# Section B

Friday, March 24, 2023 11:46 AM

Ex: Tunnel Closures

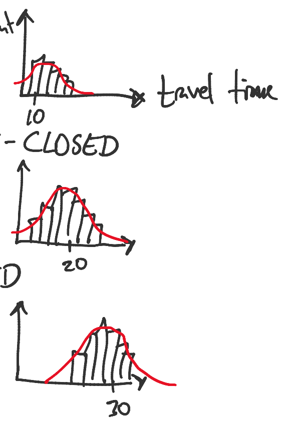
$x_t$ = Totoro's travel time on day $t$
$y_t$ = state of the tunnel on day $t$

T = 365 days of data

$y_t$ = OPEN

$y_t$ = SEMI-CLOSED

$y_t$ = CLOSED




$p(x_t | y_t)$

$$\hat{p}_{OPEN} = \frac{\#(y_t = OPEN)}{T}$$

$$\hat{p}_{SC} = \frac{\#(y_t = SC)}{T}$$

$$\hat{p}_{C} = \frac{\#(y_t = C)}{T}$$

$p(y_t)$

Gaussian Naive Bayes

Model: $\underbrace{p(x_t, y_t)} = p(x_t | y_t)\, p(y_t)$

## 1st Order Markov Assumption

Let $y_t$ be the state of the system at time $t$

$$p(y_t | y_{t-1}, y_{t-2}, \dots, y_1) = p(y_t | y_{t-1}) \leftarrow \text{by assumption}$$

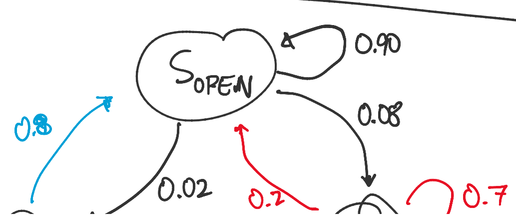
This means:

① $\Rightarrow y_t \perp\!\!\!\perp y_j \mid y_{t-1}, \ \forall j \le t-2$

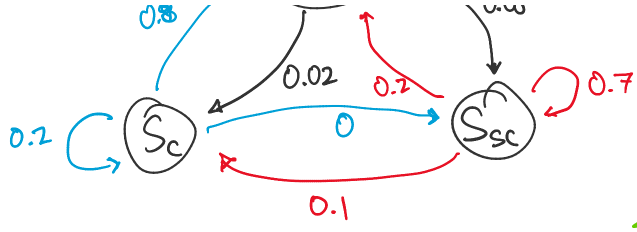
② $\Rightarrow p(y_1, \dots, y_T) = \prod_{t=1}^{T} p(y_t | y_{t-1}, \dots, y_1)$ by Chain Rule

$$= \prod_{t=1}^{T} p(y_t | y_{t-1}) \quad \text{by assumption}$$

## 1st Ord. Mark. Assump. as Finite State Machine




$p(y_t | y_{t-1})$




$p(y_t | y_{t-1})$

OH

Color $= x_1$  Size $= x_2$  HI $= y$

$$p(x_1 = Cy,\ x_2 = Med,\ y = N) = p(y = N)\, p(x_1 = Cy | y = N)\, p(x_2 = Med | y = N)$$

$$p(x_1, x_2, y) = p(y)\, p(x_1 | y)\, p(x_2 | y)$$

$$p(x_1 = Rain \mid y = N) = \theta_{x_1 = Rain,\ y = N}$$

| A | B | C | y |
|---|---|---|---|
| 0 | 0 | 1 | + |
| 1 | 0 | 1 | + |
| 0 | 1 | 1 | − |
| 1 | 1 | 0 | − |
| 2 | 2 | 2 | + |

