From Clauses to Klauses

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CAV 2024

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Conjunctive Normal Form (CNF): conjunction of clauses

$$(x_1 \vee x_2 \vee x_3) \wedge \ldots$$

- Standard input to satisfiability (SAT) solvers for $30+$ years

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Conjunctive Normal Form (CNF): conjunction of clauses

$$(x_1 \lor x_2 \lor x_3) \land \dots$$

- Standard input to satisfiability (SAT) solvers for $30+$ years

Cardinality Conjunctive Normal Form (KNF): conjunction of cardinality constraints (klauses)

$$(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 3) \land \dots$$

- Extend input for more flexibility in solving
- Incremental change without sacrificing general usage

Cardinality Constraints Extend Clauses

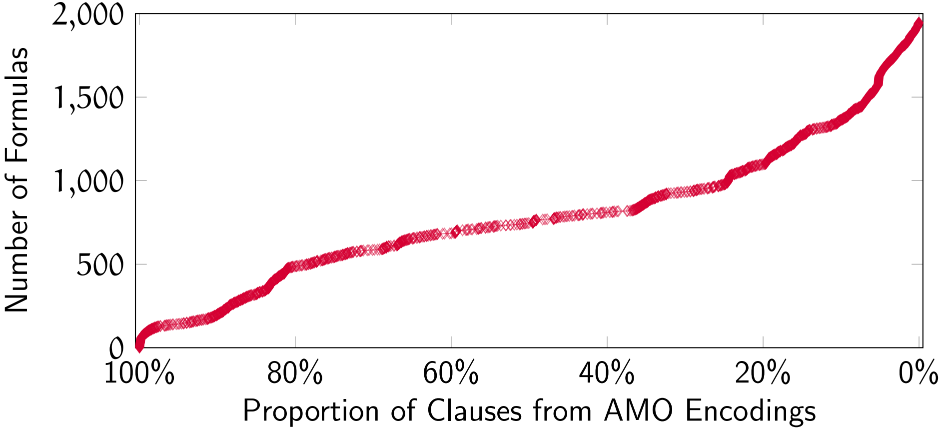
- A clause can be represented as a cardinality constraint

$$(x_1 \vee x_2 \vee x_3) = (x_1 + x_2 + x_3 \geq 1)$$

- Comparison operators $(=, >, \geq, <, \leq)$ can be represented with one or two $\geq$ constraints
- At-Most-One (AMO): at most one literal is true

$$AMO(x_1, x_2, x_3) = (x_1 + x_2 + x_3 \leq 1) = (\overline{x}_1 + \overline{x}_2 + \overline{x}_3 \geq 2)$$

AMO Cardinality Constraints in Competition Formulas



- Anniversary Track: two decades of SAT competition
- 36% of 5,300 formulas with AMO of size 5 or larger

Historic Motivations for Cardinality Input

- Easier for users to create problems
  - Complex encoding types (e.g. modulus k-totalizer) error-prone
- Can use stronger reasoning techniques
  - e.g., PHP from exponential to linear solving
- Smaller formulas
  - e.g., Magic Squares 6×6: 4k constraints $\rightarrow$ 600k clauses
- Faster constraint propagation
  - e.g., AMO can propagate everything in single step

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Failed to replace CDCL-based CNF solvers for general use

- Clausal reasoning: optimizations, heuristics, inprocessing
- Clausal encodings: better clause learning, shorter proofs

Cardinality Input for CDCL: an Incremental Change

CNF: CDCL

- Auxiliary variables to encode high-level constraints

$$\mathrm{AMO}(x_1, x_2, x_3, x_4, x_5) \rightarrow \mathrm{AMO}(x_1, x_2, x_3, y) \wedge \mathrm{AMO}(\overline{y}, x_4, x_5)$$

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KNF: Cardinality-CDCL

- Lift inprocessing, proof checking, constraint propagation
- Leverage auxiliary variables with clausal encodings
- No stronger reasoning, no separate propagation engine

