definitions
  - Def: a **weak learner** is one that returns a hypothesis that is not much better than random guessing
  - Def: a **strong learner** is one that returns a hypothesis of arbitrarily low error

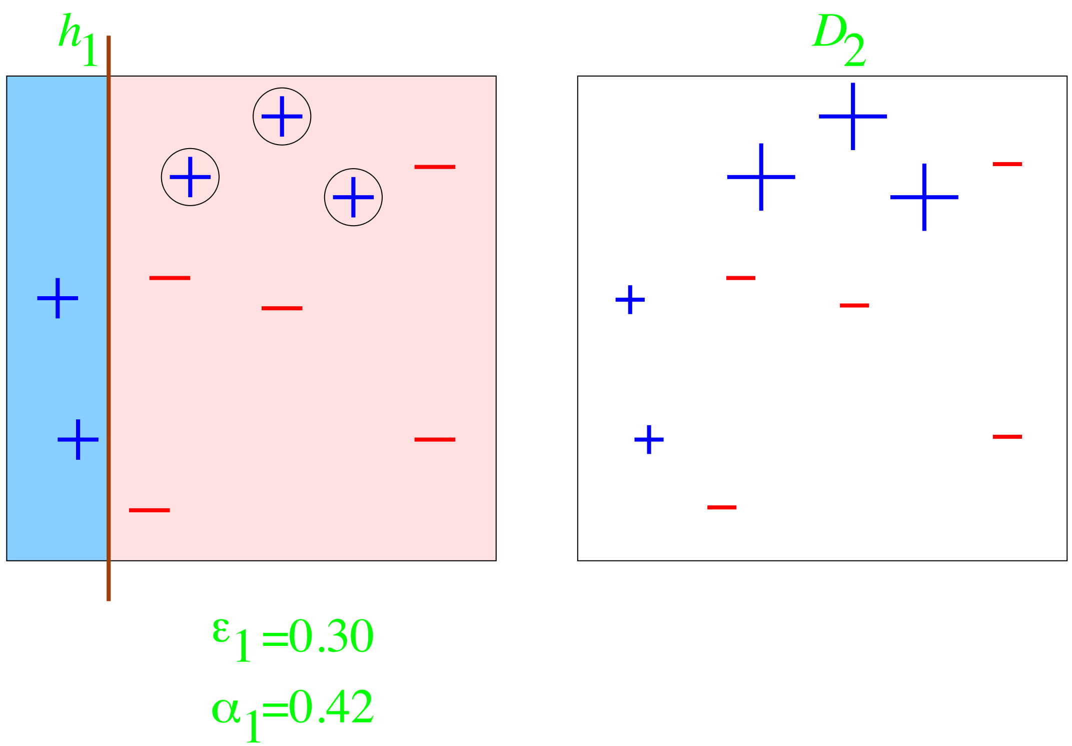
- AdaBoost answers the following question:
  - *Does that exist an efficient learning algorithm that can combine weak learners to obtain a strong learner?*

