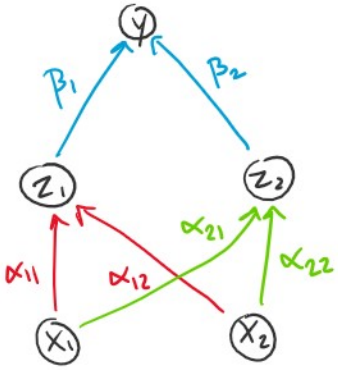
Section B

Wednesday, February 22, 2023 12:26 PM

Neural Network Models

Ex #1: NN w/ 1 hidden layer and 2 hidden units



(output)

(hidden)

(input)

Let σ be the activation function

If σ is sigmoid: $\sigma(a) = \frac{1}{1+\exp(-a)}$

$$z_1 = \sigma(\alpha_{11} x_1 + \alpha_{12} x_2 + \alpha_{10})$$

$$z_2 = \sigma(\alpha_{21} x_1 + \alpha_{22} x_2 + \alpha_{20})$$

Each is a "Log. Reg. Model" function

$$y = \sigma(\beta_1 z_1 + \beta_2 z_2 + \beta_0)$$

$$= \sigma(\beta_1 \sigma(\alpha_{11} x_1 + \alpha_{12} x_2 + \alpha_{10}) + \beta_2 \sigma(\alpha_{21} x_1 + \alpha_{22} x_2 + \alpha_{20}) + \beta_0)$$

Don't forget the intercept terms!

$x_m \in \mathbb{R}$

$z_i \in [0,1]$ if σ is sigmoid

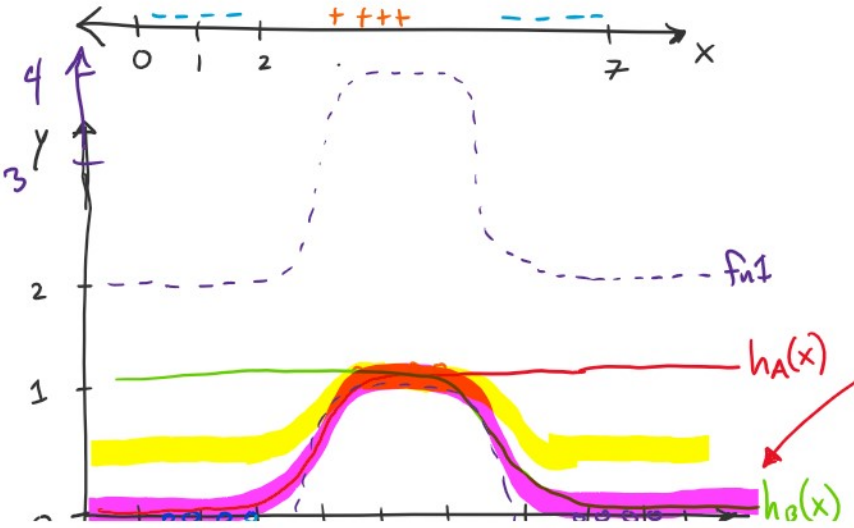
$z_i \in \mathbb{R}$ more generally

$$\Pr[Y=1 \mid \vec{x}, \alpha, \beta] \triangleq y$$

To make a prediction, use the Bayes Optimal Classifier:

$$\hat{y} = h_{\alpha,\beta}(\vec{x}) = \begin{cases} 1 & \text{if } y \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

1D Face Recognition

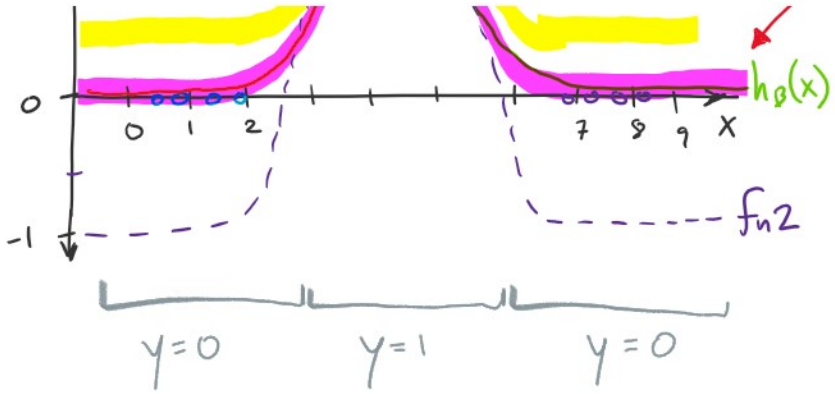


Q: What is $z_c(x) = \frac{h_A(x) + h_B(x)}{2}$ (fn1)

Q: What is

$h_c(x) = \sigma(w_A h_A(x) + w_B h_B(x) + w_0)$ (fn2)

where $w_A = 2 \quad w_B = 2 \quad w_0 = -3$


0
-1
0
1
2
7
8
9
X
$h_\beta(x)$
f_{n2}
y=0
y=1
y=0