Contributions

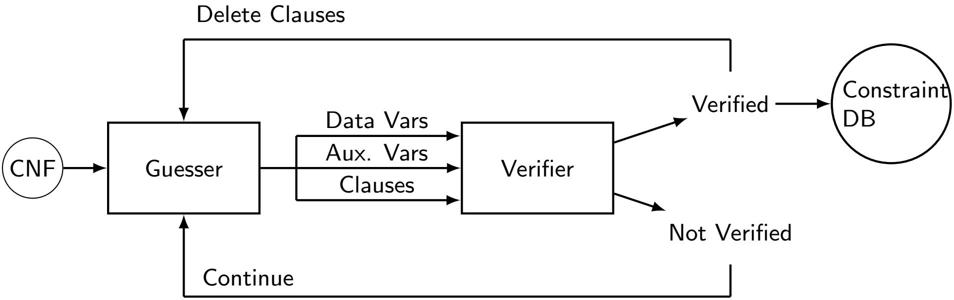
Long Term Goal: User generated KNF formulas

- Problem specific encoding optimizations
- Efficient KNF proof checking
- More paradigms: local search, MaxSAT, parallel solving

Short Term Goal: Backwards compatibility with CNF

- Cardinality constraint extraction producing KNF
- Multiple configurations for KNF solving
- End-to-end proof checking for KNF extraction and solving

Cardinality Constraint Extraction Framework



- Guess a candidate cardinality constraint
- Verify the constraint structure by constructing a BDD and filter out non-constraints

Extractor Comparison on PySAT AMO Encodings

Table: Size 10 AMO on 8 PySAT encodings. ✓ if complete AMO is extracted.

Tool	Pair	SCnt	CNet	SNet	Tot	mTot	mkTot	Lad
Guess-and-Verify	✓	✓	✓	✓	✓	✓	✓	X
Lingeling	✓	X	X	X	X	X	X	X
Riss	✓	X	X	X	X	X	X	X

- Lingeling and Riss only find smaller sub constraints
- BDD verifier works for general cardinality constraints
- Need more sophisticated heuristics for guesser

Cardinality Constraint Solving Options

NATIVE:

- Propagate natively on cardinality constraints (CCDCL)
  - Extends CDCL watch-pointers and conflict analysis
  - Faster propagation, no aux. variables generally better on SAT

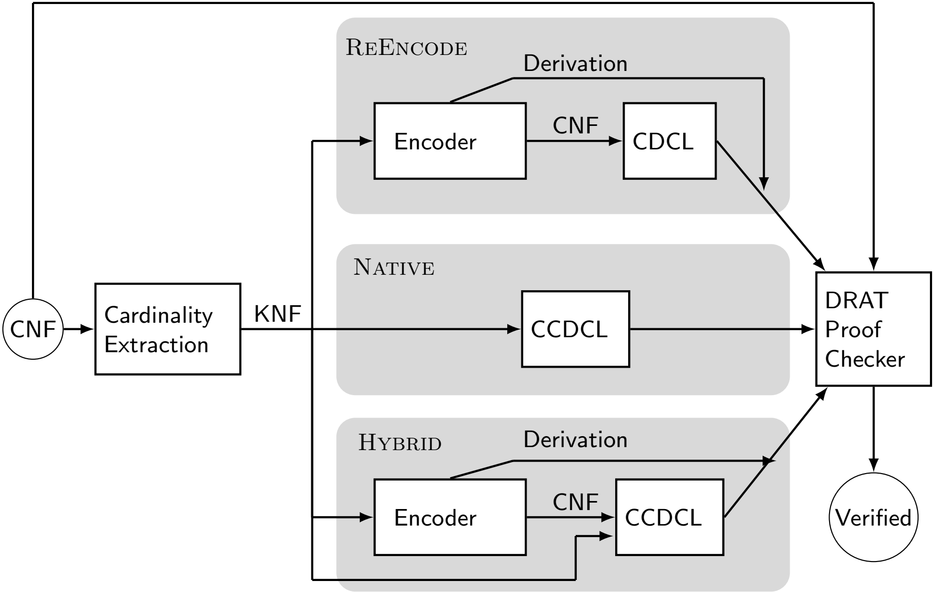
REENCODE:

