Robotic Motion Planning: Configuration Space

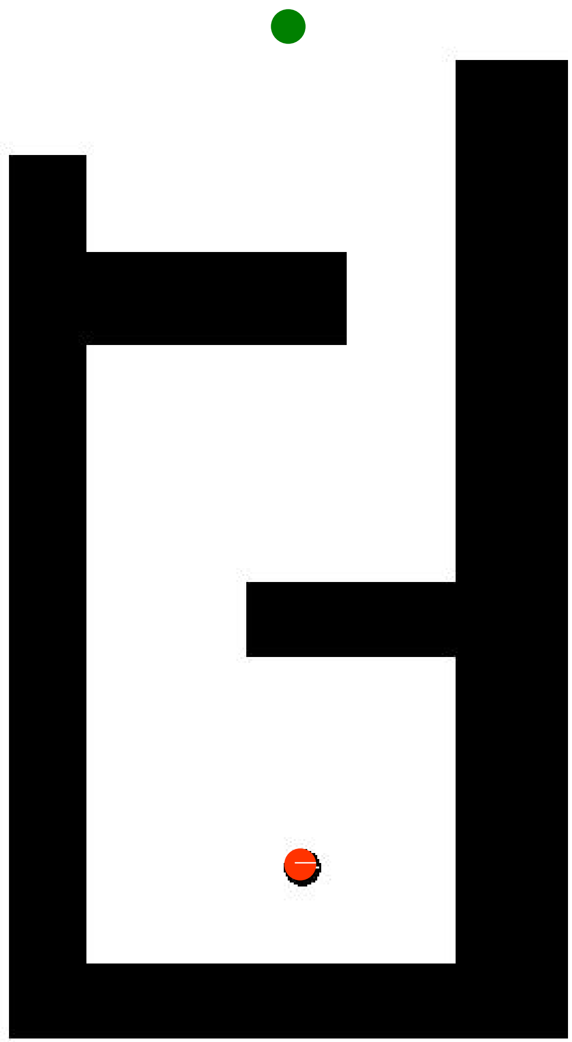
Robotics Institute 16-735

http://www.cs.cmu.edu/~motionplanning

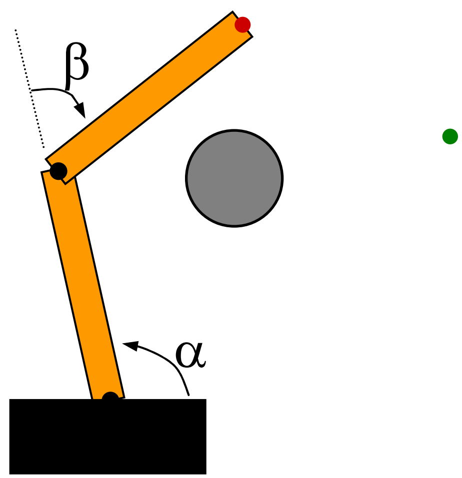
Howie Choset

http://www.cs.cmu.edu/~choset

What if the robot is not a point?

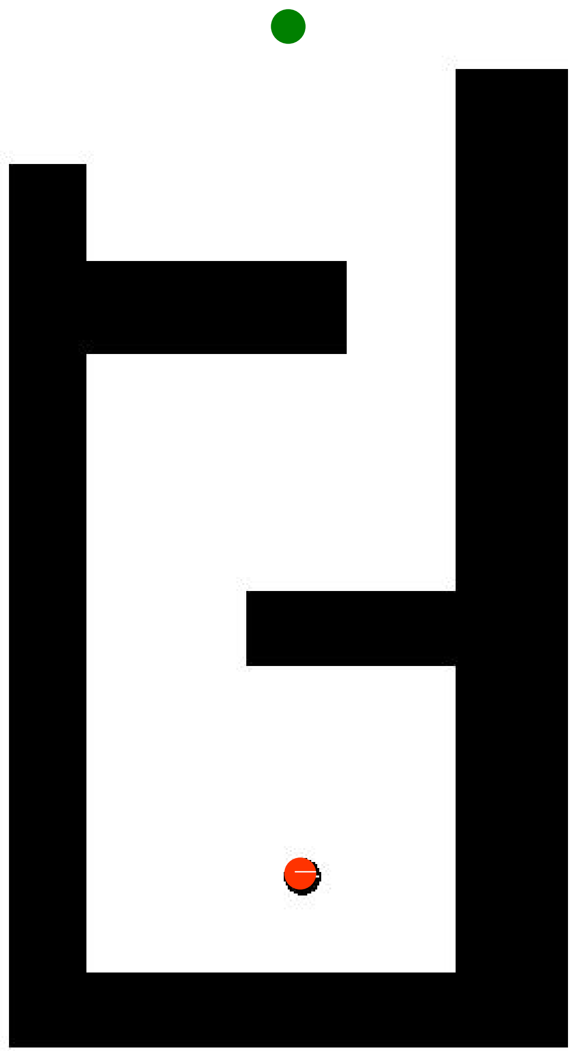


The Scout should probably not be modeled as a point...



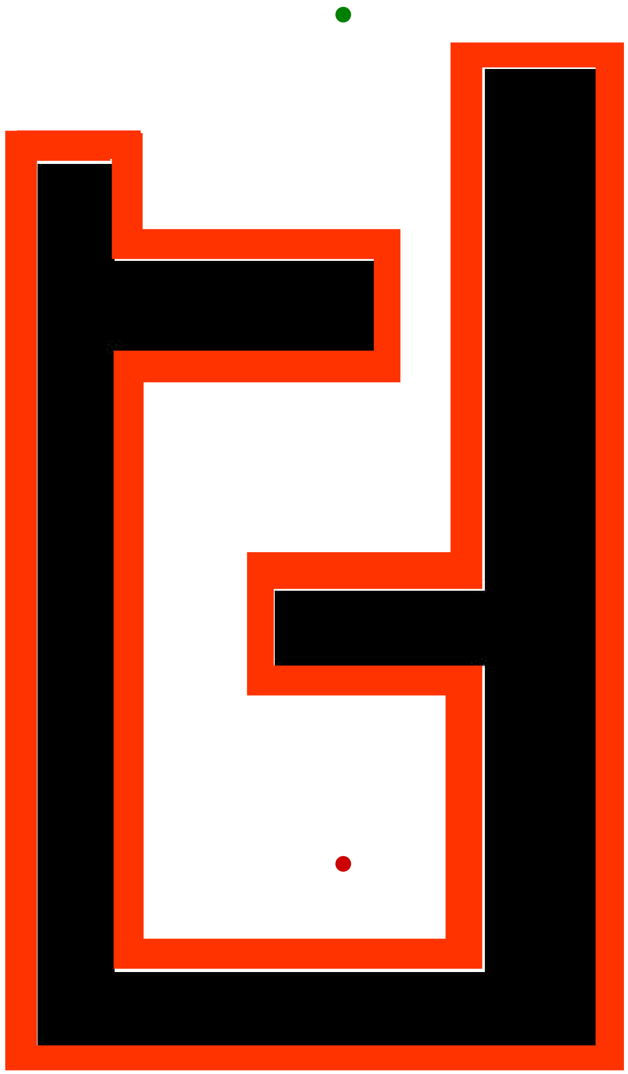
Nor should robots with extended linkages that may contact obstacles...

What is the position of the robot?



Expand obstacle(s)

Reduce robot



not quite right ...

Configuration Space

- A key concept for motion planning is a **configuration**:
  - *a complete specification of the position of every point in the system*

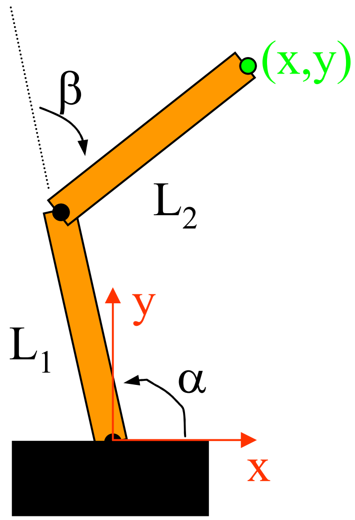
- A simple example: a robot that translates but does not rotate in the plane:

  - what is a sufficient representation of its configuration?

- The space of all configurations is the **configuration space** or **C-space.**

Robot Manipulators

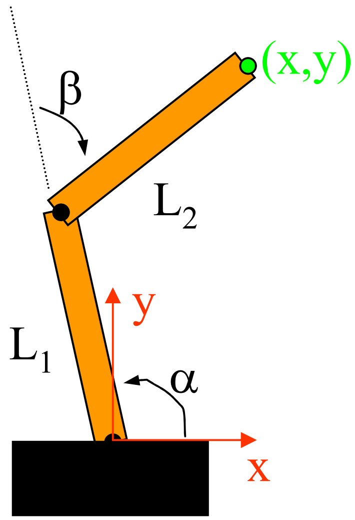
What are this arm's forward kinematics?



(How does its position depend on its joint angles?)

Robot Manipulators

What are this arm's forward kinematics?



Find **(x,y)** in terms of α and β ...

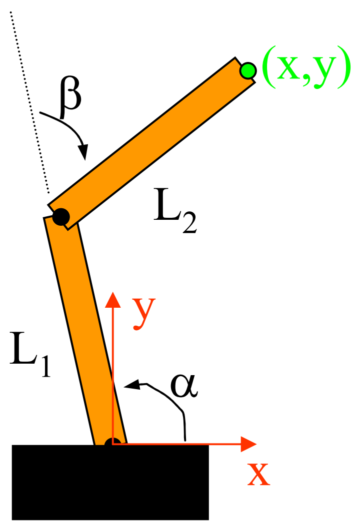
Keeping it "simple"

$c_\alpha = \cos(\alpha)$, $s_\alpha = \sin(\alpha)$

$c_\beta = \cos(\beta)$, $s_\beta = \sin(\beta)$

$c_+ = \cos(\alpha+\beta)$, $s_+ = \sin(\alpha+\beta)$

Manipulator kinematics



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} L_1 c_\alpha \\ L_1 s_\alpha \end{pmatrix} + \begin{pmatrix} L_2 c_+ \\ L_2 s_+ \end{pmatrix}$$ Position

Keeping it "simple"

$c_\alpha = \cos(\alpha)$, $s_\alpha = \sin(\alpha)$

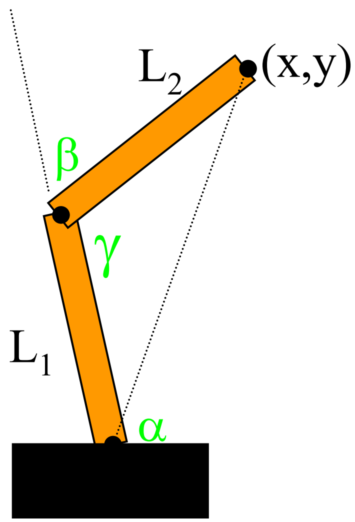
$c_\beta = \cos(\beta)$, $s_\beta = \sin(\beta)$

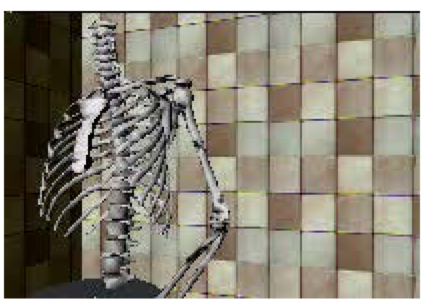
$c_+ = \cos(\alpha+\beta)$, $s_+ = \sin(\alpha+\beta)$

Inverse Kinematics

Inverse kinematics -- finding joint angles from Cartesian coordinates

via a geometric or algebraic approach...



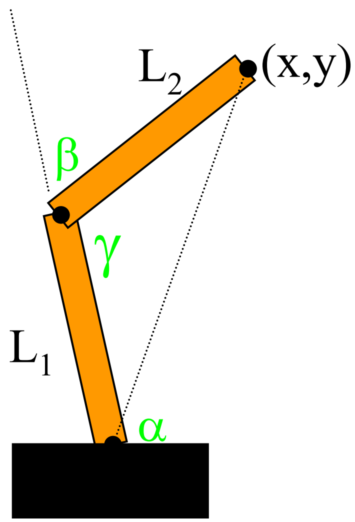


Given (x,y) and L_1 and L_2 , what are the values of α, β, γ ?