AdaBoost: Toy Example

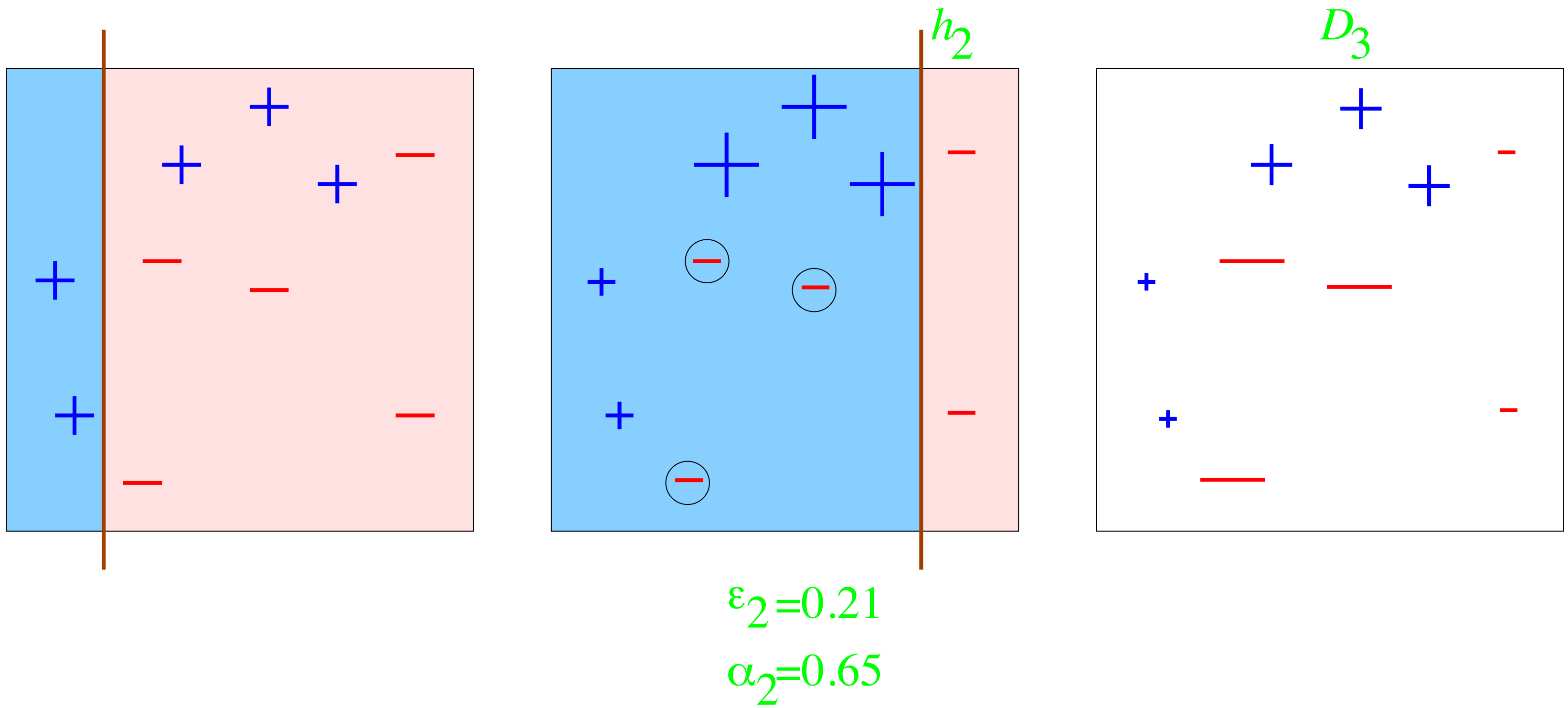


weak classifiers = vertical or horizontal half-planes

AdaBoost: Toy Example

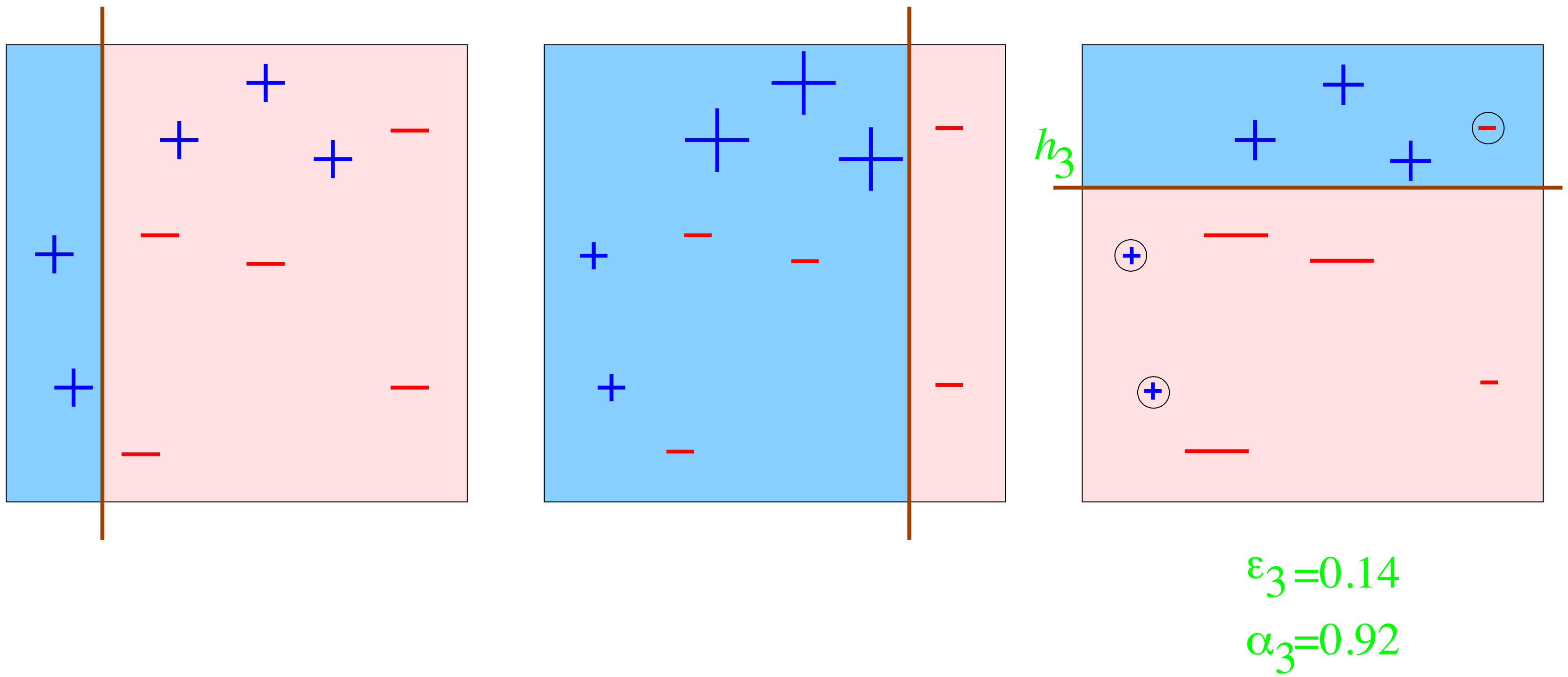


AdaBoost: Toy Example

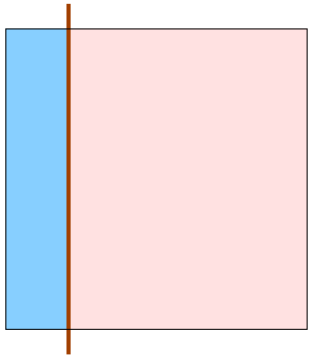
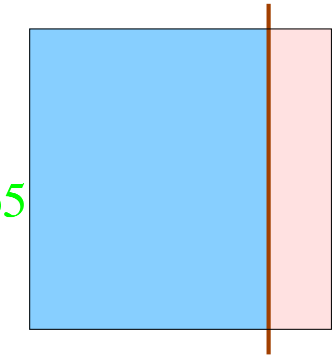
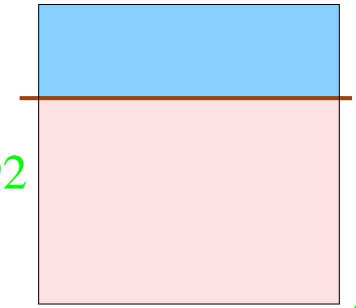


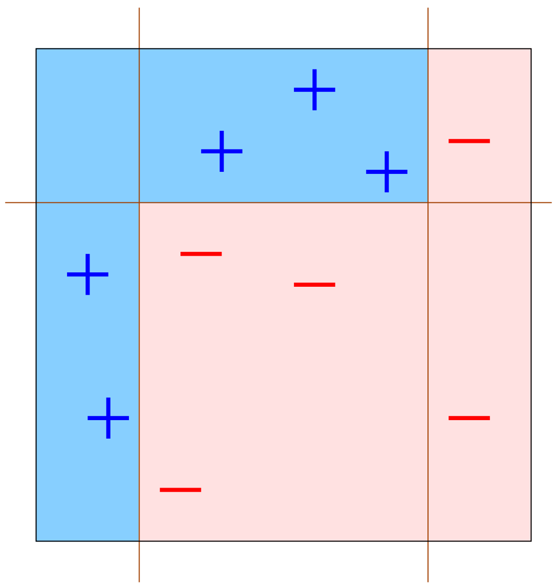
Slide from Schapire NIPS Tutorial

AdaBoost: Toy Example



AdaBoost: Toy Example

$H_{\text{final}} = \text{sign}\Big($ 0.42  $+\, 0.65$  $+\, 0.92$  $\Big)$

$=$ 

Slide from Schapire NIPS Tutorial

AdaBoost

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in X$, $y_i \in Y = \{-1, +1\}$
Initialize $D_1(i) = 1/m$.
For $t = 1, \ldots, T$:

- Train weak learner using distribution D_t.
- Get weak hypothesis $h_t : X \to \{-1, +1\}$ with error

$$\epsilon_t = \Pr_{i \sim D_t}\left[h_t(x_i) \neq y_i\right].$$

- Choose $\alpha_t = \frac{1}{2} \ln\left(\frac{1 - \epsilon_t}{\epsilon_t}\right)$.
- Update:

$$\begin{aligned} D_{t+1}(i) &= \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} & \text{if } h_t(x_i) = y_i \\ e^{\alpha_t} & \text{if } h_t(x_i) \neq y_i \end{cases} \\ &= \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t} \end{aligned}$$

  where Z_t is a normalization factor (chosen so that D_{t+1} will be a distribution).

Output the final hypothesis:

$$H(x) = \operatorname{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right).$$

Algorithm from (Freund & Schapire, 1999)

AdaBoost: Theory

(Training) Mistake Bound

The most basic theoretical property of AdaBoost concerns its ability to reduce the training error. Let us write the error ϵ_t of h_t as $\frac{1}{2} - \gamma_t$. Since a hypothesis that guesses each instance's class at random has an error rate of $1/2$ (on binary problems), γ_t thus measures how much better than random are h_t's predictions. Freund and Schapire [23] prove that the training error (the fraction of mistakes on the training set) of the final hypothesis H is at most

$$\prod_t \left[2\sqrt{\epsilon_t(1-\epsilon_t)}\right] = \prod_t \sqrt{1-4\gamma_t^2} \leq \exp\left(-2\sum_t \gamma_t^2\right). \tag{1}$$

Thus, if each weak hypothesis is slightly better than random so that $\gamma_t \geq \gamma$ for some $\gamma > 0$, then the training error drops exponentially fast.

Figure from (Freund & Schapire, 1999)

AdaBoost: Theory

Generalization Error

Freund and Schapire [23] showed how to bound the generalization error of the final hypothesis in terms of its training error, the sample size m, the VC-dimension d of the weak hypothesis space and the number of boosting rounds T. (The VC-dimension is a standard measure of the "complexity" of a space of hypotheses. See, for instance, Blumer et al. [5].) Specifically, they used techniques from Baum and Haussler [4] to show that the generalization error, with high probability, is at most

$$\hat{\Pr}\left[H(x) \neq y\right] + \tilde{O}\left(\sqrt{\frac{Td}{m}}\right)$$

where $\hat{\Pr}\left[\cdot\right]$ denotes empirical probability on the training sample. This bound suggests that boosting will overfit if run for too many rounds, i.e., as T becomes large. In fact, this sometimes does happen. However, in early experiments, several authors [9, 15, 36] observed empirically that boosting often does *not* overfit, even when run for thousands of rounds. Moreover, it was observed that AdaBoost would sometimes continue to drive down the generalization error long after the training error had reached zero, clearly contradicting the spirit of the bound above. For instance, the left

Figure from (Freund & Schapire, 1999)

AdaBoost

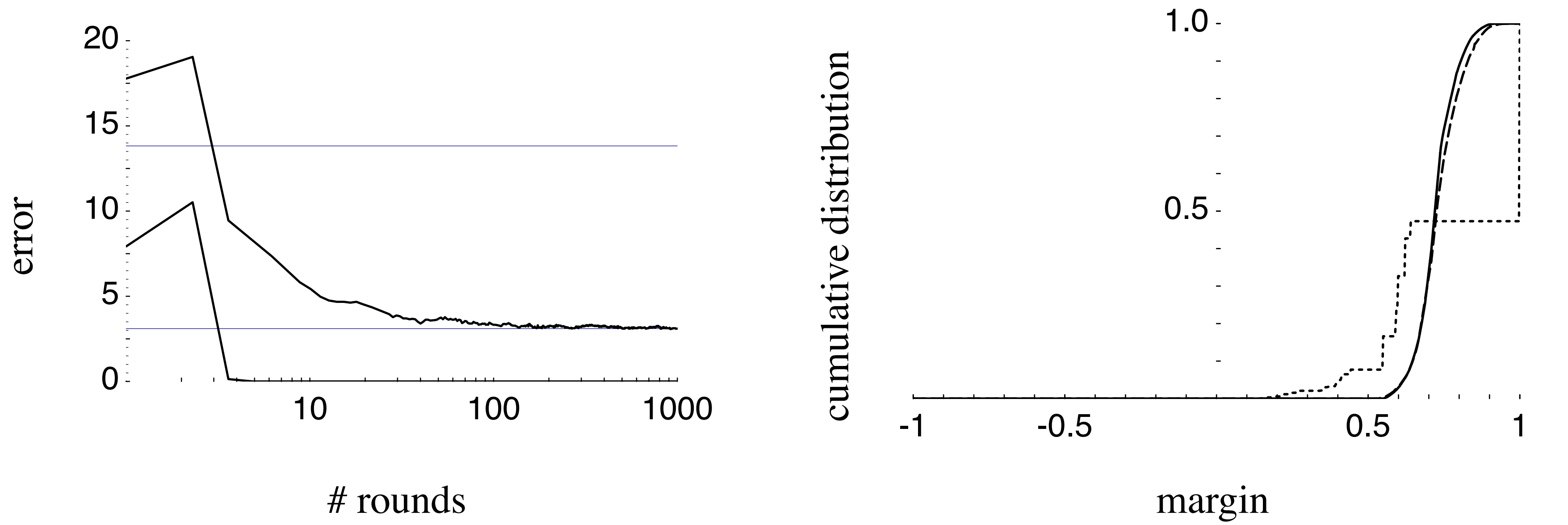


Figure 2: Error curves and the margin distribution graph for boosting C4.5 on the letter dataset as reported by Schapire et al. [41]. *Left*: the training and test error curves (lower and upper curves, respectively) of the combined classifier as a function of the number of rounds of boosting. The horizontal lines indicate the test error rate of the base classifier as well as the test error of the final combined classifier. *Right*: The cumulative distribution of margins of the training examples after 5, 100 and 1000 iterations, indicated by short-dashed, long-dashed (mostly hidden) and solid curves, respectively.

Figure from (Freund & Schapire, 1999)

BAGGING VS. BOOSTING

Bagging vs. Boosting

- **Bagging** tends to be most useful when your classifiers exhibit **high variance** from one training sample to the next
- **Boosting** tends to be most useful when your classifiers exhibit **high bias** (i.e. are very simple)

Bias-Variance Tradeoff

Suppose we have a regression dataset $\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^{N}$ for which $y^{(i)} = c^*(\mathbf{x}^{(i)}) + \epsilon$ where $\epsilon \sim$ Gaussian$(0, 1)$.