$p(A | y = +) = \text{Gaussian}(\mu_{+,A}, \sigma^2_{+,A})$

$p(A | y = -) = \text{Gau}(\mu_{-,A}, \sigma^2_{-,A})$

$p(B | y = +)$

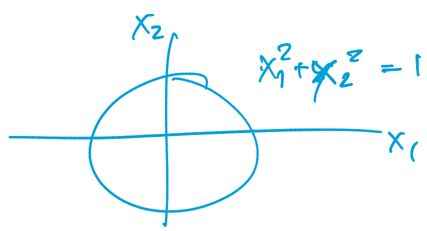
$p(B | y = -)$

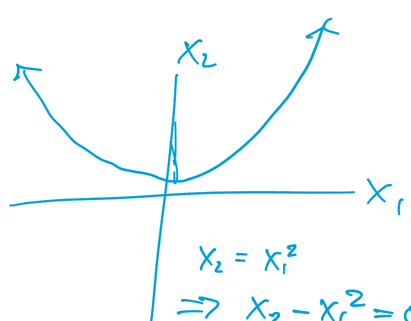
$$p(A = 0 | y = +) = \frac{\#(A = 0, y = +)}{\#(y = +)}$$

$$p(y = +) = \frac{\#(y = +)}{N}$$

$p(y = -) =$

$p(y = ?) =$



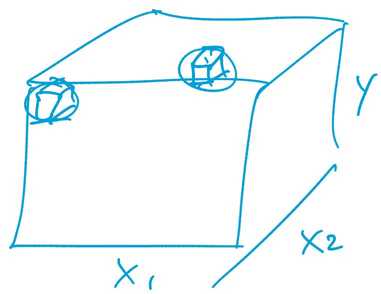

$x_2 = x_1^2$

$\Rightarrow x_2 - x_1^2 = 0$

$p(y = Y \mid \cancel{x_1} = Cy,\ \cancel{x_2} = Med)$

$J \to x_2 - x_1 = 0$

$$\frac{p(y=Y \mid \cancel{x_1} = Cy, \cancel{x_2} = Med) + p(y=N \mid x_2 = Cy, x_2\ Med)}{1.0}$$



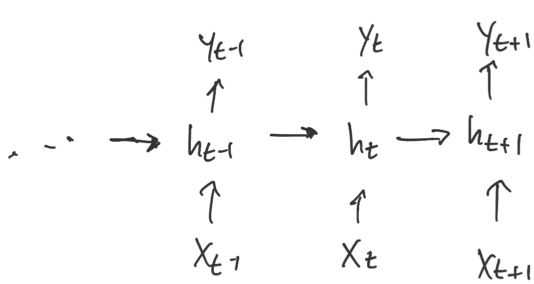

$$p(y=Y, x_1=Cy, x_2=Med) + p(y=N, x_1=Cy, x_2=Med) \leq 1.0$$

- $p(y=N) = \dfrac{\#(y=N)}{\#\ \text{rows}}$
- $p(x_1=Cy \mid y=N) = \dfrac{\#(x_1=Cy \wedge y=N)}{\#(y=N)}$
- $p(x_2=Med \mid y=N) = \dfrac{\#(\ )}{\#(\ )}$

$$\frac{\partial}{\partial \gamma}\left(\gamma^2 \exp(-2\gamma)\right) = 2\gamma \exp(-2\gamma) + \gamma^2(-2)\exp(-2\gamma)$$

$$= 0$$

$$\gamma - \gamma^2 = 0$$




$y_t$

$X_{t-1}$ $\quad\quad$ $X_t$ $\quad\quad$ $X_{t+1}$

$$\begin{array}{c} y_t \\ \uparrow \\ h_t \\ \uparrow \\ x_t \end{array}$$

Sum $\dfrac{\partial J_t}{\partial W_{hh}}$

$$J = \sum_{t=1}^{T} J_t$$

$$J_t = \ell\left(y_t^{(i)}, y_t\right)$$

$$= \underbrace{\sum_{i=1}^{2} y_{t,i} \log(\hat{y}_{t,i})}$$