- Encode cardinality constraints into clauses
  - Encoded constraints generally better on UNSAT

HYBRID:

- Combination of NATIVE and REENCODE
  - Reencoded clauses kept throughout solving
  - Native propagation enabled half of the time
  - Good for both SAT and UNSAT instances

Cardinality Solving and Proof Checking for CNF



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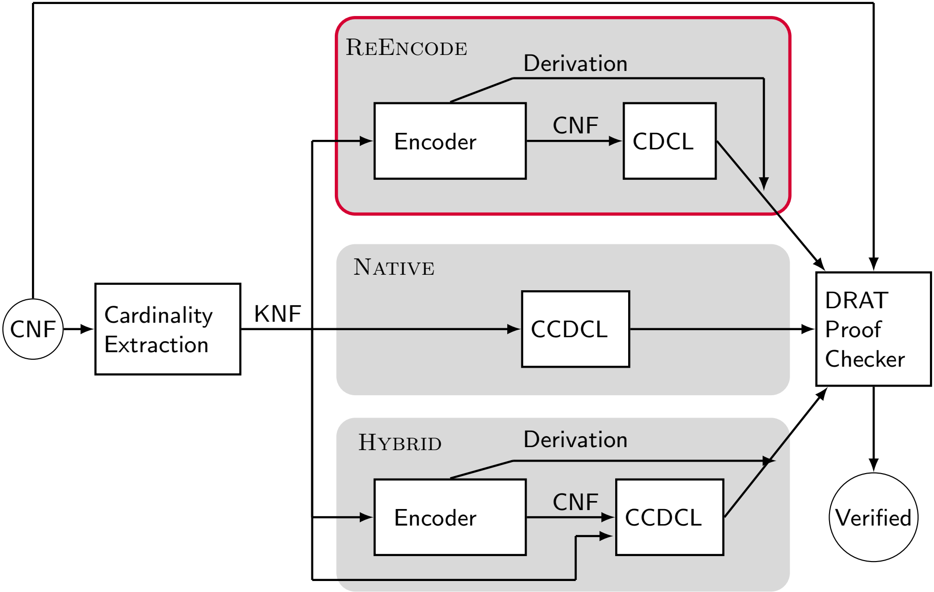

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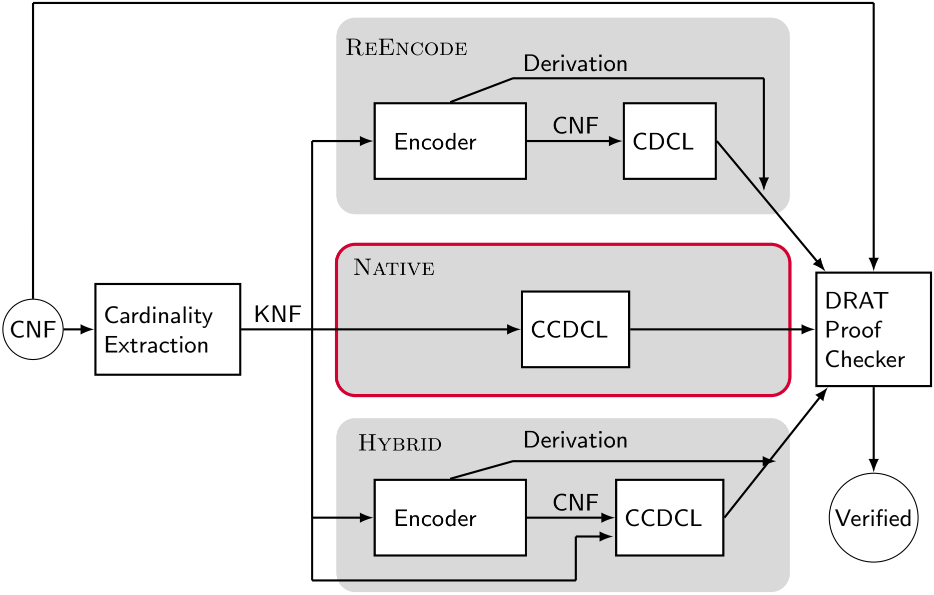