We can decompose the mean squared error of a classifier $h_{\boldsymbol{\theta}}(\cdot)$ as follows:

$$\mathbb{E}[(y - h_{\boldsymbol{\theta}}(\mathbf{x}))^2] = \text{Bias}^2 + \text{Variance} + \sigma^2$$

where

$$\text{Bias} = \mathbb{E}[h_{\boldsymbol{\theta}}(\mathbf{x})] - \mathbb{E}[c^*(\mathbf{x})]$$

$$\text{Variance} = \mathbb{E}[(\mathbb{E}[h_{\boldsymbol{\theta}}(\mathbf{x})] - h_{\boldsymbol{\theta}}(\mathbf{x}))^2]$$

Above, all the expectations are under the uniform distribution over the dataset $(\mathbf{x}, y) \sim \mathcal{D}$.

For binary classification with $y^{(i)} \in \{0, 1\}$, we can take $h_\theta(\mathbf{x}) = p(y = 1 \mid \mathbf{x})$ and the same decomposition applies.

Bias-Variance Tradeoff

Suppose you have two regressors with the same MSE, but…

- classifier A is heavily **overfitting**
- classifier B is heavily **underfitting**

$$\mathbb{E}[(y - h_{\boldsymbol{\theta}}(\mathbf{x}))^2] = \text{Bias}^2 + \text{Variance} + \boxed{\sigma^2}$$

the irreducible error

where

$$\text{Bias} = \boxed{\mathbb{E}[h_{\boldsymbol{\theta}}(\mathbf{x})] - \mathbb{E}[c^*(\mathbf{x})]}$$

bad assumptions in learner

$$\text{Variance} = \boxed{\mathbb{E}[(\mathbb{E}[h_{\boldsymbol{\theta}}(\mathbf{x})] - h_{\boldsymbol{\theta}}(\mathbf{x}))^2]}$$

sensitivity to noise in data

We can interpret this as a tradeoff between…

- classifier A: achieving **low bias** at the expense of **high variance**
- classifier B: achieving **low variance** at the expense of **high bias**

Learning Objectives

Ensemble Methods: Boosting

You should be able to…

1. Explain how a weighted majority vote over linear classifiers can lead to a non-linear decision boundary
2. Implement AdaBoost
3. Describe a surprisingly common empirical result regarding Adaboost train/test curves

RECOMMENDER SYSTEMS

Recommender Systems

Problem Setup

- 500,000 users
- 20,000 movies
- 100 million ratings
- Goal: To obtain lower root mean squared error (RMSE) than Netflix's existing system on 3 million held out ratings

Recommender Systems

NETFLIX

Netflix Prize

COMPLETED

Home | Rules | Leaderboard | Update

Leaderboard

Showing Test Score. Click here to show quiz score

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
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Recommender Systems

- **Setup:**
  - Items:
    movies, songs, products, etc.
    (often many thousands)
  - Users:
    watchers, listeners, purchasers, etc.
    (often many millions)
  - Feedback:
    5-star ratings, not-clicking 'next', purchases, etc.
- **Key Assumptions:**
  - Can represent ratings numerically as a user/item matrix
  - Users only rate a small number of items (the matrix is sparse)

	Doctor Strange	Star Trek: Beyond	Zootopia
Alice	1		5
Bob	3	4	
Charlie	3	5	2

Two Types of Recommender Systems

Content Filtering

- *Example*: Pandora.com music recommendations (Music Genome Project)
- **Con:** Assumes access to side information about items (e.g. properties of a song)
- **Pro:** Got a new item to add? No problem, just be sure to include the side information

Collaborative Filtering

- *Example*: Netflix movie recommendations
- **Pro:** Does not assume access to side information about items (e.g. does not need to know about movie genres)
- **Con:** Does not work on new items that have no ratings

COLLABORATIVE FILTERING

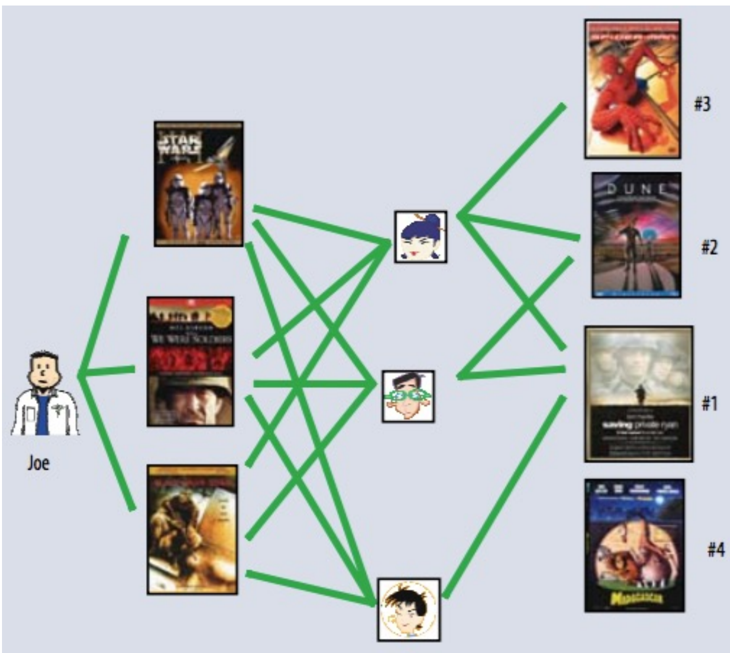
Collaborative Filtering

- **Everyday Examples of Collaborative Filtering...**
  - Bestseller lists
  - Top 40 music lists
  - The "recent returns" shelf at the library
  - Unmarked but well-used paths thru the woods
  - The printer room at work
  - "Read any good books lately?"
  - ...
- **Common insight:** personal tastes are correlated
  - If Alice and Bob both like X and Alice likes Y then Bob is more likely to like Y
  - especially (perhaps) if Bob knows Alice