Inverse Kinematics

Inverse kinematics -- finding joint angles from Cartesian coordinates

via a geometric or algebraic approach...



$$\gamma = \cos^{-1}\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

$$\beta = 180 - \gamma$$

$$\alpha = \sin^{-1}\left(\frac{L_2 \sin(\gamma)}{x^2 + y^2}\right) + \tan^{-1}(y/x)$$

atan2(y,x)

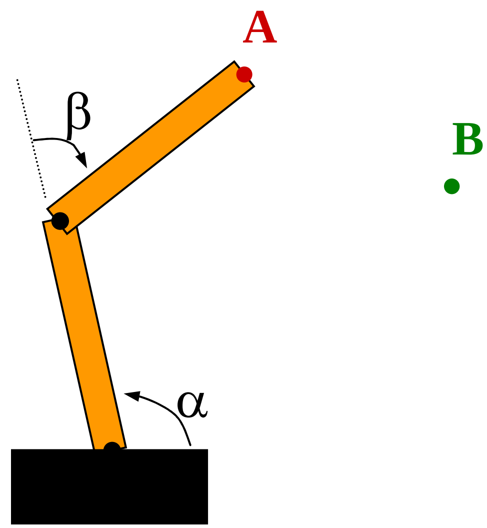
(1,0) = 1.3183, -1.06

(-1,0) = 1.3183, 4.45

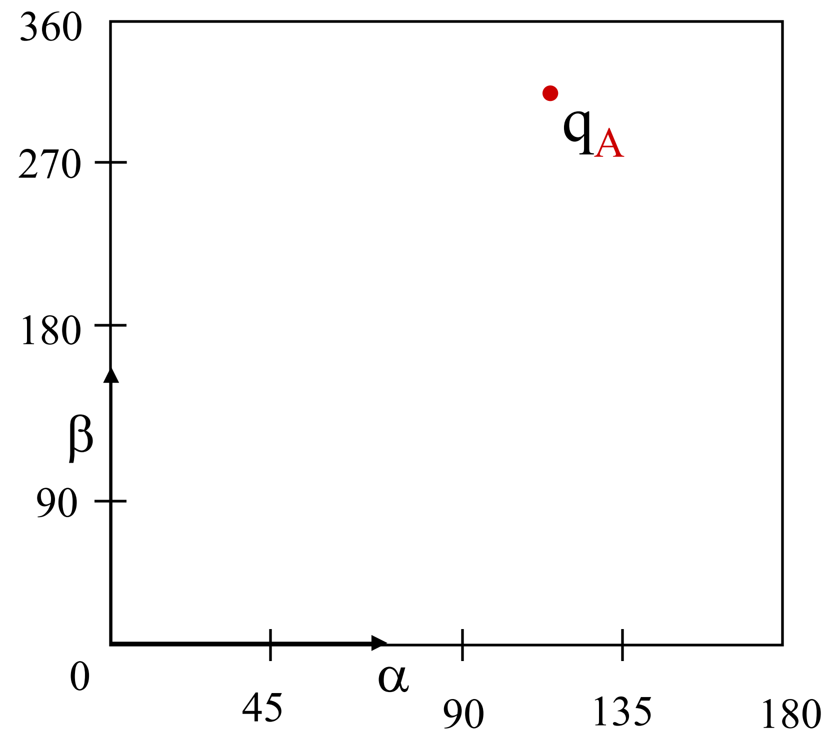
But it's not usually this ugly...

Configuration Space

Where can we put • q_B ?



An obstacle in the robot's workspace



Torus

(wraps horizontally and vertically)

Puma Inv. Kinematics

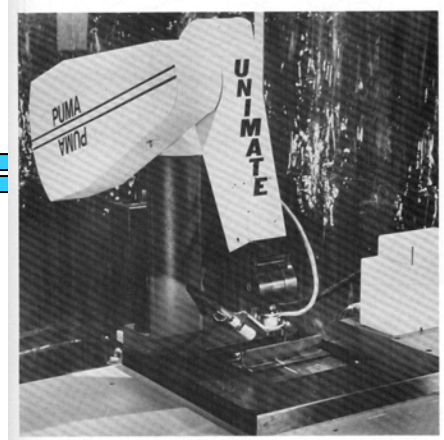


FIGURE 3.17 The Unimation PUMA 560.

```
%
% Solve for theta(1)

r=sqrt(Px^2 + Py^2);
if (n1 == 1),
    theta(1)= atan2(Py,Px) + asin(d3/r);
else
    theta(1)= atan2(Py,Px) + pi - asin(d3/r);
end

%
% Solve for theta(2)

V114= Px*cos(theta(1)) + Py*sin(theta(1));
r=sqrt(V114^2 + Pz^2);
Psi = acos((a2^2-d4^2-a3^2+V114^2+Pz^2)/
           (2.0*a2*r));
theta(2) = atan2(Pz,V114) + n2*Psi;

%
% Solve for theta(3)

num = cos(theta(2))*V114+sin(theta(2))*Pz-a2;
den = cos(theta(2))*Pz - sin(theta(2))*V114;
theta(3) = atan2(a3,d4) - atan2(num, den);
```

```
% Solve for theta(4)

V113 = cos(theta(1))*Ax + sin(theta(1))*Ay;
V323 = cos(theta(1))*Ay - sin(theta(1))*Ax;
V313 = cos(theta(2)+theta(3))*V113 +
       sin(theta(2)+theta(3))*Az;
theta(4) = atan2((n4*V323),(n4*V313));

% Solve for theta(5)

num = -cos(theta(4))*V313 - V323*sin(theta(4));
den = -V113*sin(theta(2)+theta(3)) +
       Az*cos(theta(2)+theta(3));
theta(5) = atan2(num,den);

% Solve for theta(6)

V112 = cos(theta(1))*Ox + sin(theta(1))*Oy;
V132 = sin(theta(1))*Ox - cos(theta(1))*Oy;
V312 = V112*cos(theta(2)+theta(3)) +
       Oz*sin(theta(2)+theta(3));
V332 = -V112*sin(theta(2)+theta(3)) +
        Oz*cos(theta(2)+theta(3));
V412 = V312*cos(theta(4)) - V132*sin(theta(4));
V432 = V312*sin(theta(4)) + V132*cos(theta(4));
num = -V412*cos(theta(5)) - V332*sin(theta(5));
den = - V432;
theta(6) = atan2(num,den);
```

it's usuall much worse!

Some Other Examples of C-Space

- A rotating bar fixed at a point
  - what is its C-space?
  - what is its workspace
- A rotating bar that translates along the rotation axis
  - what is its C-space?
  - what is its workspace
- A two-link manipulator
  - what is its C-space?
  - what is its workspace?
  - Suppose there are joint limits, does this change the C-space?
  - The workspace?

What is the Dimension of Configuration Space?

- The dimension is the number of parameter necessary to uniquely specify configuration

- One way to do this is to explicitly generate a parameterization (e.g with our 2-bar linkage)

- Another is to start with too many parameters and add (independent) constraints
  - suppose I start with 4 points in the plane (= 8 parameters), A, B, C, D
  - Rigidity requires d(A,B) = c_1 (1 constraints)
  - Rigidity requires d(A,C) = c_2 and d(B,C) = c_3 (2 constraints)
  - Rigidity requires d(A,D) = c_4 and d(B,D) = c_5 and ??? (?? constraints)

  - HOW MANY D.O.F?

- QUIZ:
  - HOW MANY DOF DO YOU NEED TO MOVE FREELY IN 3-space?

Obstacles in C-Space

- Let q denote a point in a configuration space Q
- The path planning problem is to find a mapping $c:[0,1] \rightarrow Q$ s.t. no configuration along the path intersects an obstacle
- Recall a workspace obstacle is WO_i
- A *configuration space obstacle* QO_i is the set of configurations q at which the robot intersects WO_i, *that is*
  - $\mathcal{QO}_i = \{q \in \mathcal{Q} \mid R(q) \bigcap \mathcal{WO}_i \neq \emptyset\}.$
- The *free configuration space (*or just *free space*) $\mathcal{Q}_{\text{free}}$ *is*

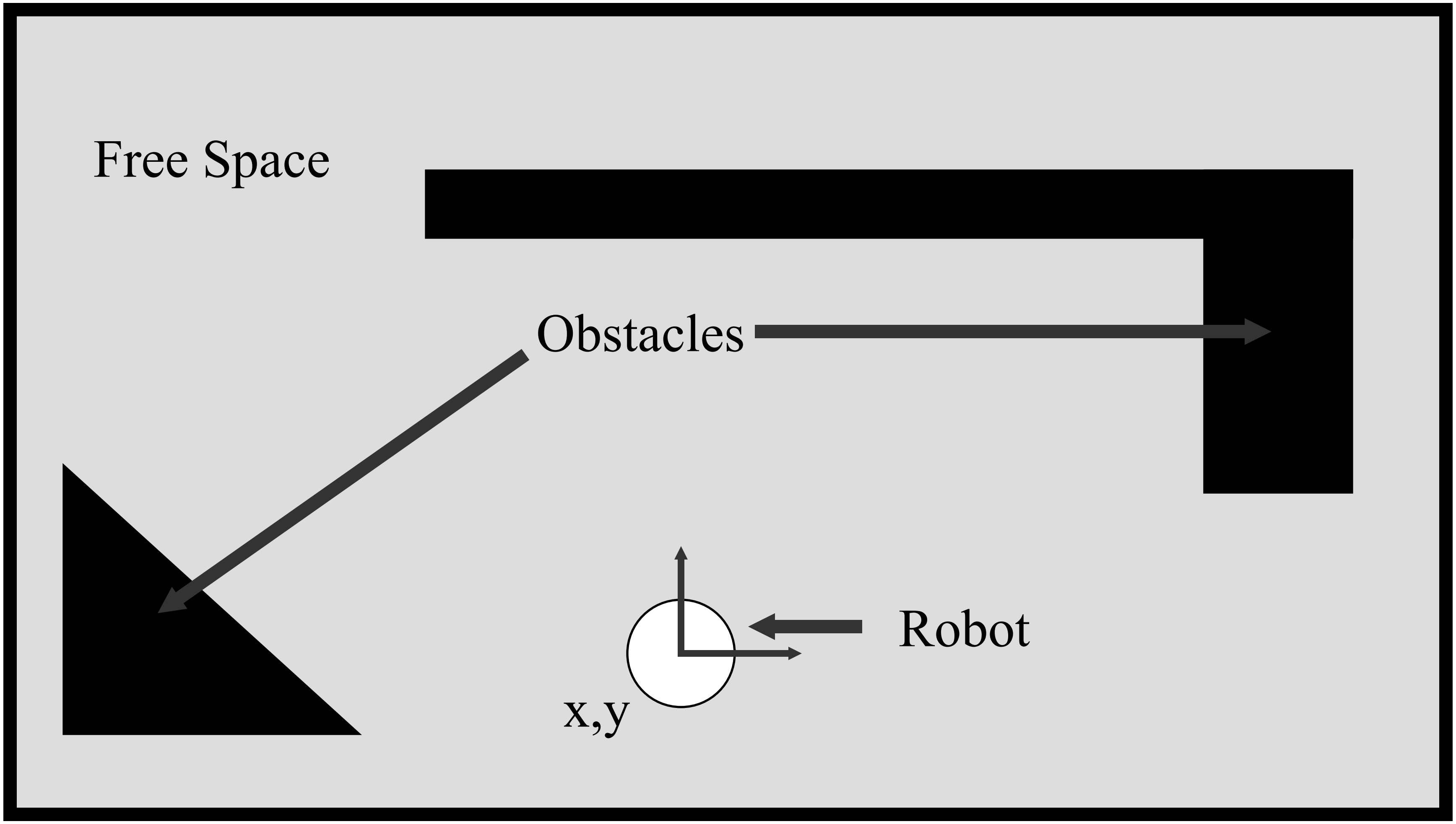
$$\mathcal{Q}_{\text{free}} = \mathcal{Q} \setminus \left(\bigcup \mathcal{QO}_i \right).$$