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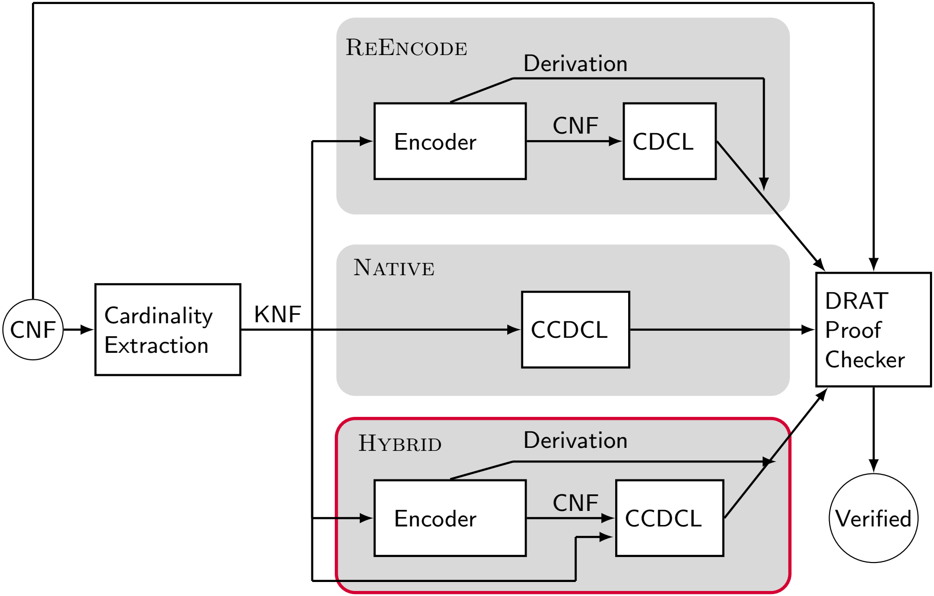
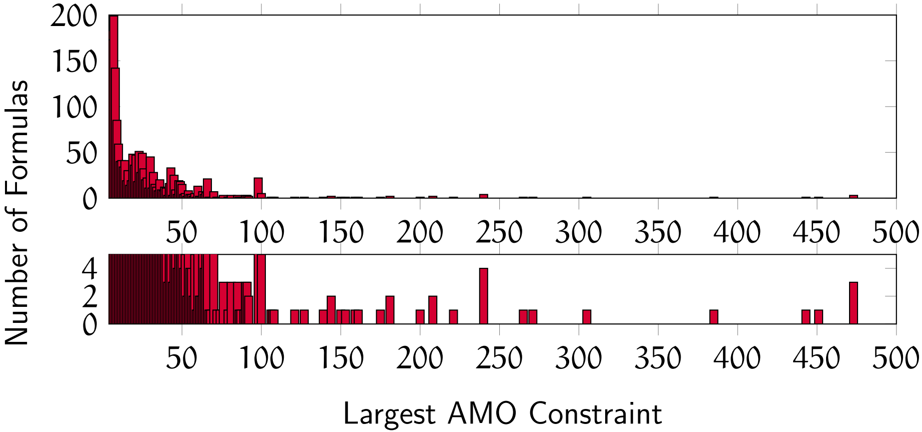
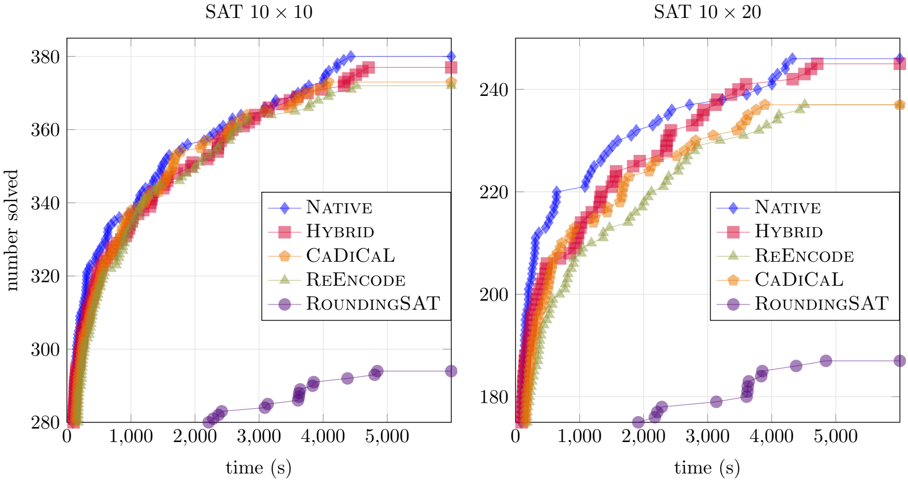