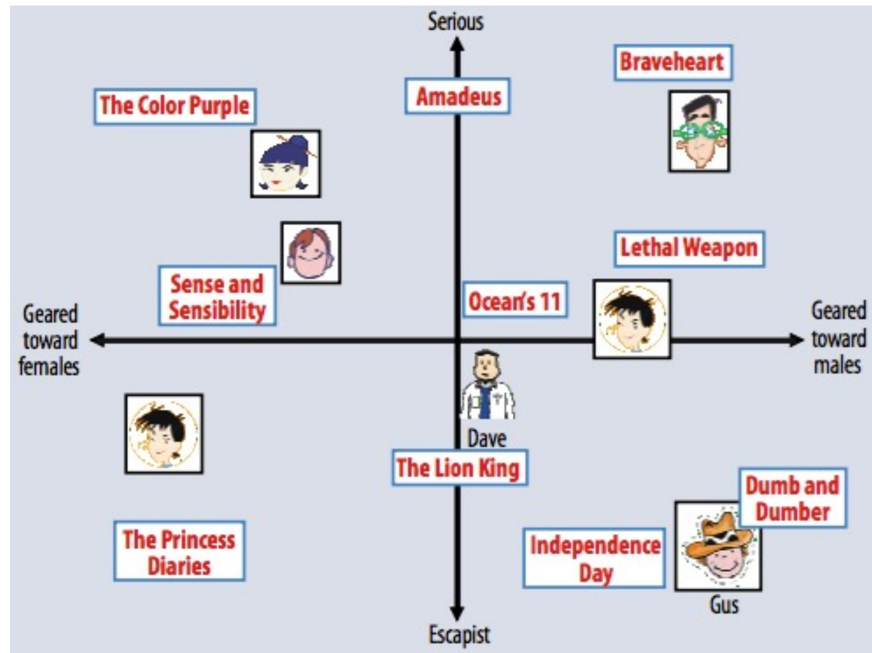
Slide from William Cohen

Two Types of Collaborative Filtering

1. Neighborhood Methods



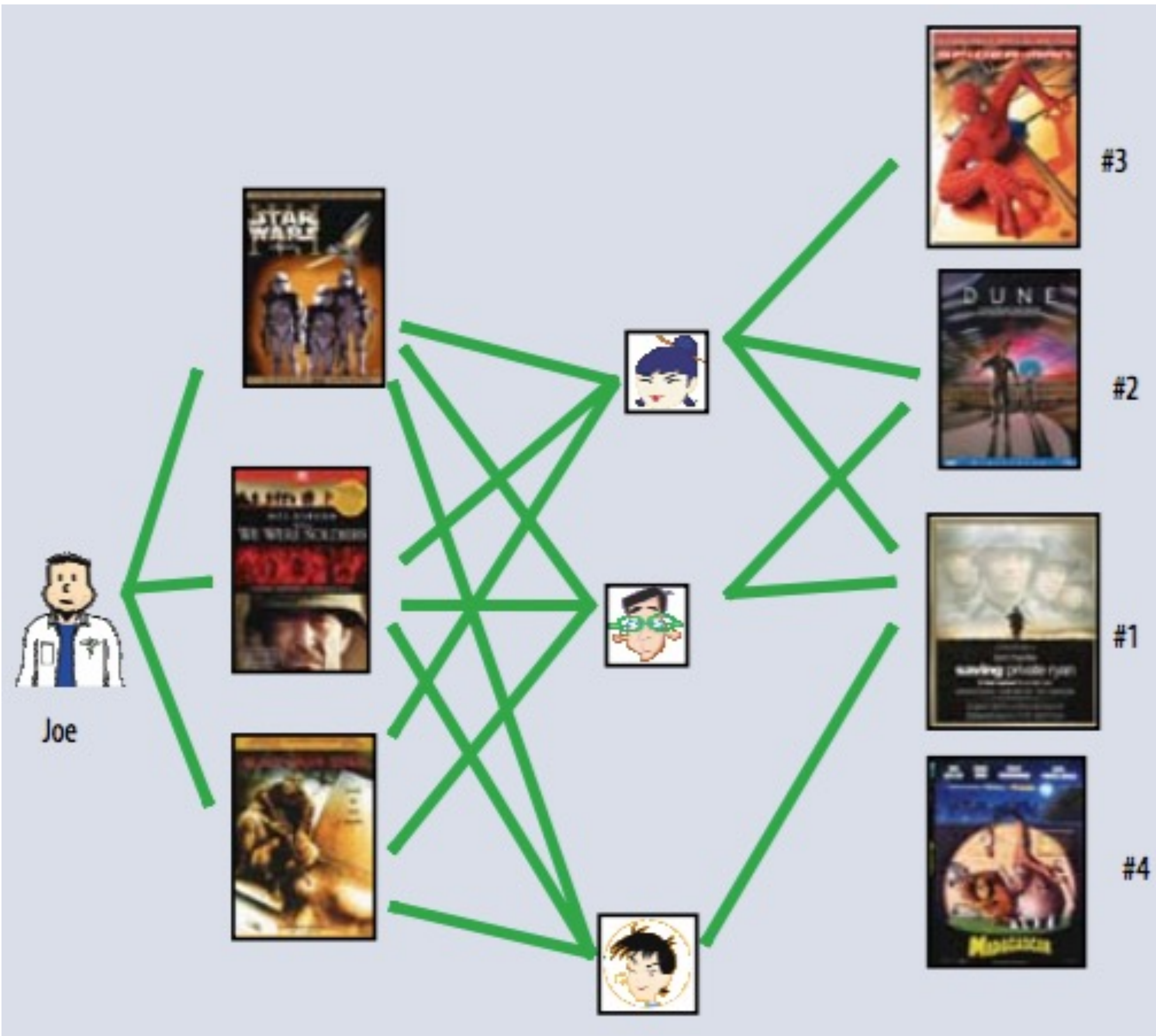
2. Latent Factor Methods



Figures from Koren et al. (2009)

Two Types of Collaborative Filtering

1. Neighborhood Methods



In the figure, assume that a green line indicates the movie was **watched**

Algorithm:

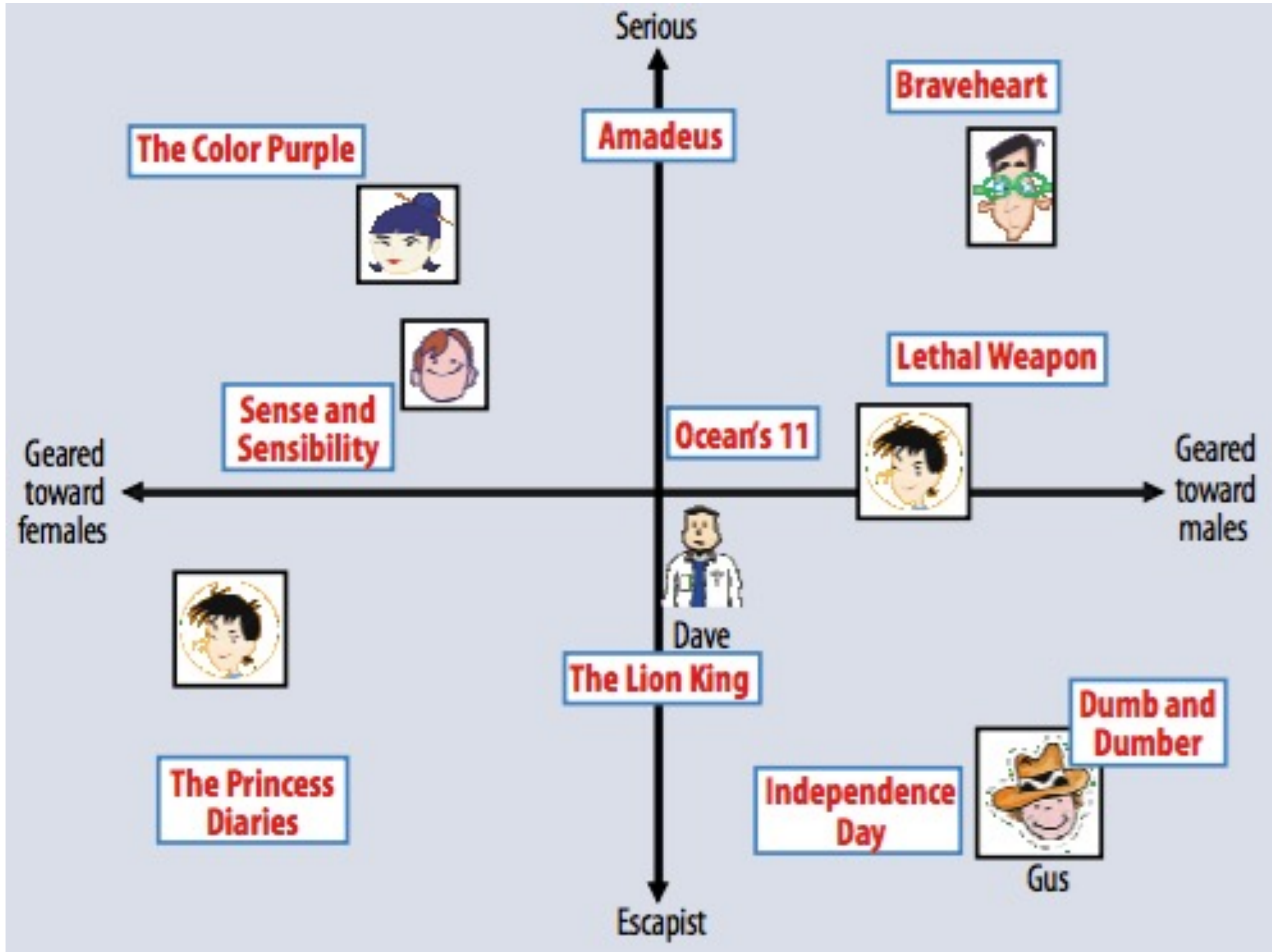
1. **Find neighbors** based on similarity of movie preferences
2. **Recommend** movies that those neighbors watched

Figures from Koren et al. (2009)

Two Types of Collaborative Filtering

2. Latent Factor Methods

- Assume that both movies and users live in some **low-dimensional space** describing their properties
- **Recommend** a movie based on its **proximity** to the user in the latent space
- **Example Algorithm:** Matrix Factorization



Figures from Koren et al. (2009)

Recommending Movies

Question:

Applied to the Netflix Prize problem, which of the following methods *always* requires side information about the users and movies?

Select all that apply

A. principal component analysis
B. collaborative filtering
C. latent factor methods
D. ensemble methods
E. content filtering
F. neighborhood methods
G. recommender systems

Answer:

MATRIX FACTORIZATION

Matrix Factorization

- Many different ways of factorizing a matrix
- We'll consider three:
    1. Unconstrained Matrix Factorization
    2. Singular Value Decomposition
    3. Non-negative Matrix Factorization
- MF is just another example of a **common recipe**:
    1. define a model
    2. define an objective function
    3. optimize with SGD

Matrix Factorization

Whiteboard

– Background: Low-rank Factorizations

– Residual matrix

Example: MF for Netflix Problem

		NERO	JULIUS CAESAR	CLEOPATRA	SLEEPLESS IN SEATTLE	PRETTY WOMAN	CASABLANCA
HISTORY	1	1	1	1	0	0	0
HISTORY	2	1	1	1	0	0	0
HISTORY	3	1	1	1	0	0	0
BOTH	4	1	1	1	1	1	1
ROMANCE	5	-1	-1	-1	1	1	1
ROMANCE	6	-1	-1	1	1	1	1
ROMANCE	7	-1	-1	-1	1	1	1

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	HISTORY	ROMANCE
1	1	0
2	1	0
3	1	0
4	1	1
5	-1	1
6	-1	1
7	-1	1

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	NERO	JULIUS CAESAR	CLEOPATRA	SLEEPLESS IN SEATTLE	PRETTY WOMAN	CASABLANCA
HISTORY	1	1	1	0	0	0
ROMANCE	0	0	1	1	1	1

