SAT and SMT Solvers in Practice

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https://github.com/marijnheule/sat-examples.git

Automated Reasoning and Satisfiability

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DIMACS: SAT solver input format

The DIMACS format for SAT solvers has three types of lines:

- header: `p cnf n m` in which `n` denotes the highest variable index and `m` the number of clauses
- clauses: a sequence of integers ending with "0"
- comments: any line starting with "c "

	`c example`
	`p cnf 4 7`
$(a \vee b \vee \overline{c}) \wedge$	`1 2 -3 0`
$(\overline{a} \vee \overline{b} \vee c) \wedge$	`-1 -2 3 0`
$(b \vee c \vee \overline{d}) \wedge$	`2 3 -4 0`
$(\overline{b} \vee \overline{c} \vee d) \wedge$	`-2 -3 4 0`
$(a \vee c \vee d) \wedge$	`1 3 4 0`
$(\overline{a} \vee \overline{c} \vee \overline{d}) \wedge$	`-1 -3 -4 0`
$(\overline{a} \vee b \vee d)$	`-1 2 4 0`

DIMACS: SAT solver output format

The solution line of a SAT solver starts with "`s` ":

- `s SATISFIABLE`: The formula is satisfiable
- `s UNSATISFIABLE`: The formula is unsatisfiable
- `s UNKNOWN`: The solver cannot determine satisfiability

In case the formula is satisfiable, the solver emits a certificate:

- lines starting with "`v` "
- a list of integers ending with 0
- e.g. `v -1 2 4 0`

In case the formula is unsatisfiable, then most solvers support emitting a proof of unsatisfiability to a separate file

CaDiCaL: download and install

Most SAT solvers are implemented in C/C++

CaDiCaL is one of the strongest SAT solvers. As the name suggests it is based on CDCL. Recommended for Linux and macOS users.

obtain CaDiCaL:

- `git clone https://github.com/arminbiere/cadical.git`
- cd cadical
- ./configure; make

to run: `./build/cadical` formula.cnf

Kissat: download and install

Most SAT solvers are implemented in C/C++

Kissat is successor of CaDiCaL and it is written in C.
Recommended for Linux and macOS users.

obtain Kissat:

- `git clone https://github.com/arminbiere/kissat.git`
- cd kissat
- ./configure; make

to run: `./build/kissat` formula.cnf

SAT4J: download and install

SAT4J is a SAT solver in Java. It is also based on CDCL.
Recommended for windows users.

obtain SAT4J:

- `git clone https://github.com/marijnheule/sat-examples.git`
- cd sat-examples

to run: `java -jar org.sat4j.core-2.3.1.jar` formula.cnf

UBCSAT: download and install

UBCSAT is a collection of local search SAT solvers.

obtain UBCSAT:

- download and unzip `http://ubcsat.dtompkins.com/downloads/ubcsat-beta-12-b18.tar.gz`
- cd ubcsat-beta-12-b18
- make clean; make

to run: `./ubcsat -alg ddfw -i` formula.cnf

there are many LS algorithms to choose from (`-alg`)
`./ubcsat -ha` (shows the available algorithms)

YalSAT: download and install

YalSAT: **y**et **a**nother **l**ocal search **SAT** solver:

obtain YalSAT:

- `git clone https://github.com/arminbiere/yalsat.git`
- cd yalsat
- ./configure.sh; make

to run: `./yalsat` formula.cnf

A powerful local search solver from the author of CaDiCaL and Kissat

Many SAT solvers

Many SAT solvers have been developed

Lots of them participate in the annual SAT competition

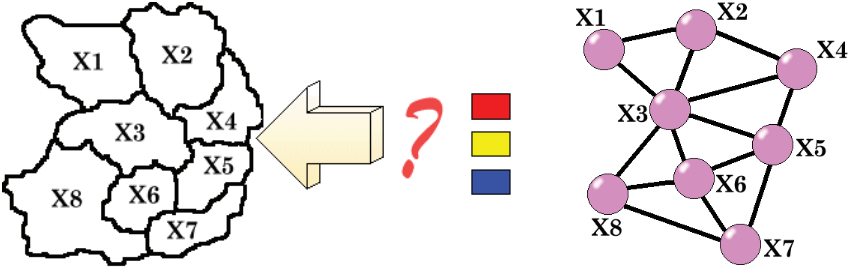
- All code of participants in open source
- Each solver is run on hundreds of benchmarks
- Large timeout 5000 seconds

For details and downloading more solvers visit
`http://satcompetition.org/`

Demo: SAT Solving

Graph coloring

Given a graph $G(V, E)$, can the vertices be colored with k colors such that for each edge $(v, w) \in E$, the vertices v and w are colored differently.