The free space is generally an open set

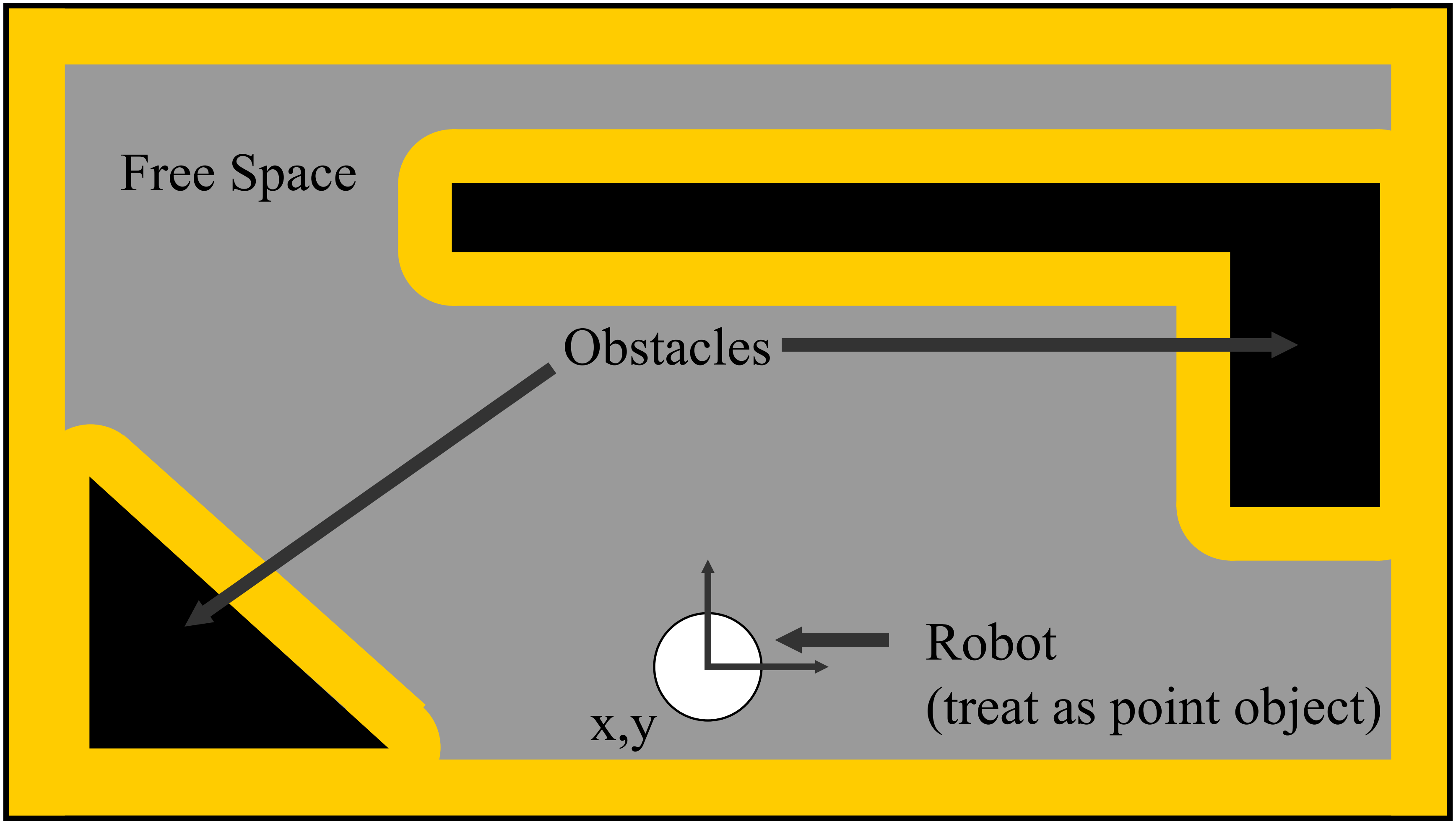
A *free path* is a mapping $c:[0,1] \rightarrow Q_{free}$

A *semifree path* is a mapping $c:[0,1] \rightarrow \text{cl}(Q_{free})$

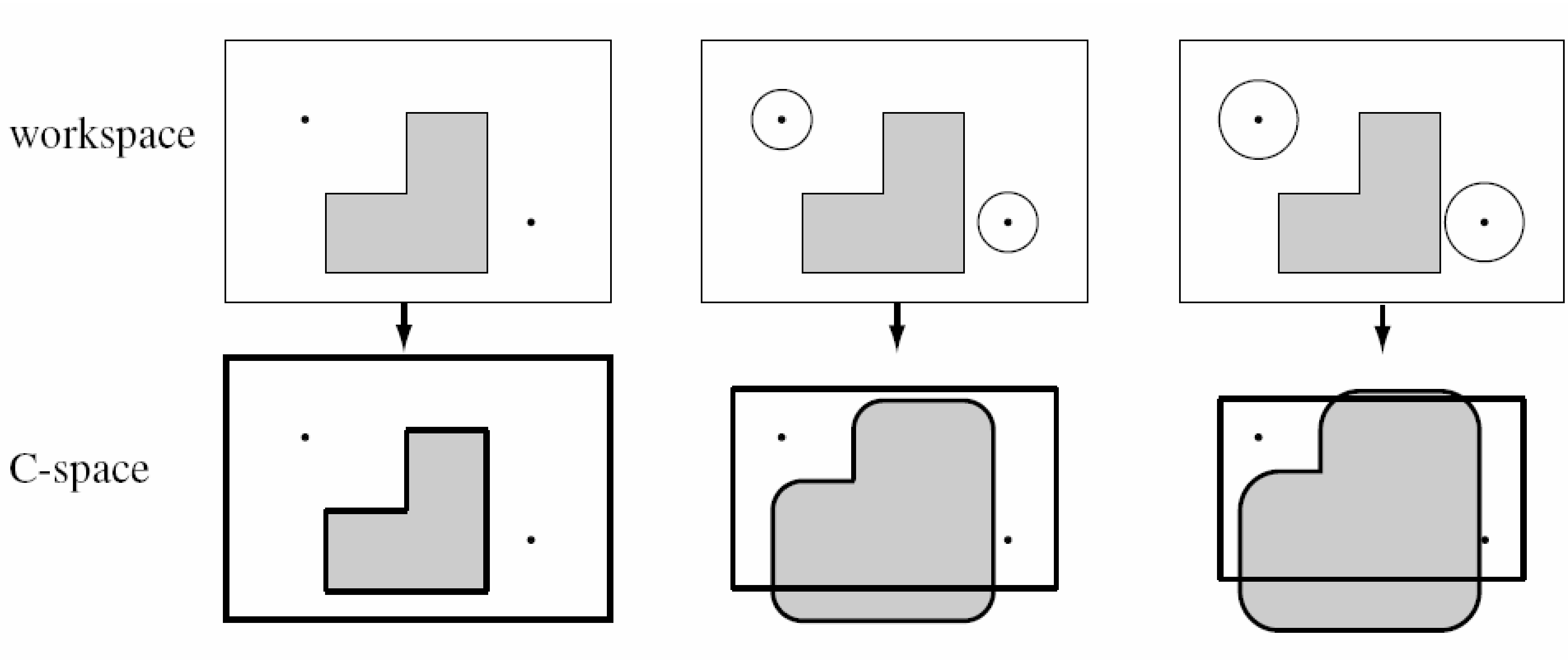
Example of a World (and Robot)



Configuration Space: Accommodate Robot Size



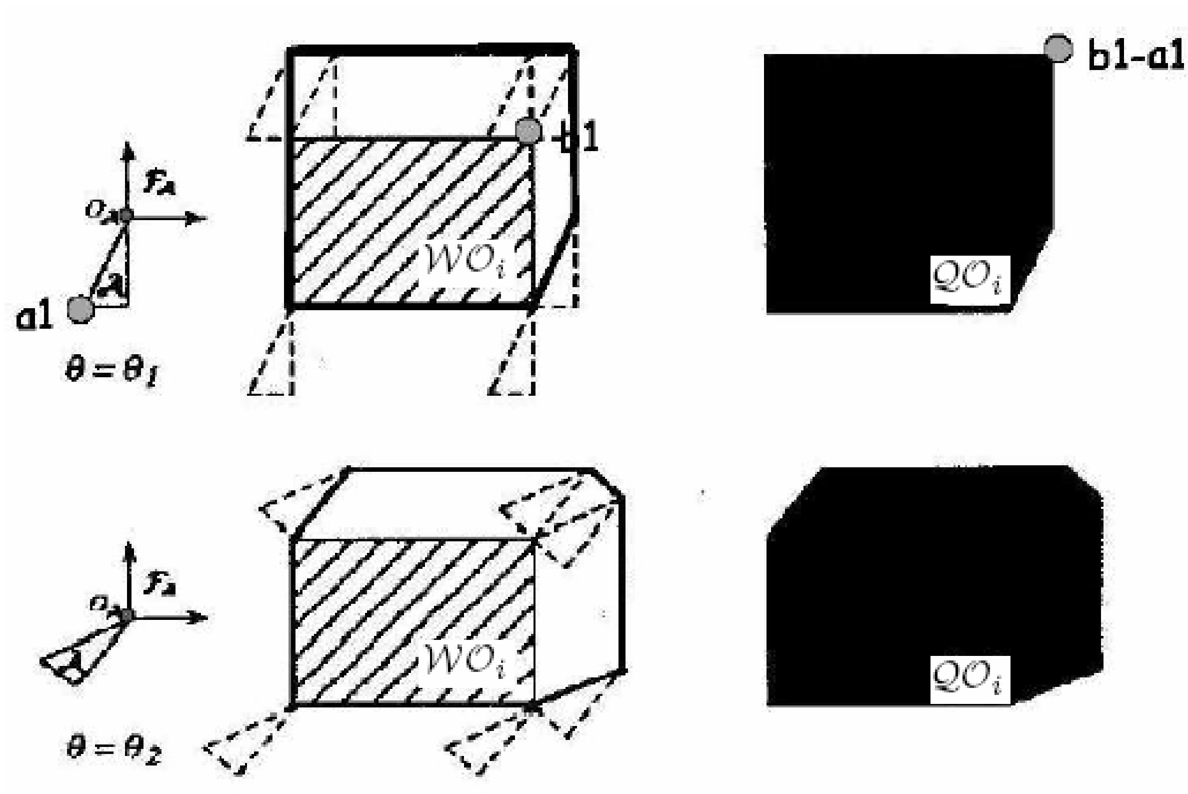
Trace Boundary of Workspace



$$QO_i = \{q \in Q \mid R(q) \bigcap WO_i \neq \emptyset\}.$$

Pick a reference point…

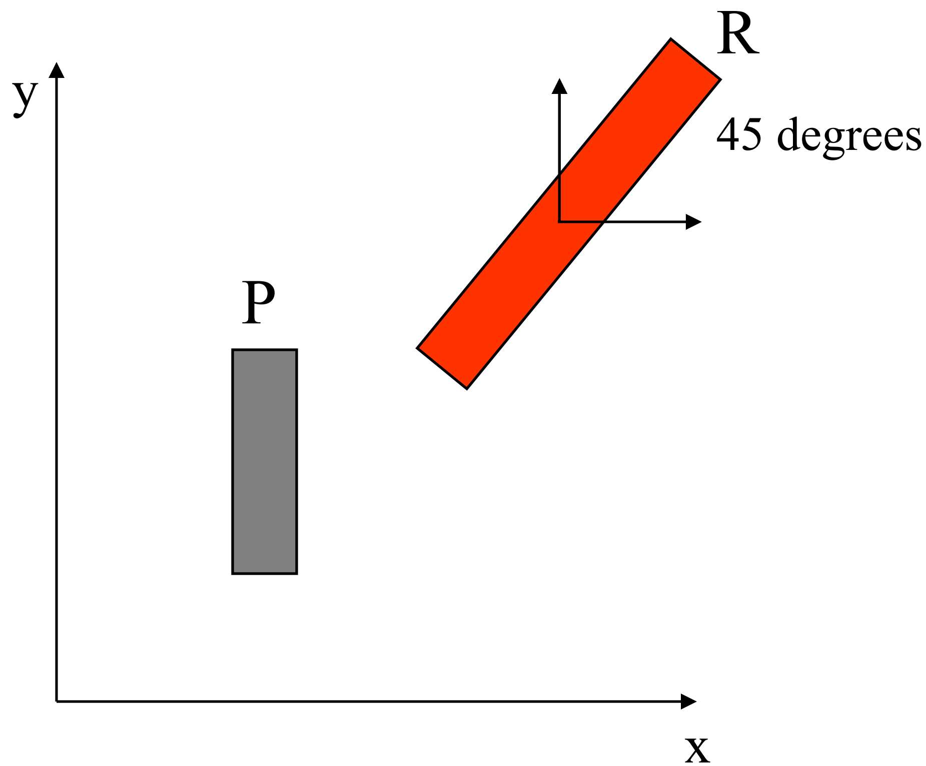
Translate-only, non-circularly symmetric



$$\mathcal{QO}_i = \{q \in \mathcal{Q} \mid R(q) \bigcap \mathcal{WO}_i \neq \emptyset\}.$$