V^T

(a) Example of rank-2 matrix factorization

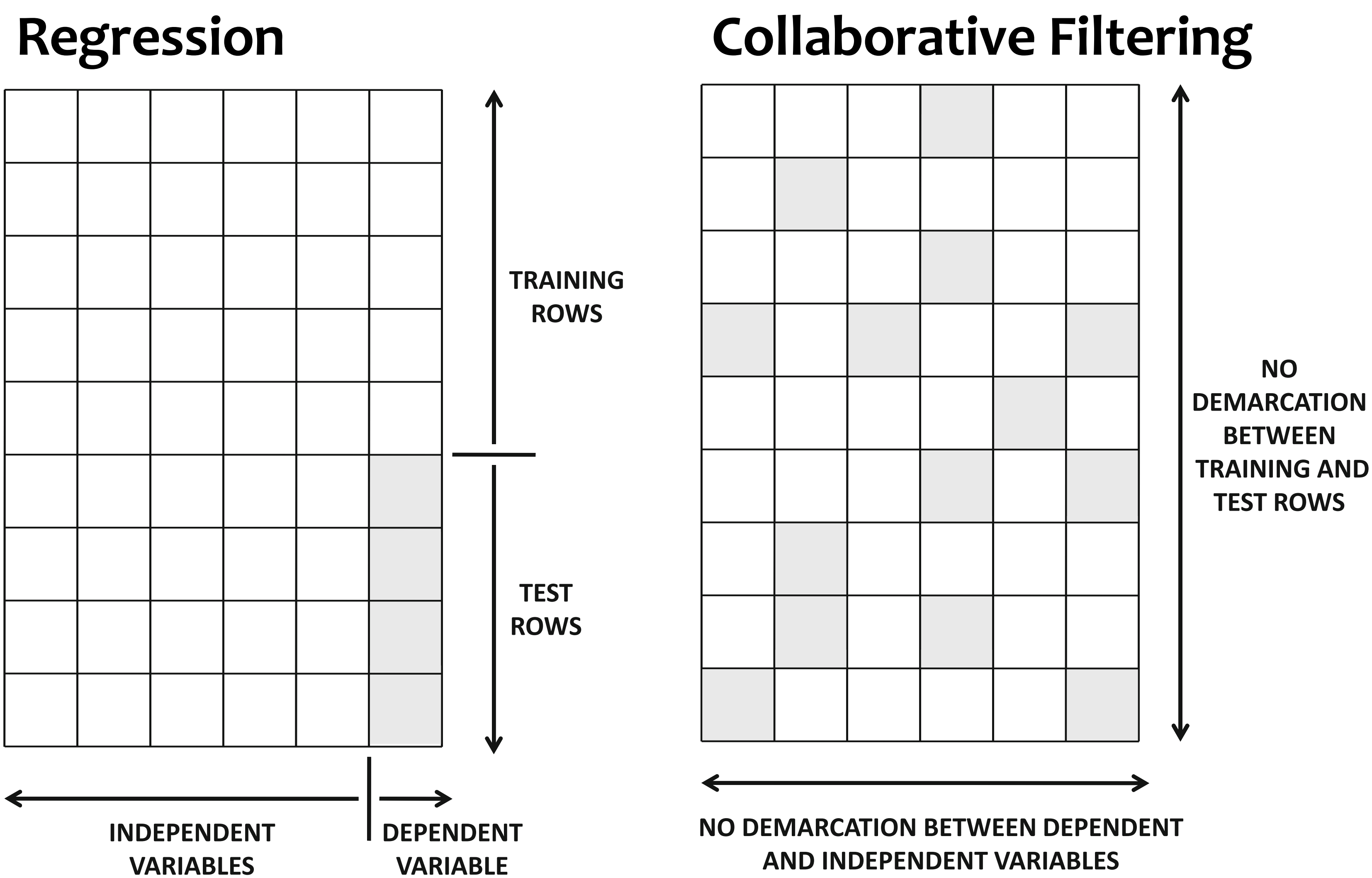
		NERO	JULIUS CAESAR	CLEOPATRA	SLEEPLESS IN SEATTLE	PRETTY WOMAN	CASABLANCA
HISTORY	1	0	0	0	0	0	0
HISTORY	2	0	0	0	0	0	0
HISTORY	3	0	0	0	0	0	0
BOTH	4	0	0	-1	0	0	0
ROMANCE	5	0	0	-1	0	0	0
ROMANCE	6	0	0	1	0	0	0
ROMANCE	7	0	0	-1	0	0	0

E

(b) Residual matrix

Figures from Aggarwal (2016)

Regression vs. Collaborative Filtering



Figures from Aggarwal (2016)

UNCONSTRAINED MATRIX FACTORIZATION

Unconstrained Matrix Factorization

Whiteboard

- Optimization problem
- SGD
- SGD with Regularization
- Alternating Least Squares
- User/item bias terms (matrix trick)

Unconstrained Matrix Factorization

SGD for UMF:

While not converged:

1. Sample (i,j) from Z uniformly at random
2. Compute $e_{ij} = r_{ij} - \vec{u}_i^T \vec{v}_j$
3. Update

$$\vec{u}_i \leftarrow \vec{u}_i - \gamma \nabla_{\vec{u}_i} J_{ij}(U,V)$$

$$\vec{v}_j \leftarrow \vec{v}_j - \gamma \nabla_{\vec{v}_j} J_{ij}(U,V)$$

W/ Regularization

$$J_{ij}(U,V) = \frac{1}{2}\left(r_{ij} - \vec{u}_i^T \vec{v}_j\right)^2 + \lambda\left(\|u_i\|_2^2 + \|v_j\|_2^2\right)$$

$$\nabla_{\vec{u}_i} J_{ij}(U,V) = -e_{ij}\,\vec{v}_j + \lambda \vec{u}_i$$

$$\nabla_{\vec{v}_j} J_{ij}(U,V) = -e_{ij}\,\vec{u}_i + \lambda \vec{v}_j$$

where $e_{ij} = r_{ij} - \vec{u}_i^T \vec{v}_j$

Unconstrained Matrix Factorization

SGD for UMF:

User/Item Bias terms

$$\hat{r}_{ij} = o_i + p_j + \vec{u}_i^T \vec{v}_j$$

matrix trick:

$$U = \begin{bmatrix} \cdots & o_1 & 1 \\ \cdots & o_2 & 1 \\ & \vdots & \vdots \\ \cdots & o_n & 1 \end{bmatrix} \qquad V = \begin{bmatrix} \cdots & 1 & p_1 \\ \cdots & 1 & p_2 \\ & \vdots & \vdots \\ \cdots & 1 & p_n \end{bmatrix}$$

Unconstrained Matrix Factorization

Alternating Least Squares (ALS) for UMF:

Block Coord. Descent:

while not converged:

① $U = \underset{U}{\operatorname{argmin}}\ J(U,V)$

② $V = \underset{V}{\operatorname{argmin}}\ J(U,V)$

convex and easy to solve in closed form

Lin. Reg.

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{N}\left(y_i - \vec{\theta}^T \vec{x}_i\right)^2$$

$$J(U,V) = \frac{1}{2}\sum_{(i,j)\in Z}\left(r_{ij} - \vec{u}_i^T \vec{v}_j\right)^2$$

if U is fixed Least Squares in V

if V is fixed Least Squares in U

Option #1: take derivatives, set to zero and solve in closed form

★ solving $J(U,V)$ in closed form directly isn't easy and $J(U,V)$ is nonconvex

Matrix Factorization

Example Factors

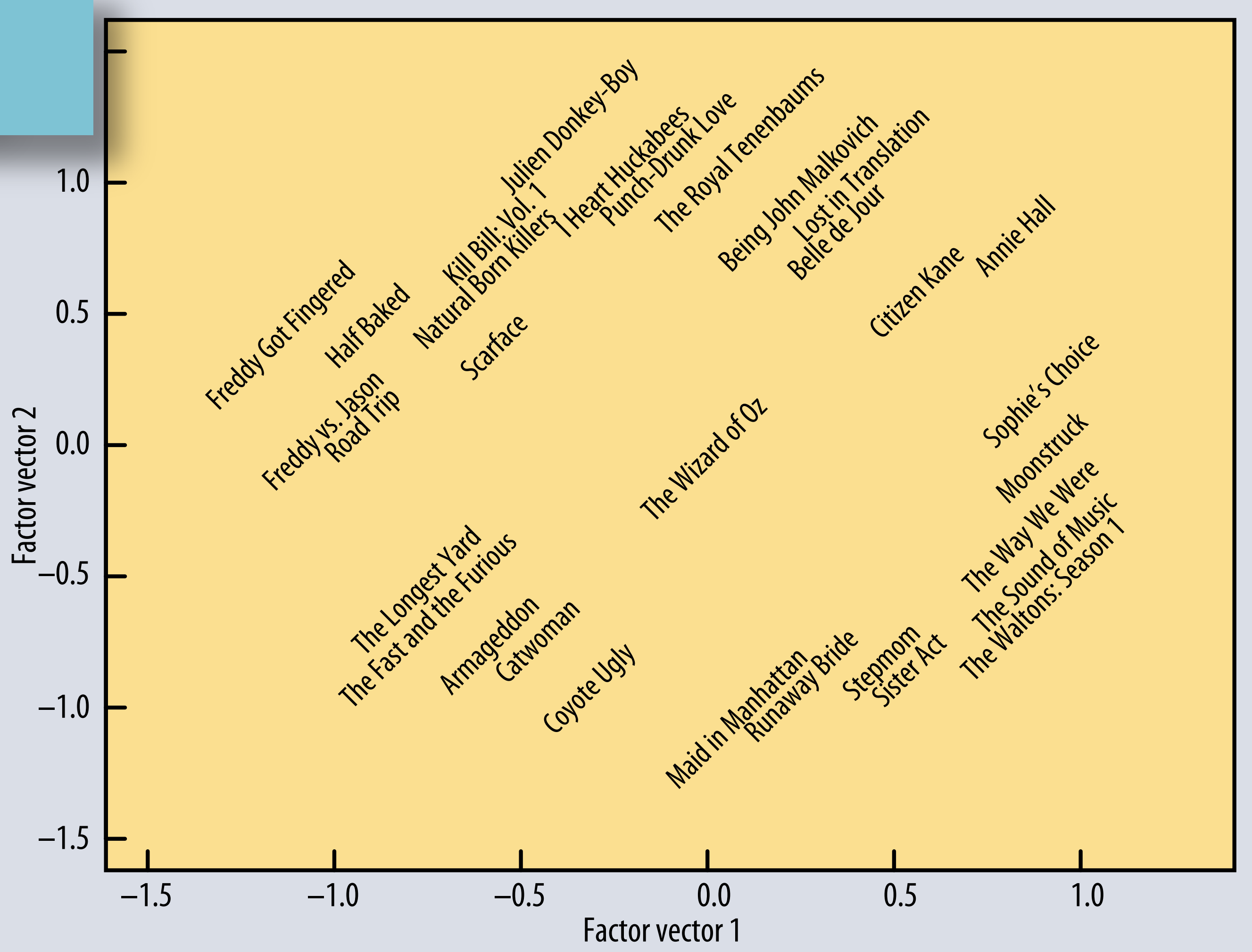


Figure 3. The first two vectors from a matrix decomposition of the Netflix Prize data. Selected movies are placed at the appropriate spot based on their factor vectors in two dimensions. The plot reveals distinct genres, including clusters of movies with strong female leads, fraternity humor, and quirky independent films.