Graph coloring encoding

Variables	Range	Meaning
$x_{v,i}$	$i \in \{1, \ldots, c\}$ $v \in \{1, \ldots, \lvert V\rvert\}$	node v has color i

Clauses	Range	Meaning
$(x_{v,1} \vee x_{v,2} \vee \cdots \vee x_{v,c})$	$v \in \{1, \ldots, \lvert V\rvert\}$	v is colored
$(\overline{x}_{v,s} \vee \overline{x}_{v,t})$	$s \in \{1, \ldots, c-1\}$ $t \in \{s+1, \ldots, c\}$	v has at most one color
$(\overline{x}_{v,i} \vee \overline{x}_{w,i})$	$(v, w) \in E$	v and w have a different color

Graph coloring encoding code

```c
#include <stdio.h>
#include <stdlib.h>

int main (int argc, char** argv) {
  FILE* graph  = fopen (argv[1], "r");
  int i, j, a, b, nVertex, nEdge, nColor = atoi  (argv[2]);
  fscanf (graph, " p edge %i %i ", &nVertex, &nEdge);

  printf ("p cnf %i %i\n", nVertex * nColor, nVertex + nEdge * nColor);

  for (i = 0; i < nVertex; i++) {
    for (j = 1; j <= nColor; j++)
      printf ("%i ", i * nColor + j);
    printf ("0\n"); }

  while (1) {
    int tmp = fscanf (graph, " e %i %i ", &a, &b);
    if (tmp == 0 || tmp == EOF) break;
    for (j = 1; j <= nColor; j++)
      printf ("-%i -%i 0\n", (a-1) * nColor + j, (b-1) * nColor + j);
  }
}
```

Demo: Encode, Decode

Unsatisfiable cores

An unsatisfiable core of an unsatisfiable formula F is a subset of F that is unsatisfiable.

An minimal unsatisfiable core of an unsatisfiable formula such that the removal of any clause makes the formula satisfiable.

Extracting a minimal unsatisfiable core from a formula has many applications, but the computational costs could be high.

- maxSAT
- diagnosis
- formal verification

Proofs

A proof of unsatisfiability is a certificate that a given formula is unsatisfiable.

Various proof producing methods exists (another lecture).

Proof checking tools cannot only validate a proof but also produce additional information about the formula:

- unsatisfiable core
- optimized proof

DRAT-trim is a tool that validates proofs and produces such information

Demo: Core Extraction

StarExec

StarExec is a cross community logic solving service

- Great to evaluate solvers/heuristics in parallel
- Also used to run the SAT/SMT competitions

Register at `https://www.starexec.org/`

- select SAT as your community

Demo: StarExec

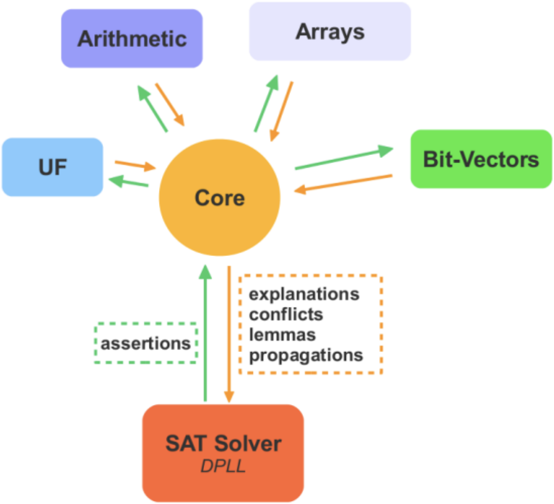
Tools for making SAT-based modeling easier

PySAT is a Python toolkit that makes it easier for users to call SAT solvers and build encodings using Python:

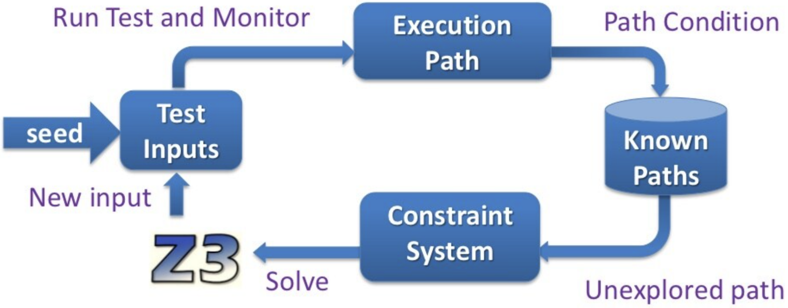
- `https://pysathq.github.io/`
- SAT solver is still written in C, C++
- Interface includes several encodings for linear constraints:
  - At-most-one constraints
  - Cardinality constraints
  - AIGER circuits to CNF
  - ...
- Well documented
- Active development