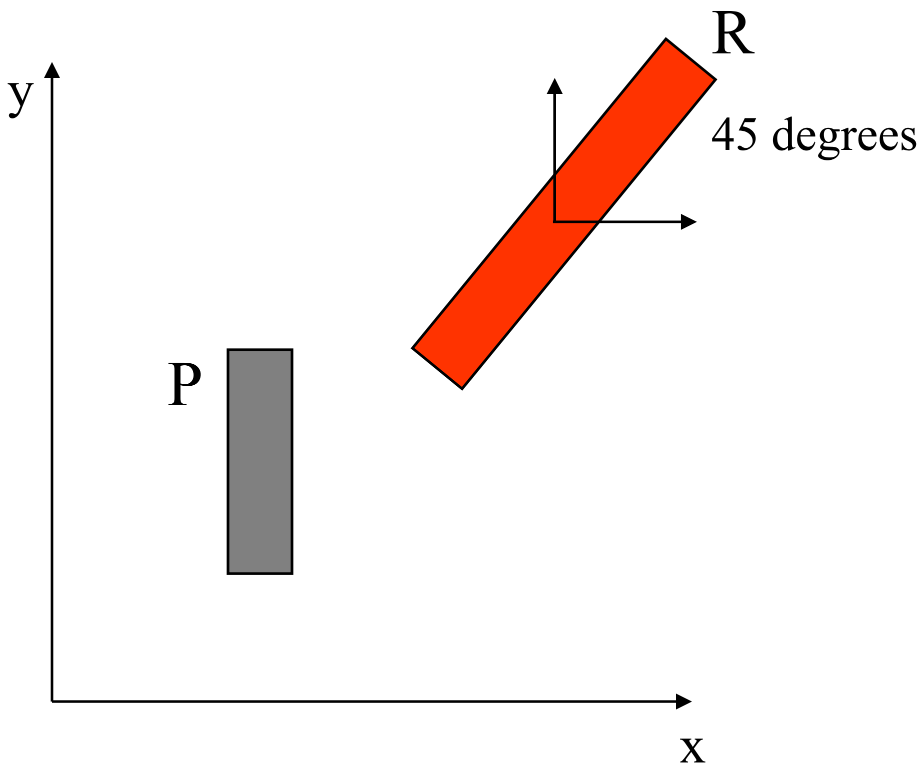
Pick a reference point…

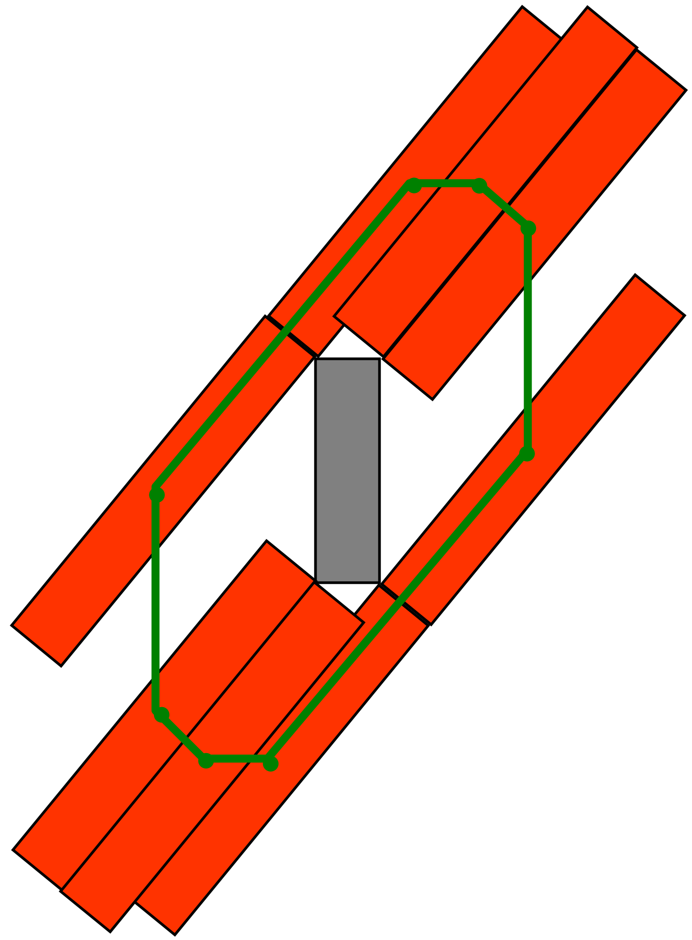
Any reference point



Any refernece point

Taking the cross section of configuration space in which the robot is rotated 45 degrees...

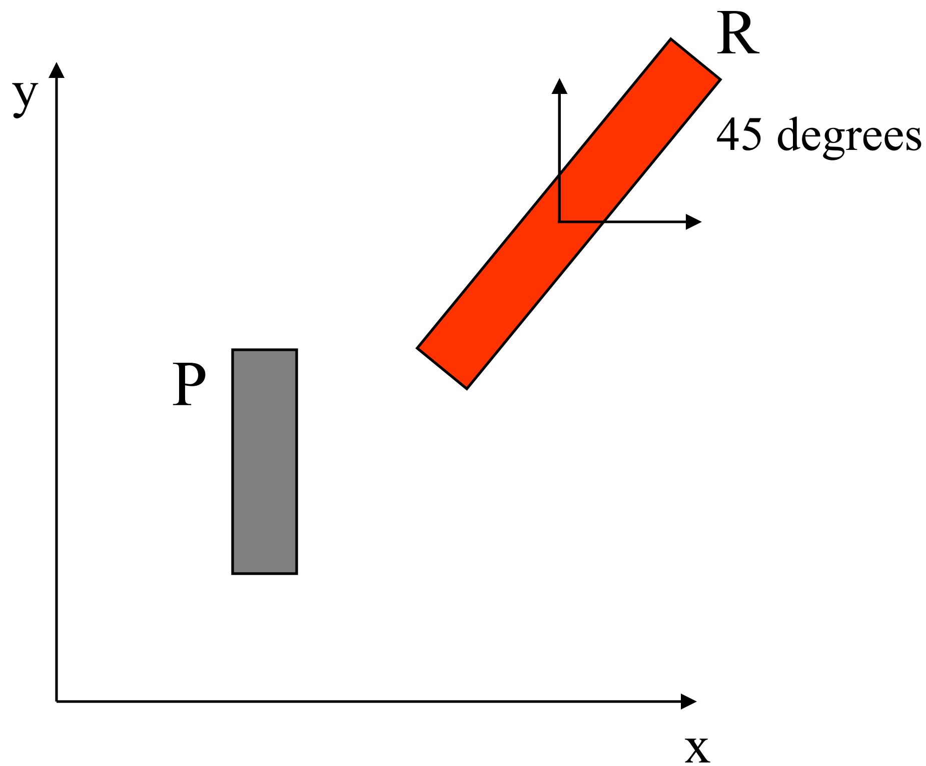


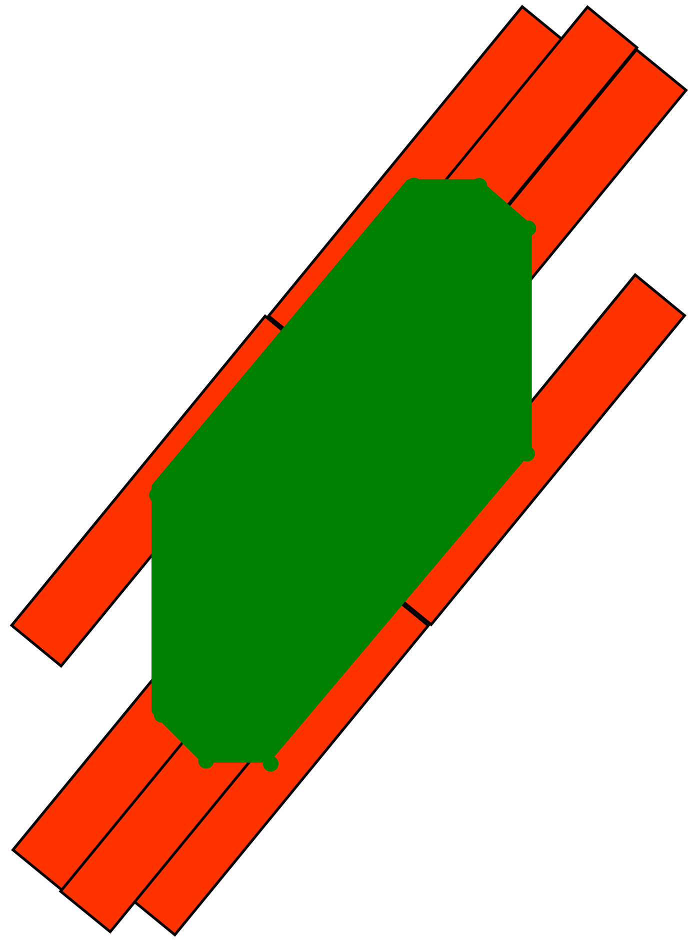


How many sides does P ⊕R have?

Any reference point

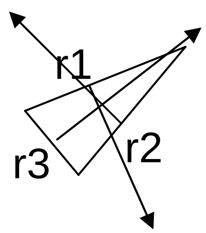
Taking the cross section of configuration space in which the robot is rotated 45 degrees...