Figure from Koren et al. (2009)

Matrix Factorization

Comparison of Optimization Algorithms

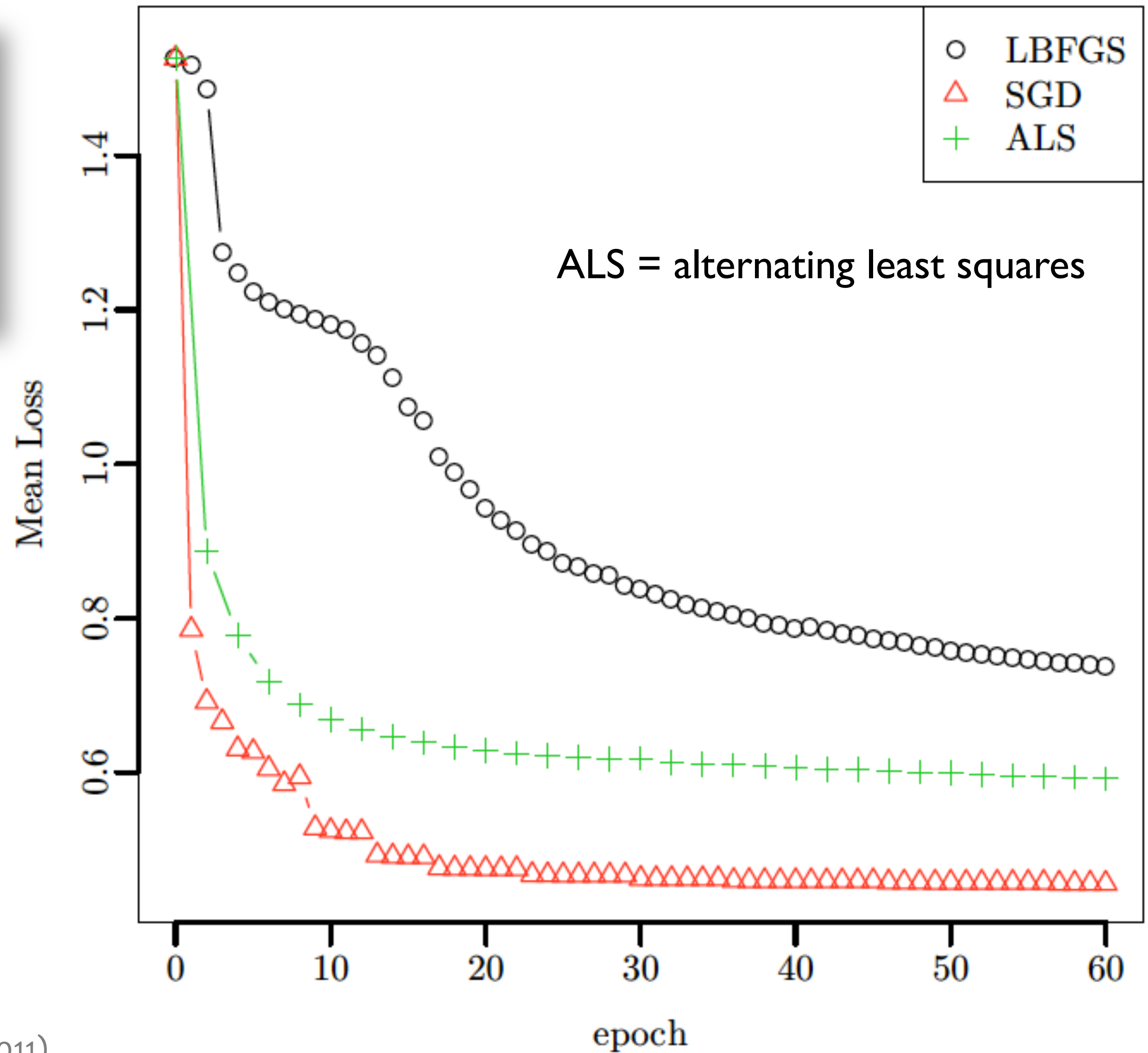


Figure from Gemulla et al. (2011)

SVD FOR COLLABORATIVE FILTERING

Singular Value Decomposition for Collaborative Filtering

For any arbitrary matrix $\mathbf{A}$, SVD gives a decomposition:

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^T$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix, and $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices.

Suppose we have the SVD of our ratings matrix

$$R = Q\Sigma P^T,$$

but then we truncate each of Q, Σ, and P s.t. Q and P have only k columns and Σ is $k \times k$:

$$R \approx Q_k \Sigma_k P_k^T$$

For collaborative filtering, let:

$$U \triangleq Q_k \Sigma_k$$

$$V \triangleq P_k$$

$$\Rightarrow U, V = \underset{U,V}{\operatorname{argmin}} \frac{1}{2} ||R - UV^T||_2^2$$

s.t. columns of U are mutually orthogonal

s.t. columns of V are mutually orthogonal

Theorem: If *R* fully observed and no regularization, the optimal UV^T from SVD equals the optimal UV^T from Unconstrained MF

NON-NEGATIVE MATRIX FACTORIZATION

Implicit Feedback Datasets

- What information does a five-star rating contain?

- Implicit Feedback Datasets:
  - In many settings, users don't have a way of expressing *dislike* for an item (e.g. can't provide negative ratings)
  - The only mechanism for feedback is to "like" something
- Examples:
  - Facebook has a "Like" button, but no "Dislike" button
  - Google's "+1" button
  - Pinterest pins
  - Purchasing an item on Amazon indicates a preference for it, but there are many reasons you might *not* purchase an item (besides dislike)
  - Search engines collect click data but don't have a clear mechanism for observing dislike of a webpage

Examples from Aggarwal (2016)

Non-negative Matrix Factorization

Constrained Optimization Problem:

$$U, V = \underset{U,V}{\operatorname{argmin}} \frac{1}{2} ||R - UV^T||_2^2$$

$$\text{s.t. } U_{ij} \geq 0$$

$$\text{s.t. } V_{ij} \geq 0$$

Multiplicative Updates: simple iterative algorithm for solving just involves multiplying a few entries together

Fighting Fire with Fire: Using Antidote Data to Improve Polarization and Fairness of Recommender Systems

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where $\mathbf{S}_j = \sum_{i \in \Omega_j} \mathbf{u}_i \mathbf{u}_i^\top + \tilde{\mathbf{U}}\tilde{\mathbf{U}}^\top + \lambda \mathbf{I}_\ell$.

By using (9) instead of the general formula in (5) we can significantly reduce the number of computations required for finding the gradient of the utility function with respect to the antidote data. Furthermore, the term $\mathbf{g}_j{}^\top \mathbf{U}^\top \mathbf{S}_j^{-1}$ appears in all the partial derivatives that correspond to elements in column j of $\tilde{\mathbf{X}}$ and can be precomputed in each iteration of the algorithm and reused for computing partial derivatives with respect to different antidote users.

5 SOCIAL OBJECTIVE FUNCTIONS

The previous section developed a general framework for improving various properties of recommender systems; in this section we show how to apply that framework specifically to issues of polarization and fairness.

As described in Section 2, polarization is the degree to which opinions, views, and sentiments diverge within a population. Recommender systems can capture this effect through the ratings that they present for items. To formalize this notion, we define polarization in terms of the variability of predicted ratings when compared across users. In fact, we note that both very high variability, and very low variability of ratings may be undesirable. In the case of high variability, users have strongly divergent opinions, leading to conflict. Recent analyses of the YouTube recommendation system have suggested that it can enhance this effect [29, 30]. On the other hand, the convergence of user preferences, i.e., very low variability of ratings given to each item across users, corresponds to increased homogeneity, an undesirable phenomenon that may occur as users interact with a recommender system [11]. As a result, in what follows we consider using antidote data in both ways: to either increase or decrease polarization.

As also described in Section 2, unfairness is a topic of growing interest in machine learning. Following the discussion in that section, we consider a recommender system fair if it provides equal quality of service (i.e., prediction accuracy) to all users or all groups of users [36].

Next we formally define the metrics that specify the objective functions associated with each of the above objectives. Since the gradient of each objective function is used in the optimization algorithm, for reproducibility we provide the details about derivation of the gradients in appendix A.2.

5.1 Polarization

To capture polarization, we seek to measure the extent to which the user ratings *disagree*. Thus, to measure user polarization we consider the estimated ratings $\hat{\mathbf{X}}$, and we define the polarization metric as the normalized sum of pairwise euclidean distances between estimated user ratings, i.e., between rows of $\hat{\mathbf{X}}$. In particular:

$$R_{pol}(\hat{\mathbf{X}}) = \frac{1}{n^2 d} \sum_{k=1}^{n} \sum_{l>k} \|\hat{\mathbf{x}}^k - \hat{\mathbf{x}}^l\|^2 \tag{10}$$

The normalization term $\frac{1}{n^2 d}$ in (10) makes the polarization metric identical to the following definition: [4]

$$R_{pol}(\hat{\mathbf{X}}) = \frac{1}{d} \sum_{j=1}^{d} \sigma_j^2 \tag{11}$$

where σ_j^2 is the variance of estimated user ratings for item j. Thus this polarization metric can be interpreted either as the average of the variances of estimated ratings in each item, or equivalently as the average user disagreement over all items.

5.2 Fairness

Individual fairness. For each user i, we define ℓ_i, the loss of user i, as the mean squared estimation error over known ratings of user i:

$$\ell_i = \frac{\|P_{\Omega^i}(\hat{\mathbf{x}}^i - \mathbf{x}^i)\|_2^2}{|\Omega^i|} \tag{12}$$

Then we define the individual unfairness as the variance of the user losses:[5]

$$R_{indv}(\mathbf{X}, \hat{\mathbf{X}}) = \frac{1}{n^2} \sum_{k=1}^{n} \sum_{l>k} (\ell_k - \ell_l)^2 \tag{13}$$

To improve individual fairness, we seek to minimize R_{indv}.