Extracting AMO Constraints on Competition Formulas



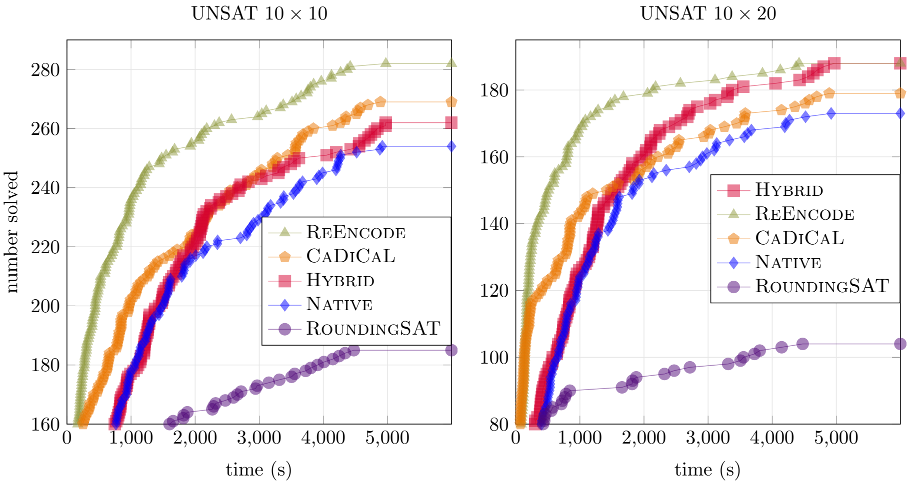
- planning (473), petrinet (451), edge-matching (140)
- Runtime average 69s, 78% below 15s

Solving Extracted Satisfiable Formulas



- 10×10: 933 formulas with at least 10 AMOs of size ≥ 10
- 10×20: 587 formulas with at least 10 AMOs of size ≥ 20

Solving Extracted Unsatisfiable Formulas



- ReEncode displays strength of auxiliary variables
- Hybrid suffers overhead from mode switching

Conclusion

- KNF input requires moderate changes to a SAT solver
- Combining clausal encodings and native propagation good for SAT and UNSAT problems
- Apply solver to KNF problems too large for CNF encoding
  - Computational geometry, e.g., point-discrepancy problem

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We'd like to hear about cardinality constraint problems you'd like to solve!