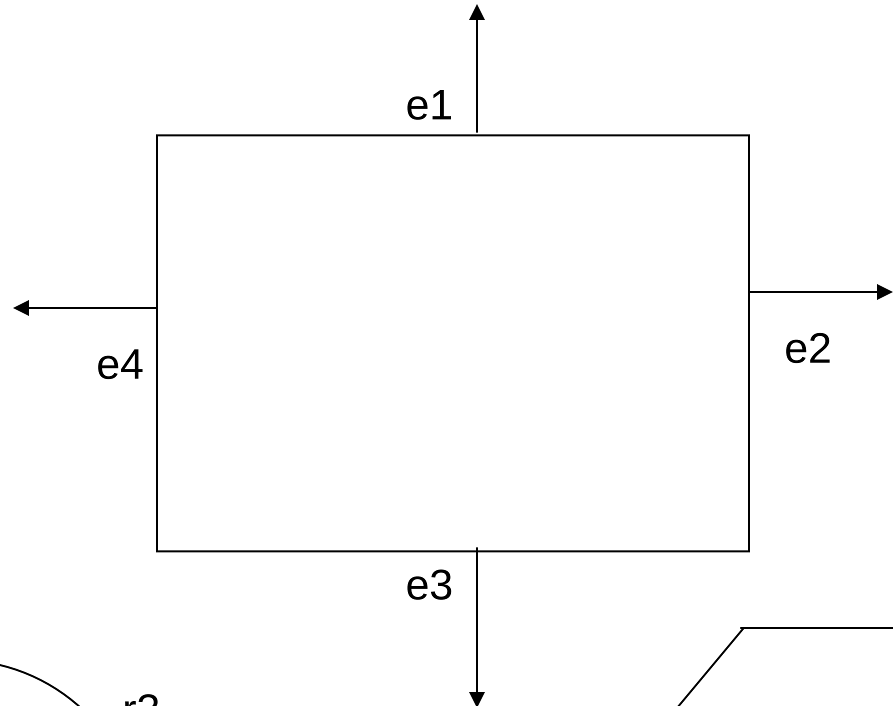


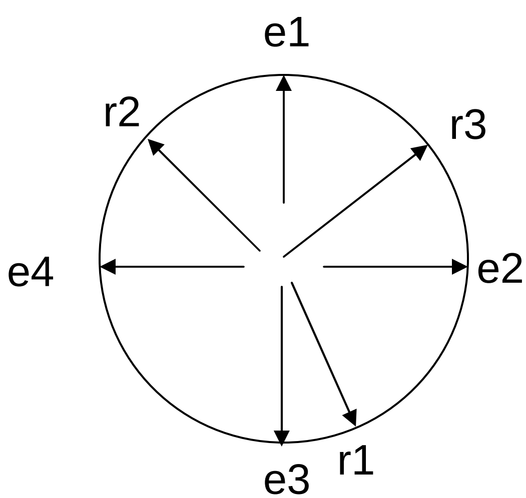


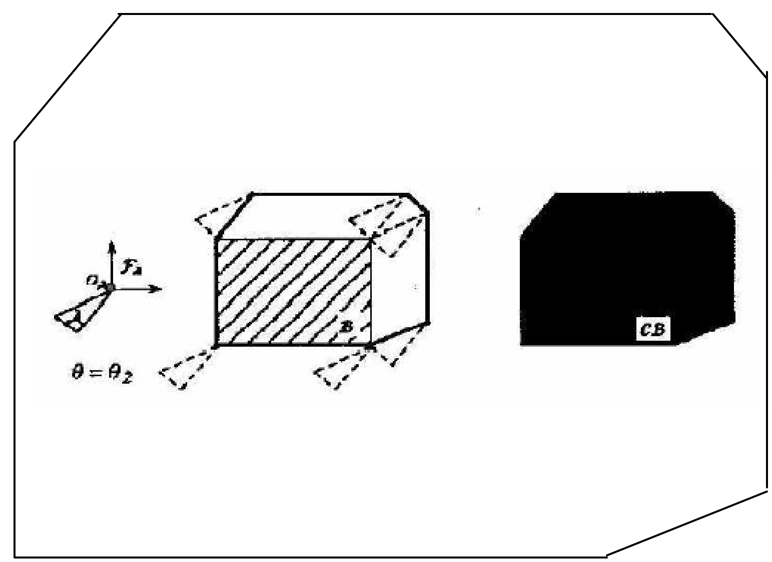
How many sides does P ⊕R have?

Star Algorithm: Polygonal Obstacles

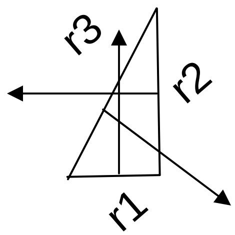


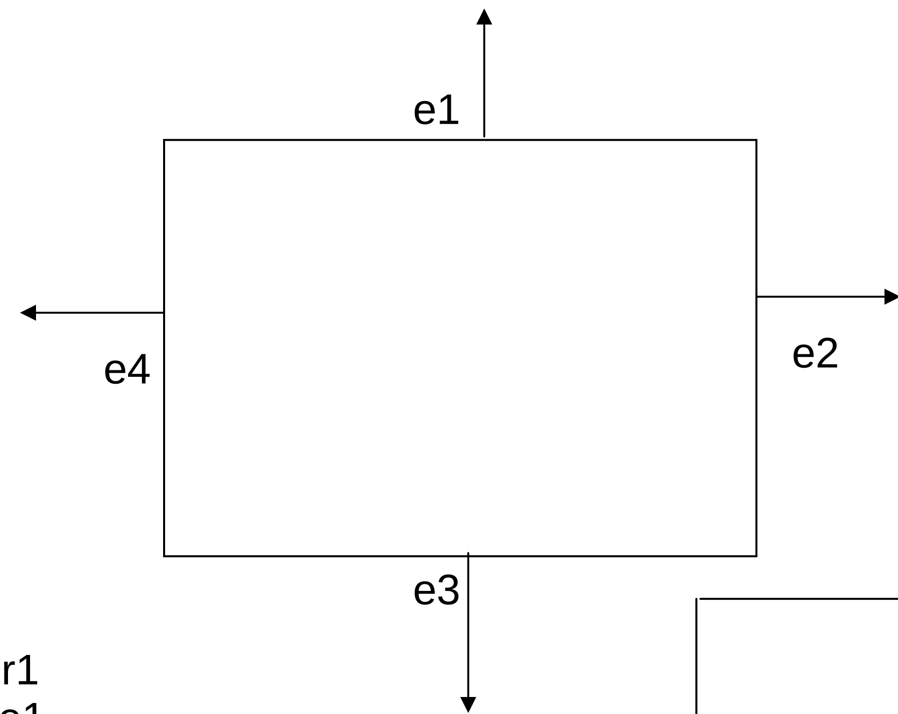


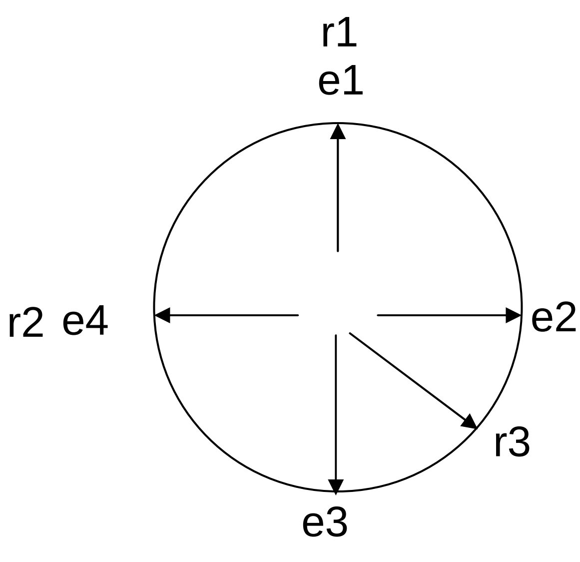


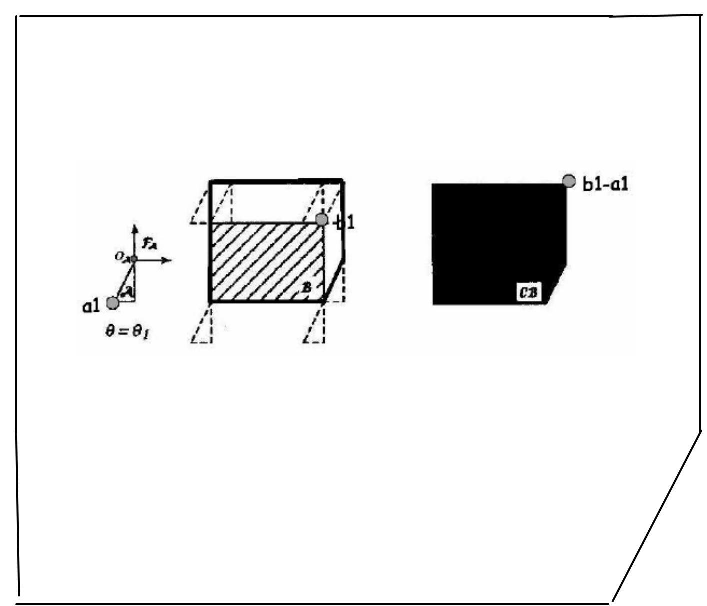


Star Algorithm





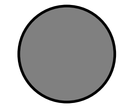


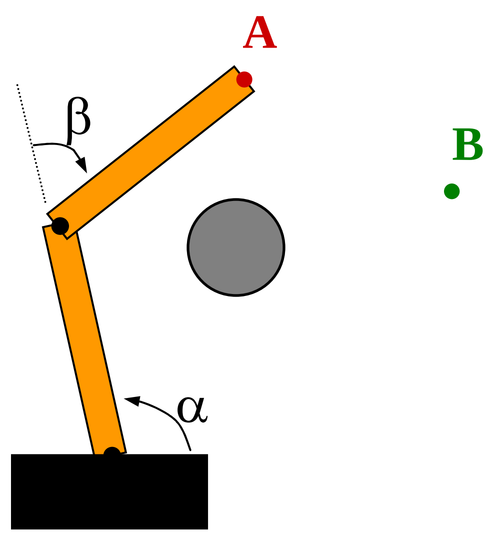


Once you have the C-obstacle, where do you put it?

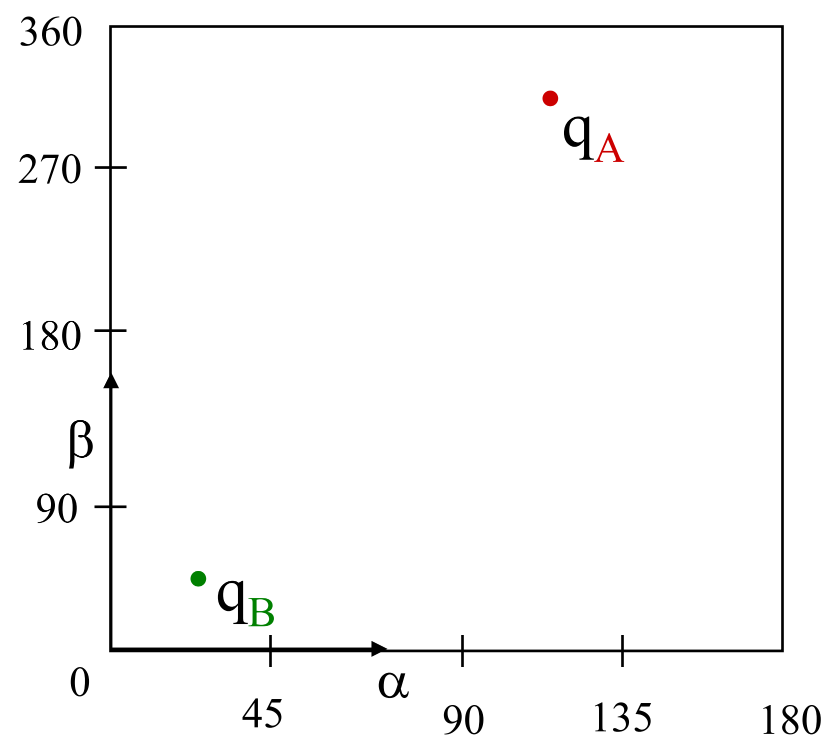
- Leave it as an exercise for homework…

Configuration Space "Quiz"

Where do we put  ?



An obstacle in the robot's workspace

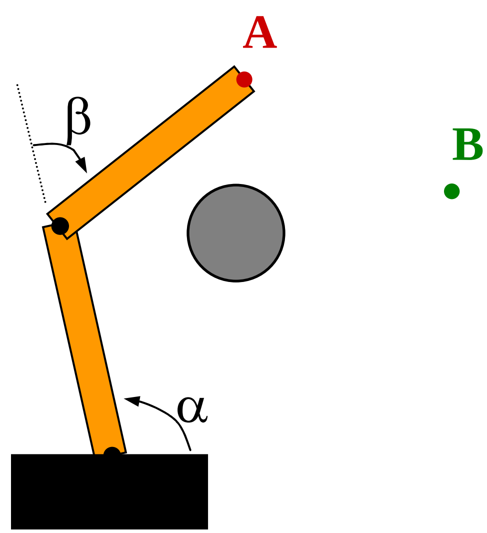


Torus

(wraps horizontally and vertically)