Group fairness. Let I be the set of all users/items and $G = \{G_1 \dots, G_g\}$ be a partition of users/items into g groups, i.e., $I = \bigcup_{i \in \{1,\dots,g\}} G_i$. We define the loss of group i as the mean squared estimation error over all known ratings in group i:

$$L_i = \frac{\|P_{\Omega_{G_i}}(\hat{\mathbf{X}} - \mathbf{X})\|_2^2}{|\Omega_{G_i}|} \tag{14}$$

For a given partition G, we define the group unfairness as the variance of all group losses:

$$R_{grp}(\mathbf{X}, \hat{\mathbf{X}}, G) = \frac{1}{g^2} \sum_{k=1}^{g} \sum_{l>k} (L_k - L_l)^2 \tag{15}$$

Again, to improve group fairness, we seek to minimize R_{grp}.

5.3 Accuracy vs. Social Welfare

Adding antidote data to the system to improve a social utility will also have an effect on the overall prediction accuracy. Previous works have considered social objectives as regularizers or constraints added to the recommender model (eg, [8, 25, 37]), implying a trade-off between the prediction accuracy and a social objective.

However, in the case of the metrics we define here, the relationship is not as simple. Considering polarization, we find that in general, increasing or decreasing polarization will tend to decrease system accuracy. In either case we find that system accuracy only declines slightly in our experiments; we report on the specific values in Section 6. Considering either individual or group unfairness, the situation is more subtle. Note that our unfairness metrics will be exactly zero for a system with zero error (perfect accuracy). As a

[4] We can derive it by rewriting (10) as $R_{pol}(\hat{\mathbf{X}}) = \frac{1}{d} \sum_{j=1}^{d} \frac{1}{n^2} \sum_{k=1}^{n} \sum_{l>k} (\hat{x}_{kj} - \hat{x}_{lj})^2$.

[5] Note that for a set of equally likely values $x_1, \dots, x_n$ the variance can be expressed without referring to the mean as: $\frac{1}{n^2} \sum_i \sum_{j>i} (x_i - x_j)^2$.

Summary

- Recommender systems solve many **real-world** (*large-scale) **problems**
- Collaborative filtering by Matrix Factorization (MF) is an **efficient** and **effective** approach
- MF is just another example of a **common recipe**:
  1. define a model
  2. define an objective function
  3. optimize with your favorite black box optimizer
     (e.g. SGD, Gradient Descent, Block Coordinate Descent aka. Alternating Least Squares)

Learning Objectives

Recommender Systems

You should be able to…

1. Compare and contrast the properties of various families of recommender system algorithms: content filtering, collaborative filtering, neighborhood methods, latent factor methods
2. Formulate a squared error objective function for the matrix factorization problem
3. Implement unconstrained matrix factorization with a variety of different optimization techniques: gradient descent, stochastic gradient descent, alternating least squares
4. Offer intuitions for why the parameters learned by matrix factorization can be understood as user factors and item factors

EXTRA SLIDES ON UMF

Unconstrained Matrix Factorization

In-Class Exercise

Derive a block coordinate descent algorithm for the Unconstrained Matrix Factorization problem.

- User vectors:

$$\mathbf{w}_u \in \mathbb{R}^r$$

- Set of non-missing entries

$$\mathcal{Z} = \{(u, i) : v_{ui} \text{ is observed}\}$$

- Item vectors:

$$\mathbf{h}_i \in \mathbb{R}^r$$

- Objective:

$$\underset{\mathbf{w},\mathbf{h}}{\operatorname{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$

- Rating prediction:

$$v_{ui} = \mathbf{w}_u^T \mathbf{h}_i$$

Matrix Factorization
(with matrices)

- User vectors:

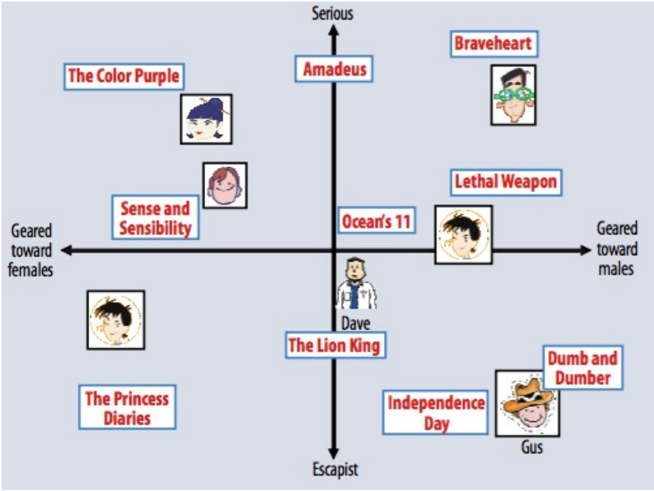
$$(W_{u*})^T \in \mathbb{R}^r$$

- Item vectors:

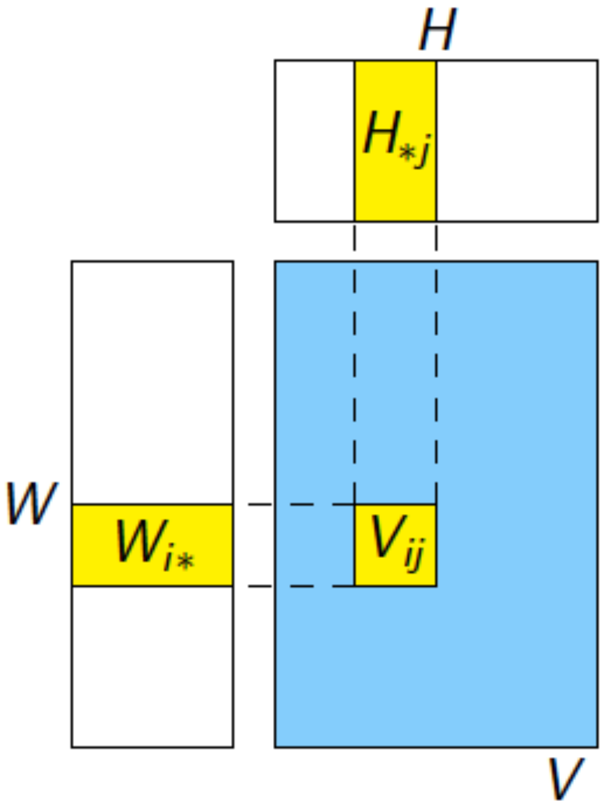
$$H_{*i} \in \mathbb{R}^r$$

- Rating prediction:

$$\begin{aligned} V_{ui} &= W_{u*} H_{*i} \\ &= [WH]_{ui} \end{aligned}$$



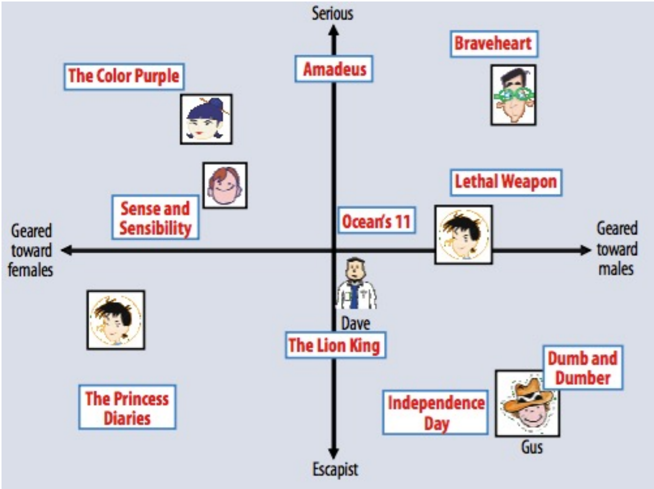
Figures from Koren et al. (2009)



Figures from Gemulla et al. (2011)

Matrix Factorization
(with vectors)

- User vectors:
$$\mathbf{w}_u \in \mathbb{R}^r$$
- Item vectors:
$$\mathbf{h}_i \in \mathbb{R}^r$$
- Rating prediction:
$$v_{ui} = \mathbf{w}_u^T \mathbf{h}_i$$



Figures from Koren et al. (2009)

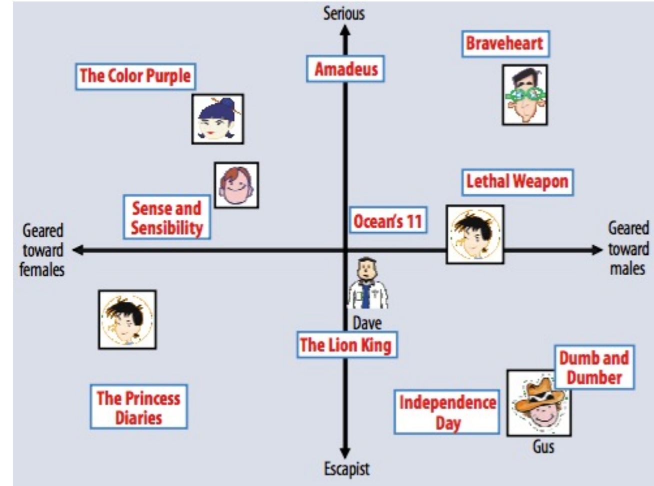
Matrix Factorization
(with vectors)

- Set of non-missing entries:

$$\mathcal{Z} = \{(u,i) : v_{ui} \text{ is observed}\}$$

- Objective:

$$\underset{\mathbf{w},\mathbf{h}}{\operatorname{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$