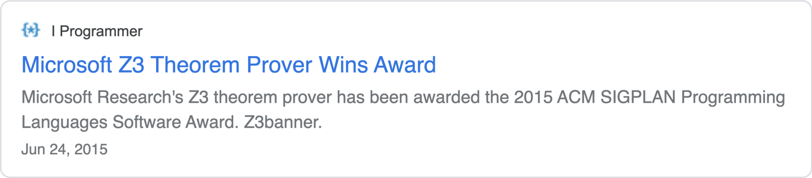
Demo: PySAT

Satisfiability Modulo Theories (SMT)



SMT at Microsoft: Test Input Generation




I Programmer

Microsoft Z3 Theorem Prover Wins Award

Microsoft Research's Z3 theorem prover has been awarded the 2015 ACM SIGPLAN Programming Languages Software Award. Z3banner.

Jun 24, 2015

SMT at Amazon Web Services: Provable Security

Automated reasoning versus machine learning: How AWS provides secure access control without data

VIDEO EXCLUSIVE BY BETSY AMY-VOGT

SMT-LIB: SMT solver input format (I)

http://smtlib.cs.uiowa.edu/

Language has similarities with functional languages and it is more readable than CNF. Theories:

- Arrays,
- Bitvectors,
- Boolean predicates,
- Floating point,
- Ints,
- Reals

SMT-LIB: SMT solver input format (II)

```
; Basic Boolean example
(set-logic QF_UF)
(declare-const p Bool)
(assert (and p (not p)))
(check-sat) ; returns 'unsat'
(exit)
```

SMT-LIB: SMT solver input format (III)

```
; Integer arithmetic
(set-logic QF_LIA)
(declare-const x Int)
(declare-const y Int)
(assert (=  (- x y) (+ x (- y) 1)))
(check-sat) ; returns 'unsat'
(exit)
```

SMT Solvers

- Z3 (Microsoft):
  `https://github.com/Z3Prover/z3/wiki`
- CVC5 (Stanford): `https://cvc5.github.io/`
- Yices (SRI): `http://yices.csl.sri.com/`
- Bitwuzla (Stanford): `https://bitwuzla.github.io/`

SMT Solvers

We recommend the use of Z3:

- Tutorial: `https://theory.stanford.edu/~nikolaj/programmingz3.html`
- APIs for Python, C++, Java
- MIT License: `https://github.com/Z3Prover/z3`
- Most used and cited SMT solver (>9,500 citations)

Proving program equivalence in SMT

```
int power3(int in)
{
    int i, out_a;
    out_a = in;
    for (i = 0; i < 2; i++)
      out_a = out_a * in;
  return out_a;
}
```

```
int power3_new(int in)
{
    int out_b;

    out_b = (in * in) * in;

    return out_b;
}
```

$$\varphi_a \equiv (out0\_a = in0\_a) \wedge (out1\_a = out0\_a \times in0\_a) \wedge$$
$$(out2\_a = out1\_a \times in0\_a)$$
$$\varphi_b \equiv out0\_b = (in0\_b \times in0\_b) \times in0\_b$$

To show these programs are equivalent, we must show the following formula is valid: $in0\_a = in0\_b \wedge \varphi_a \wedge \varphi_b \implies out2\_a = out0\_b$

Demo: Program equivalence with SMT solving (BV)

```
(declare-fun out0_a () (_ BitVec 128))
(declare-fun out1_a () (_ BitVec 128))
(declare-fun in0_a () (_ BitVec 128))
(declare-fun out2_a () (_ BitVec 128))
(declare-fun out0_b () (_ BitVec 128))
(declare-fun in0_b () (_ BitVec 128))
(define-fun phi_a () Bool
        (and (= out0_a in0_a) ; out0_a = in0_a
                (and (= out1_a (bvmul out0_a in0_a)) ; out1_a = out0_a * in0_a
                        (= out2_a (bvmul out1_a in0_a))))) ; out2_a = out1_a * in0_a
(define-fun phi_b () Bool
        (= out0_b (bvmul (bvmul in0_b in0_b) in0_b))) ; out0_b = in0_b * in0_b * in0_b
(define-fun phi_input () Bool
        (= in0_a in0_b))
(define-fun phi_output () Bool
        (= out2_a out0_b ))
(assert (not (=> (and phi_input phi_a phi_b) phi_output )))
(check-sat)
```