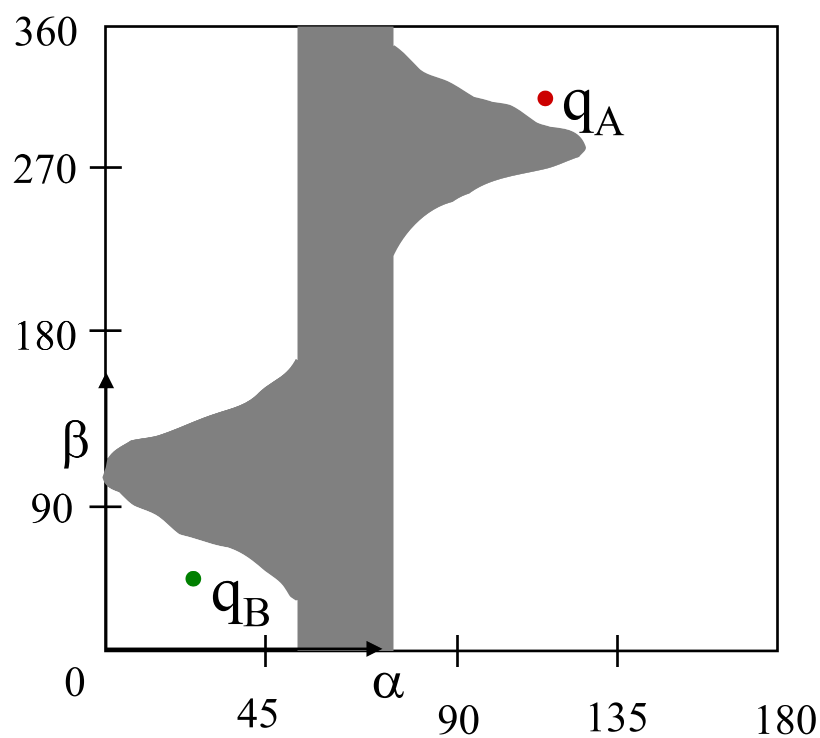
Configuration Space Obstacle

Reference *configuration*



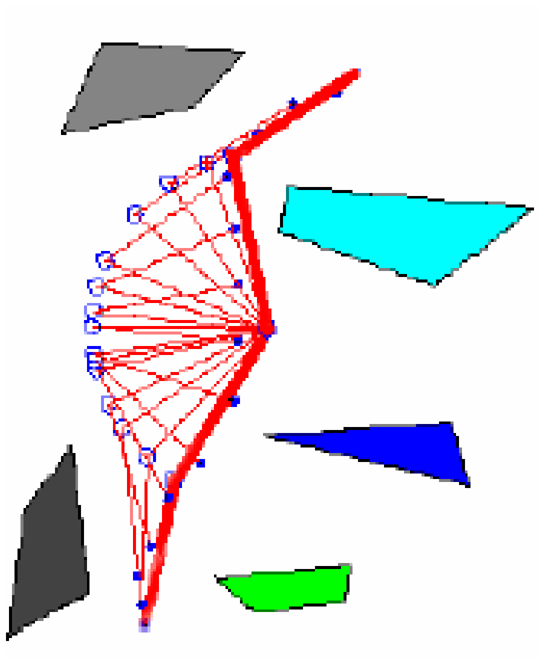
An obstacle in the robot's workspace

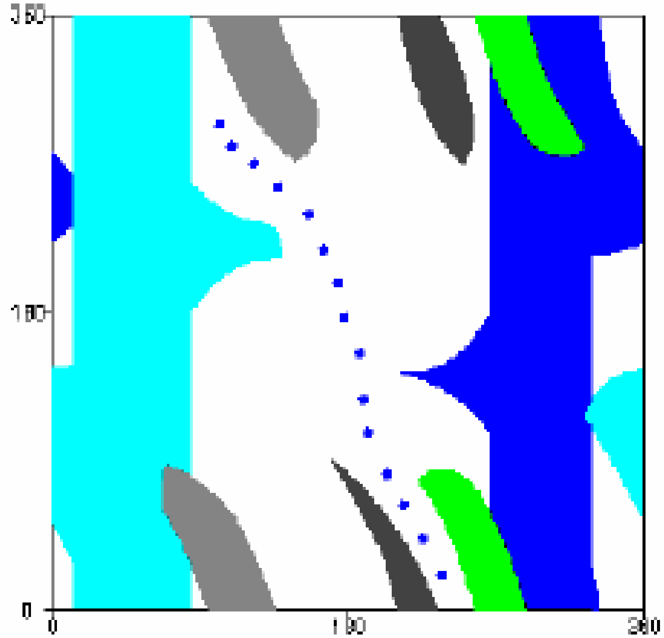
How do we get from A to B ?



The C-space representation of this obstacle…

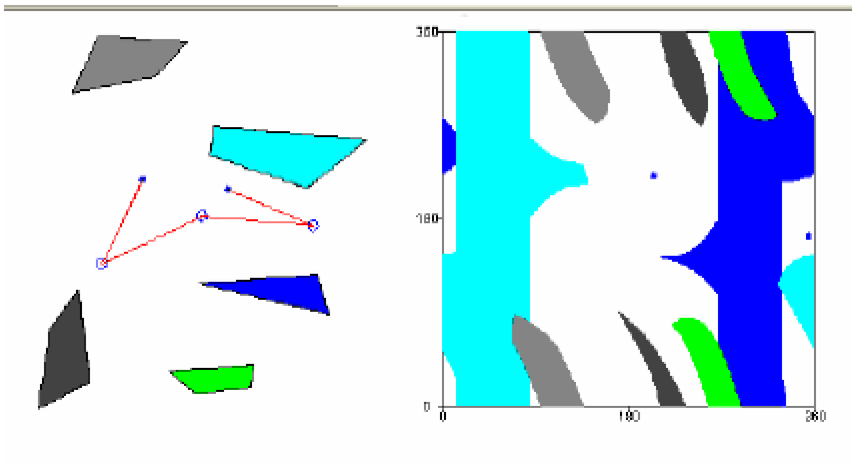
Two Link Path

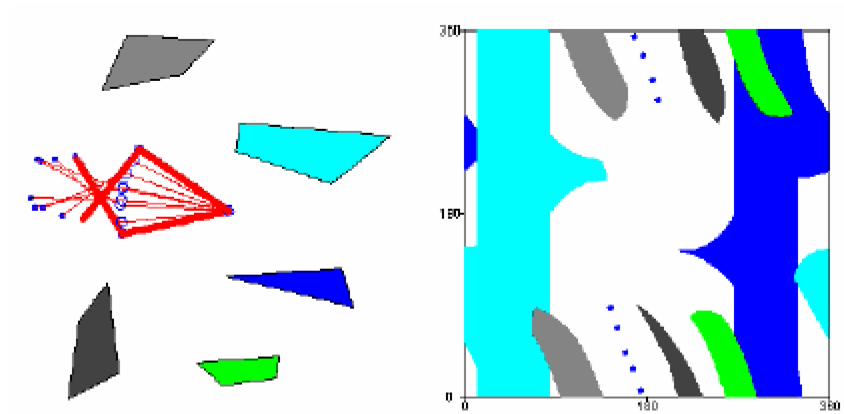




Thanks to Ken Goldberg

Two Link Path





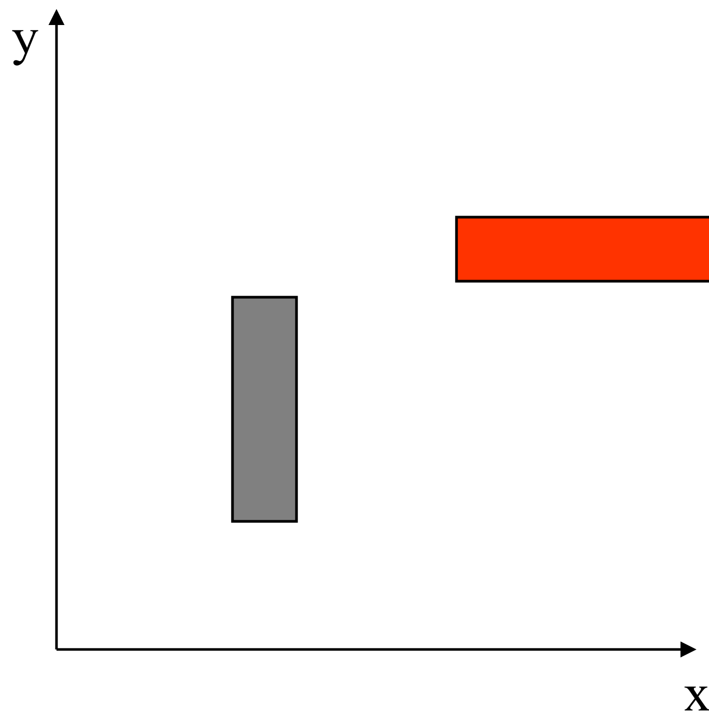
Properties of Obstacles in C-Space

- If the robot and WO_i *are __________, then*
  - *Convex then* QO_i is convex
  - *Closed then* QO_i is closed
  - *Compact then* QO_i is compact
  - *Algebraic then* QO_i is algebraic
  - *Connected then* QO_i is connected

Additional dimensions

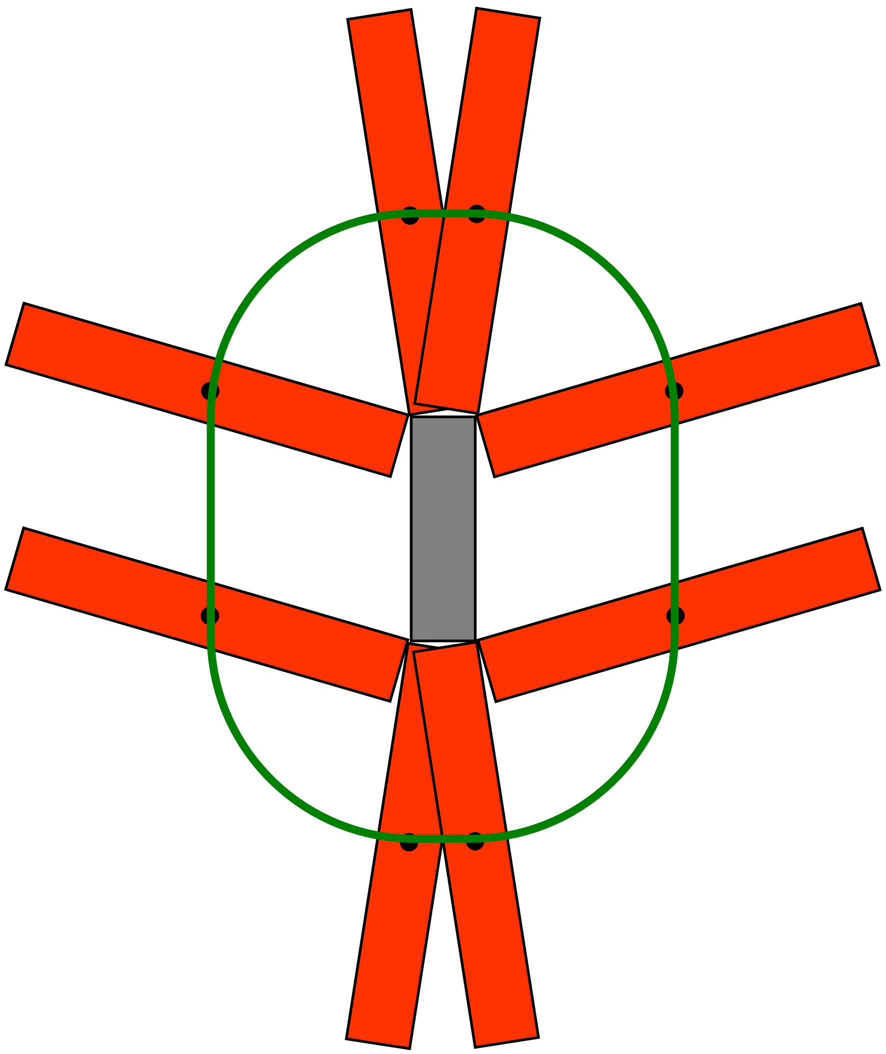
What would the configuration space of a rectangular robot (red) in this world look like? Assume it can translate *and* rotate in the plane.