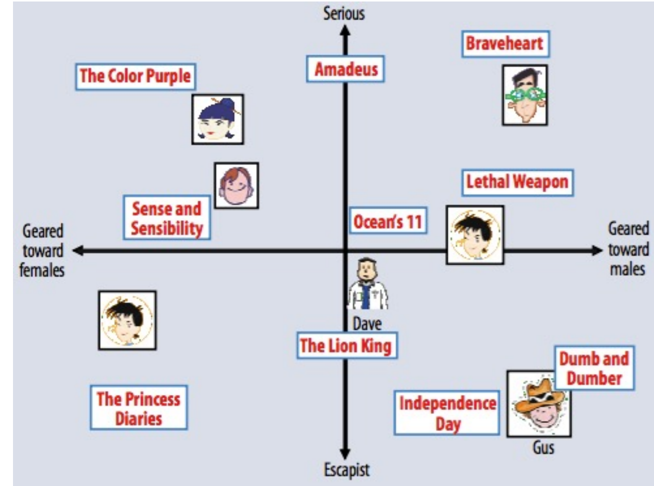
Figures from Koren et al. (2009)

Matrix Factorization
(with vectors)

- Regularized Objective:

$$\underset{\mathbf{w},\mathbf{h}}{\text{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$

$$+ \lambda(\sum_i ||\mathbf{w}_i||^2 + \sum_u ||\mathbf{h}_u||^2)$$



Figures from Koren et al. (2009)

Matrix Factorization (with vectors)

- Regularized Objective:

$$\underset{\mathbf{w},\mathbf{h}}{\operatorname{argmin}} \sum_{(u,i)\in\mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$

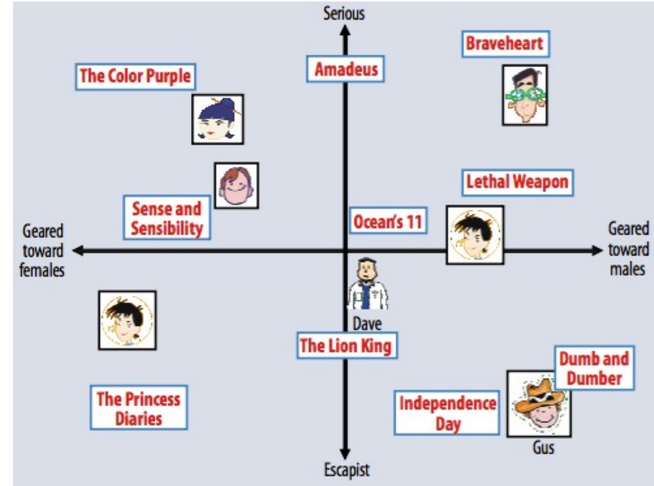
$$+ \lambda(\sum_i ||\mathbf{w}_i||^2 + \sum_u ||\mathbf{h}_u||^2)$$

- SGD update for random (u,i):

$$e_{ui} \leftarrow v_{ui} - \mathbf{w}_u^T \mathbf{h}_i$$

$$\mathbf{w}_u \leftarrow \mathbf{w}_u + \gamma(e_{ui}\mathbf{h}_i - \lambda \mathbf{w}_u)$$

$$\mathbf{h}_i \leftarrow \mathbf{h}_i + \gamma(e_{ui}\mathbf{w}_u - \lambda \mathbf{h}_i)$$



Figures from Koren et al. (2009)

Matrix Factorization

(with matrices)

- User vectors:

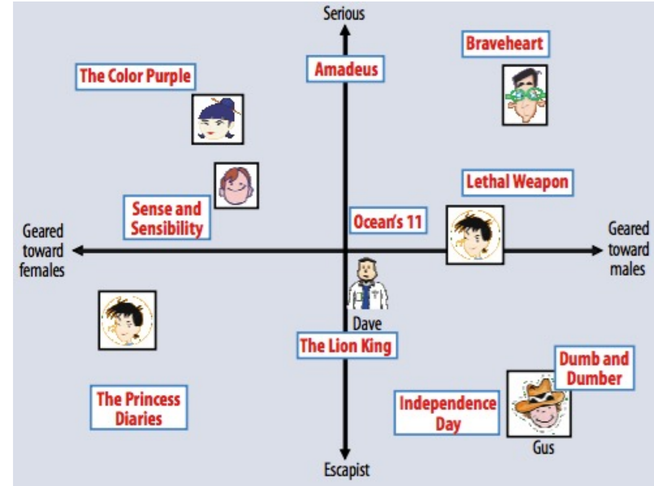
$$(W_{u*})^T \in \mathbb{R}^r$$

- Item vectors:

$$H_{*i} \in \mathbb{R}^r$$

- Rating prediction:

$$V_{ui} = W_{u*}H_{*i}$$
$$= [WH]_{ui}$$



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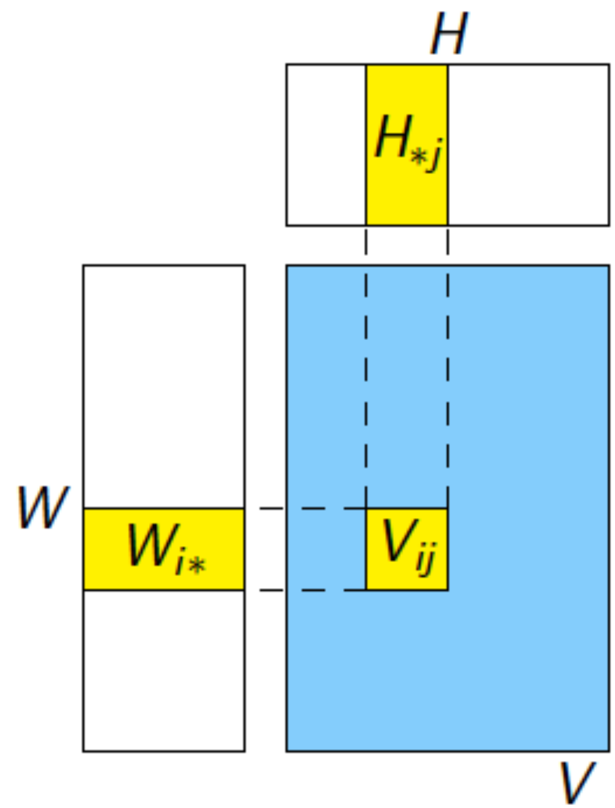


Figures from Gemulla et al. (2011)

Matrix Factorization

(with matrices)

- SGD

require that the loss can be written as

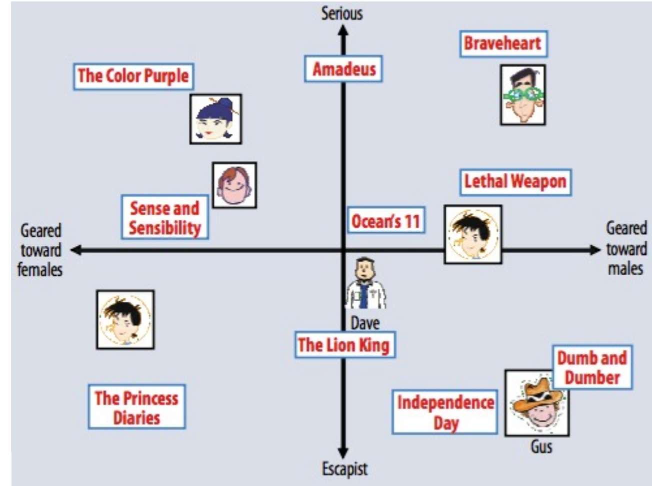
$$L = \sum_{(i,j) \in Z} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$$

Algorithm 1 SGD for Matrix Factorization

Require: A training set Z, initial values $\boldsymbol{W}_0$ and $\boldsymbol{H}_0$
  while not converged **do** {step}
    Select a training point $(i, j) \in Z$ uniformly at random.
    $\boldsymbol{W}'_{i*} \leftarrow \boldsymbol{W}_{i*} - \epsilon_n N \frac{\partial}{\partial \boldsymbol{W}_{i*}} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$
    $\boldsymbol{H}_{*j} \leftarrow \boldsymbol{H}_{*j} - \epsilon_n N \frac{\partial}{\partial \boldsymbol{H}_{*j}} l(\boldsymbol{V}_{ij}, \boldsymbol{W}_{i*}, \boldsymbol{H}_{*j})$
    $\boldsymbol{W}_{i*} \leftarrow \boldsymbol{W}'_{i*}$
  end while

step size

Figure from Gemulla et al. (2011)



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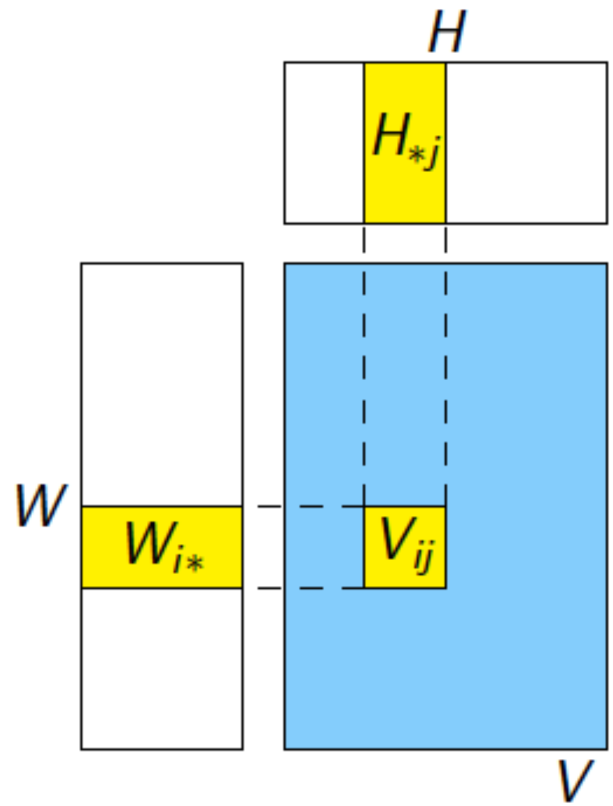

Figure from Gemulla et al. (2011)