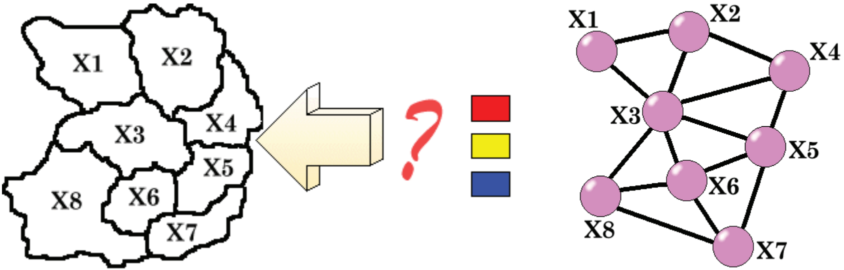
Demo: Program equivalence with SMT solving (Int)

```
(declare-fun out0_a () (Int))
(declare-fun out1_a () (Int))
(declare-fun in0_a () (Int))
(declare-fun out2_a () (Int))
(declare-fun out0_b () (Int))
(declare-fun in0_b () (Int))
(define-fun phi_a () Bool
        (and (= out0_a in0_a) ; out0_a = in0_a
                (and (= out1_a (* out0_a in0_a)) ; out1_a = out0_a * in0_a
                        (= out2_a (* out1_a in0_a))))) ; out2_a = out1_a * in0_a
(define-fun phi_b () Bool
        (= out0_b (* (* in0_b in0_b) in0_b))) ; out0_b = in0_b * in0_b * in0_b
(define-fun phi_input () Bool
        (= in0_a in0_b))
(define-fun phi_output () Bool
        (= out2_a out0_b ))
(assert (not (=> (and phi_input phi_a phi_b) phi_output )))
(check-sat)
```

Demo: Program equivalence with SMT solving (UF)

```
(declare-fun out0_a () (_ BitVec 128))
(declare-fun out1_a () (_ BitVec 128))
(declare-fun in0_a () (_ BitVec 128))
(declare-fun out2_a () (_ BitVec 128))
(declare-fun out0_b () (_ BitVec 128))
(declare-fun in0_b () (_ BitVec 128))
(declare-fun f ((_ BitVec 128) (_ BitVec 128)) (_ BitVec 128))
(define-fun phi_a () Bool
        (and (= out0_a in0_a) ; out0_a = in0_a
                (and (= out1_a (f out0_a in0_a)) ; out1_a = out0_a * in0_a
                        (= out2_a (f out1_a in0_a))))) ; out2_a = out1_a * in0_a
(define-fun phi_b () Bool
        (= out0_b (f (f in0_b in0_b) in0_b))) ; out0_b = in0_b * in0_b * in0_b
(define-fun phi_input () Bool
        (= in0_a in0_b))
(define-fun phi_output () Bool
        (= out2_a out0_b ))
(assert (not (=> (and phi_input phi_a phi_b) phi_output )))
(check-sat)
```

Graph coloring encoding in SMT



Variables:

- Integer variables x_i for each node

Constraints:

- $1 \leq x_i \leq c$
- $x_i \neq x_j$ for $(x_i, x_j) \in E$

Graph coloring encoding code

```
from z3 import *
import sys

with open(sys.argv[1]) as f:
    content = f.readlines()

nodes=int(content[0].split()[2])
edges=int(content[0].split()[3])
s = Solver()
variables = []

for id in range(1,nodes+1):
        variables.append(Int('x'+str(id)))
        # each node can be assigned a value between 1 and k
        s.add(And(1 <= variables[id-1], variables[id-1] <= int(sys.argv[2])))

for line in content:
  if line[0]=='p':
        continue
  else:
        edge=line.split()
        # if two nodes are connected then they have different colors
        s.add((variables[int(edge[1])-1])!=(variables[int(edge[2])-1]))

s.check() # check if sat/unsat
print(s.model()) # print model
```

Demo: Encoding in SMT

Unsatisfiable cores in SMT

```
; Getting unsatisfiable cores
(set-option :produce-unsat-cores true)
(set-logic QF_UF)
(declare-const p Bool) (declare-const q Bool) (declare-const r Bool)
(declare-const s Bool) (declare-const t Bool)
(assert (! (=> p q) :named PQ))
(assert (! (=> q r) :named QR))
(assert (! (=> r s) :named RS))
(assert (! (=> s t) :named ST))
(assert (! (not (=> q s)) :named NQS))
(check-sat)
; unsat
(get-unsat-core)
; (QR RS NQS)
(exit)
```