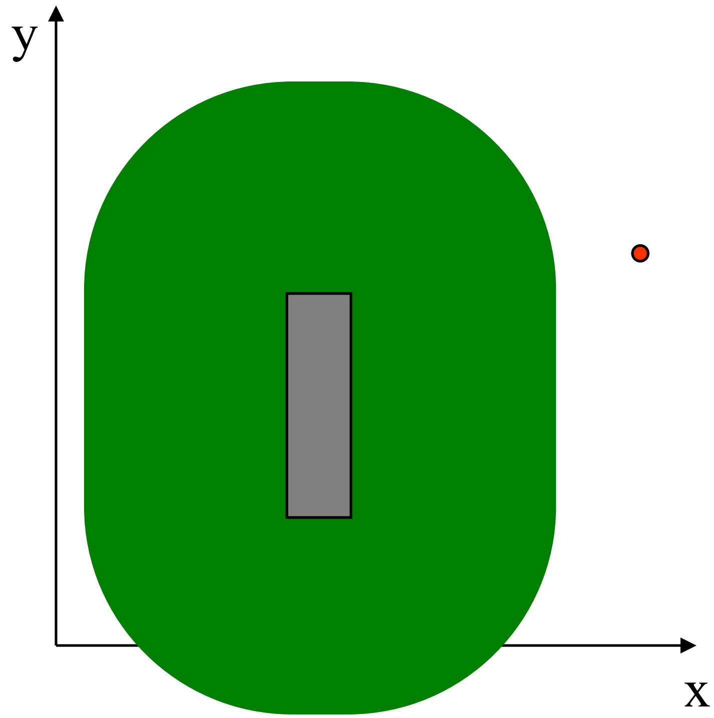
(The blue rectangle is an obstacle.)



a 2d possibility

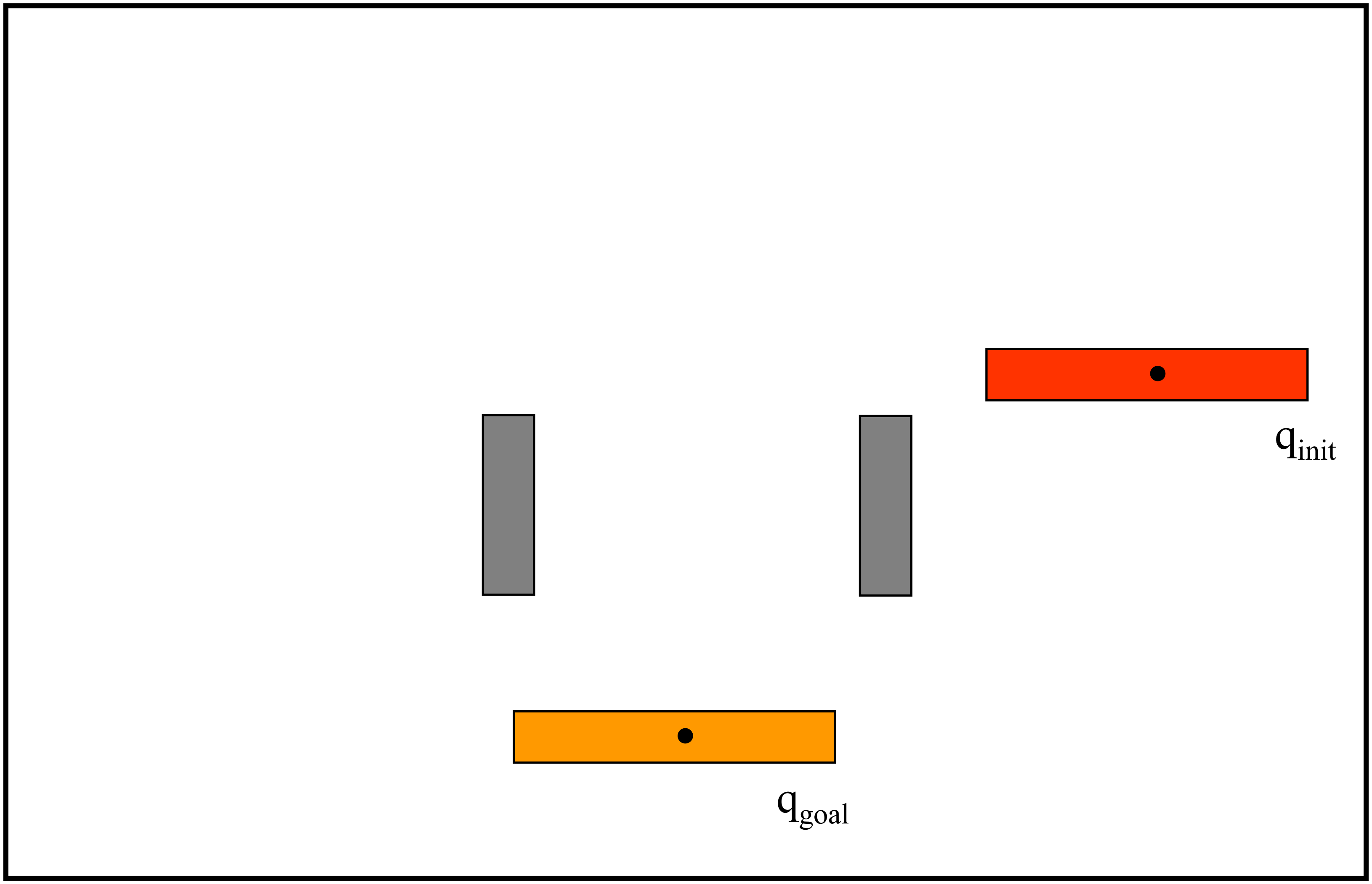
2d projection...





why not keep it this simple?

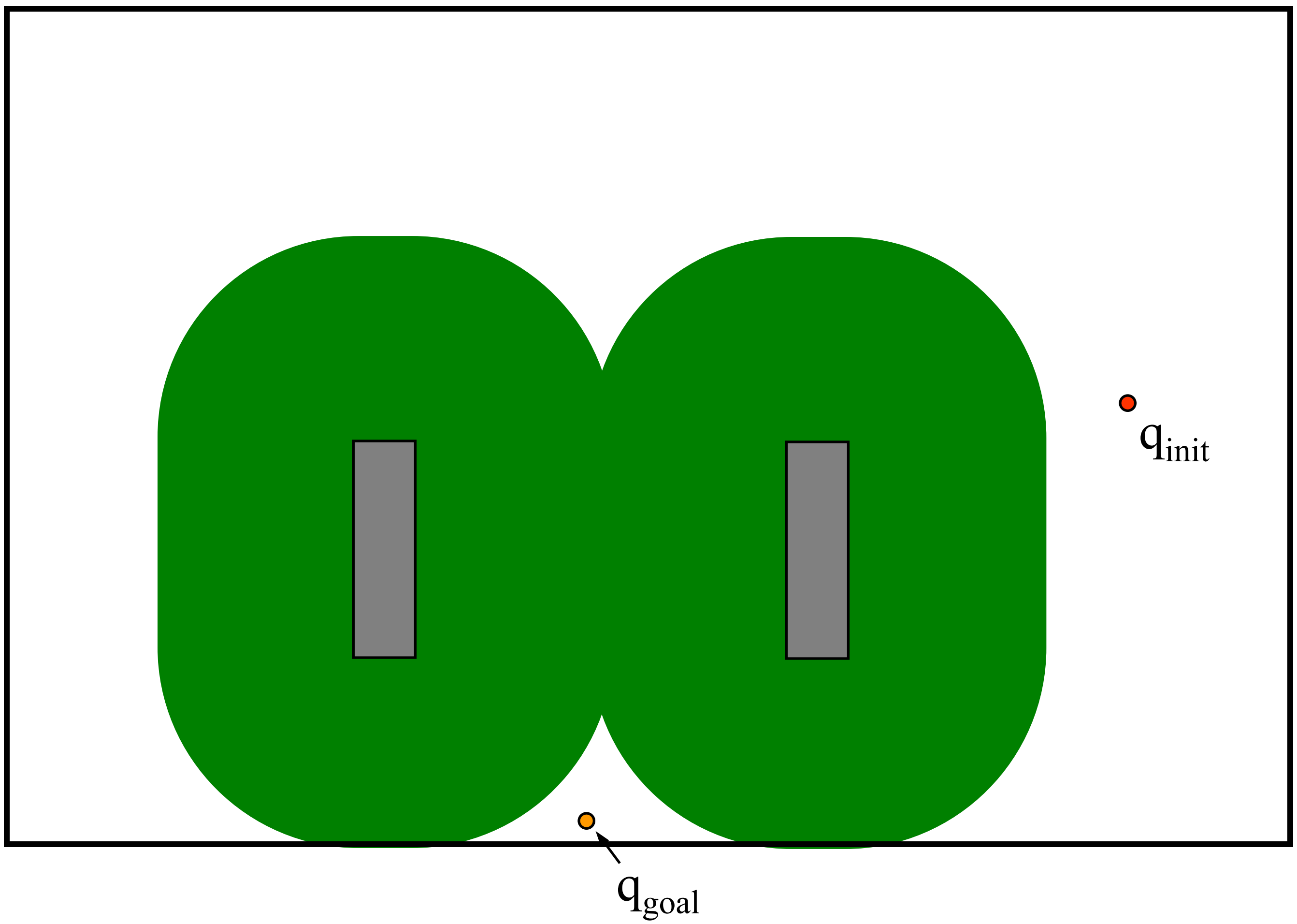
A problem?



http://www.math.berkeley.edu/~sethian/Applets/java_files_robotic_legal/robotic_legal.html

