

Chain Rule

Def #1:

$$y = f(u)$$

$$u = g(x)$$

Computation Graph



Def

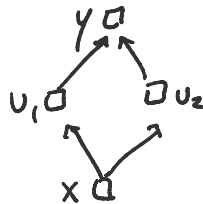
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Def #2:

$$y = f(u_1, u_2)$$

$$u_2 = g_2(x)$$

$$u_1 = g_1(x)$$

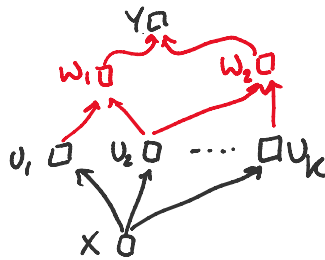


$$\frac{dy}{dx} = \frac{dy}{du_1} \frac{du_1}{dx} + \frac{dy}{du_2} \frac{du_2}{dx}$$

Def #3:

$$y = f(\vec{u})$$

$$\vec{u} = g(x)$$



$$\frac{dy}{dx} = \sum_{k=1}^K \frac{dy}{du_k} \frac{du_k}{dx}$$

Q: Does this still hold?

A: Yes!

Backprop Ex #1

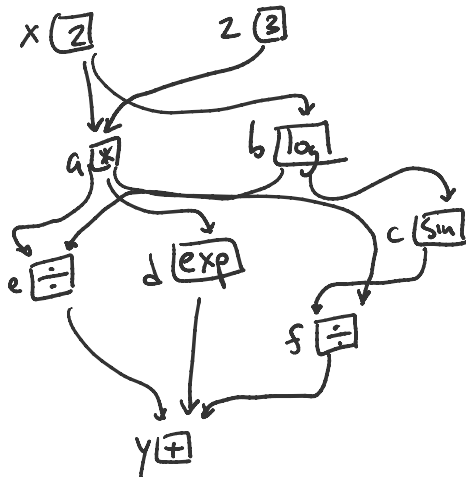
$$y = f(x, z) = \exp(xz) + \frac{xz}{\log(x)} + \frac{\sin(\log(x))}{xz}$$

Forward Computation

Given $x=2, z=3$

- $a = xz$
- $b = \log(x)$
- $c = \sin(b)$
- $d = \exp(a)$
- $e = a/b$
- $f = c/a$
- $y = d + e + f$

Computation Graph



Backward Computation

$$g_y = \frac{dy}{dy} = 1$$

$$g_f = \frac{dy}{df}$$

$$g_c = \frac{dy}{dc}$$

...

$$g_a = \frac{dy}{da}$$

$$J = \gamma/a$$

$$y = d + e + f$$

$y \oplus$

$$g_1 = \frac{\delta y}{\delta a}$$

$$g_x = \frac{\delta y}{\delta x}$$

$$g_z = \frac{\delta y}{\delta z}$$