Assignment 2

due Wednesday, September 25, 2024

The second homework is due at 9pm on Wednesday September 25, 2024. Please submit your homework via Gradescope, see https://www.gradescope.com/courses/862231.

Question 1

(a) [5 points] Write a DIMACS generator `colorA` (n) for the problem that there exists a 2-coloring of the positive numbers 1 to n such that for every integer solution of $a^2 + b^2 = c^2$ with $1 \leq a < b < c \leq n$ holds that a, b, and c do *not* all have the same color. (Hint 1: use a Boolean variable x_i for each integer. Assigning x_i to true means that i has the first color, while assigning it to false means that i has the second color. Hint 2: not all variables are required).

(b) [8 points] Use the above generator to construct `colorA` (40) and remove all clauses using blocked clause elimination. In which order can the clauses be removed? Note that there are multiple such orders. Put the blocking literal as the first literal in each eliminated clause.

(c) [4 points] Now, start with the assignment that makes all variables false. Construct a solution of `colorA` (40) by using the order in which blocked clauses were eliminated. List the variables that are assigned to true in the solution. (Hint: follow the procedure from the "Solution Reconstruction" slide of the Preprocessing lecture).

(d) [8 points] Again use `colorA` (40). Eliminate all clauses using variable elimination (either by distribution or by substitution). (Hint: the order in which blocked clauses are removed is an effective heuristic for variable elimination.)

Question 2

(a) [5 points] Write a DIMACS generator `colorB` (n) for the problem that there exists a 2-coloring of the positive numbers 1 to n such that for every integer solution of $a + b = c$ with $1 \leq a < b < c \leq n$ holds that a, b, and c do *not* all have the same color. (Hint: use a Boolean variable x_i for each integer. Assigning x_i to true means that i has the first color, while assigning it to false means that i has the second color).

(b) [10 points] Use the above generator to construct `colorB` (7). Consider the initial steps of a CDCL solver in which the first decision is $x_1 = 1$ and the second decision is $x_3 = 1$. Now unit propagation results in a conflict. Draw an *implication graph* and compute the first unique implication point of that graph. This question can be answered by scanning in a drawing of the implication graph.

(c) [10 points] Now use `colorB` (9) and construct a DPLL tree by hand (no implementation) to show that this formula is unsatisfiable. Recall that the vertices of DPLL only consist of decision variables and not variables implied by unit propagation. Only use unit propagation and chronological backtracking (i.e., no clause learning).

Bonus Question

Consider the DIMACS formula shown below with 11 variables and 20 clauses, also available on http://www.cs.cmu.edu/~mheule/15816-f24/bonusQ.cnf. Reduce the size of the formula by introducing new variables. The reduced formula should have the same set of solutions over the common (i.e., non auxiliary) variables. The number of points for a correct answer is 31 (11+20) minus the sum of the number of variables and the number of clauses of the reduced formula.

```
p cnf 11 20
-1 -2 -3  4 -5  6 -7 -8 -9 -10  11 0
-1 -2  3 -4 -5  6 -7 -8 -9 -10  11 0
-1 -2 -3  4  5 -6 -7 -8 -9 -10  11 0
-1 -2  3 -4  5 -6 -7 -8 -9 -10  11 0
-1 -2 -3  4 -5  6 -7 -8 -9  10 -11 0
-1 -2  3 -4 -5  6 -7 -8 -9  10 -11 0
-1 -2 -3  4  5 -6 -7 -8 -9  10 -11 0
-1 -2  3 -4  5 -6 -7 -8 -9  10 -11 0
-1 -2 -3  4 -5  6  7  8 -9 0
-1 -2  3 -4 -5  6  7  8 -9 0
-1 -2 -3  4  5 -6  7  8 -9 0
-1 -2  3 -4  5 -6  7  8 -9 0
 1  2 -3  4 -5  6 -7 -8  9 0
 1  2  3 -4 -5  6 -7 -8  9 0
 1  2 -3  4  5 -6 -7 -8  9 0
 1  2  3 -4  5 -6 -7 -8  9 0
 1  2 -3  4 -5  6  7  8  9 0
 1  2  3 -4 -5  6  7  8  9 0
 1  2 -3  4  5 -6  7  8  9 0
 1  2  3 -4  5 -6  7  8  9 0
```