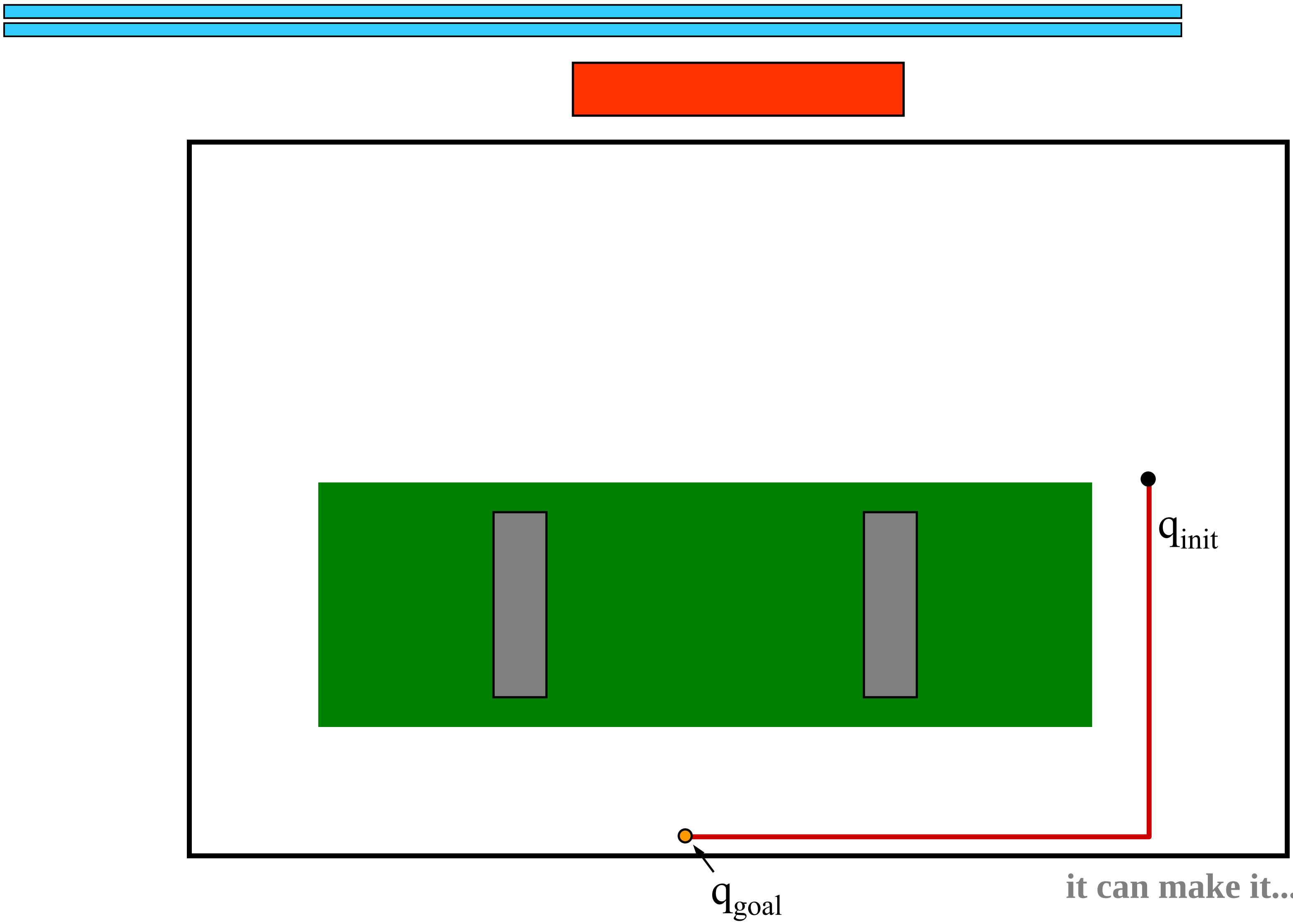
with otherwise straightforward paths

Requires one more d…



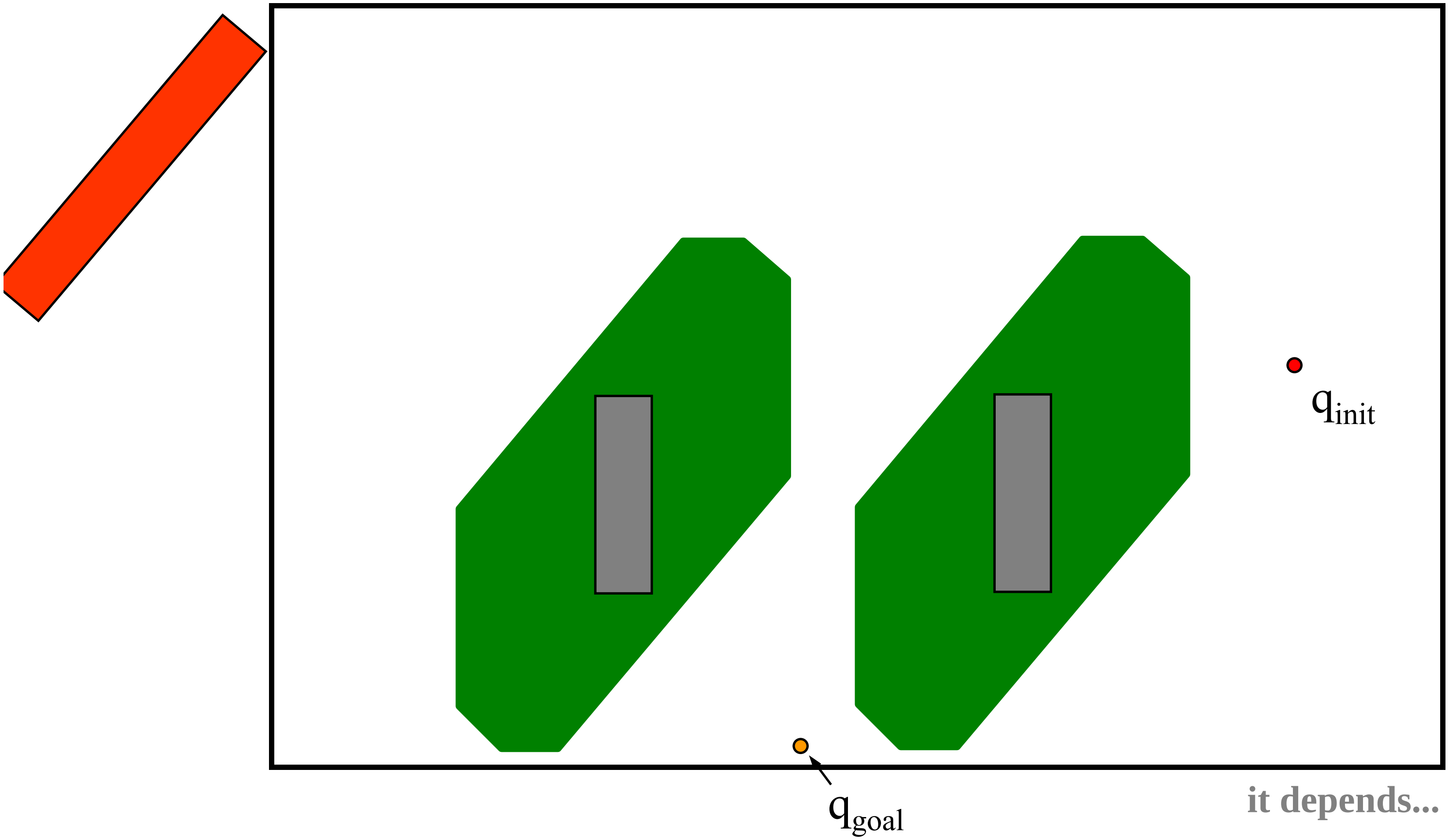
too conservative !
what instead?

When the robot is at one orientation



0º

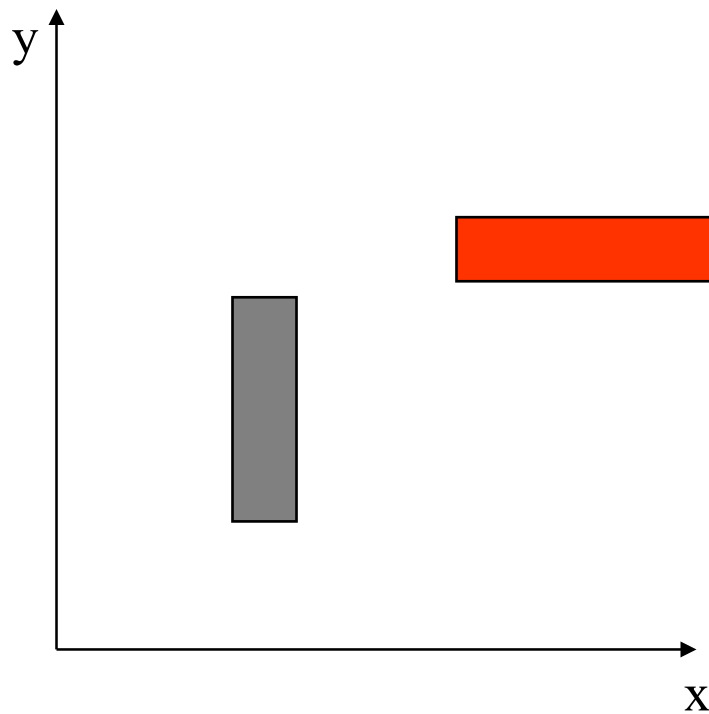
When the robot is at another orientation

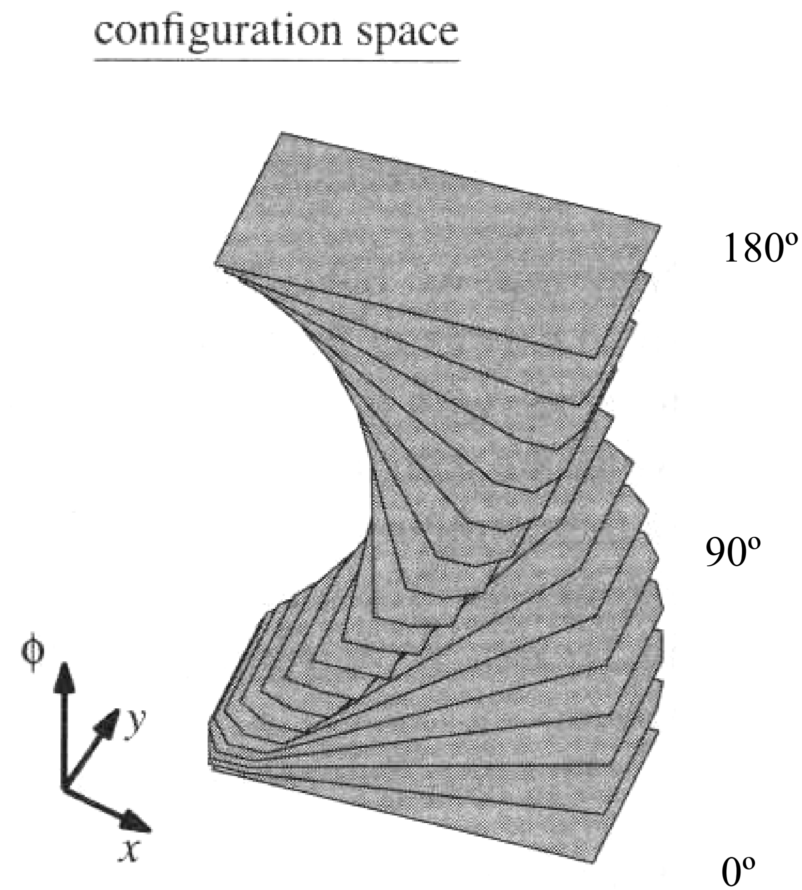


Additional dimensions

What would the configuration space of a rectangular robot (red) in this world look like?

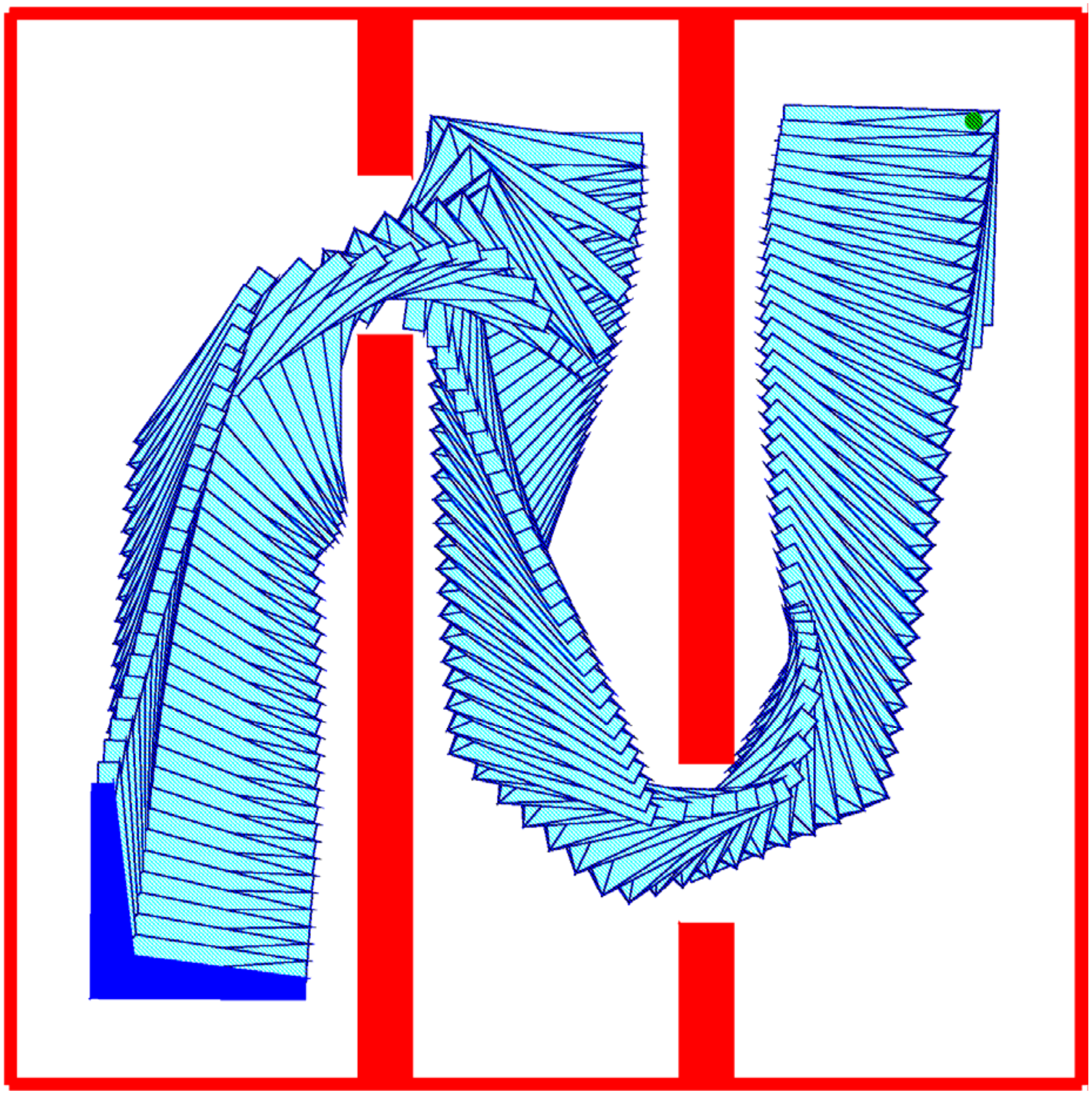
(The obstacle is blue.)



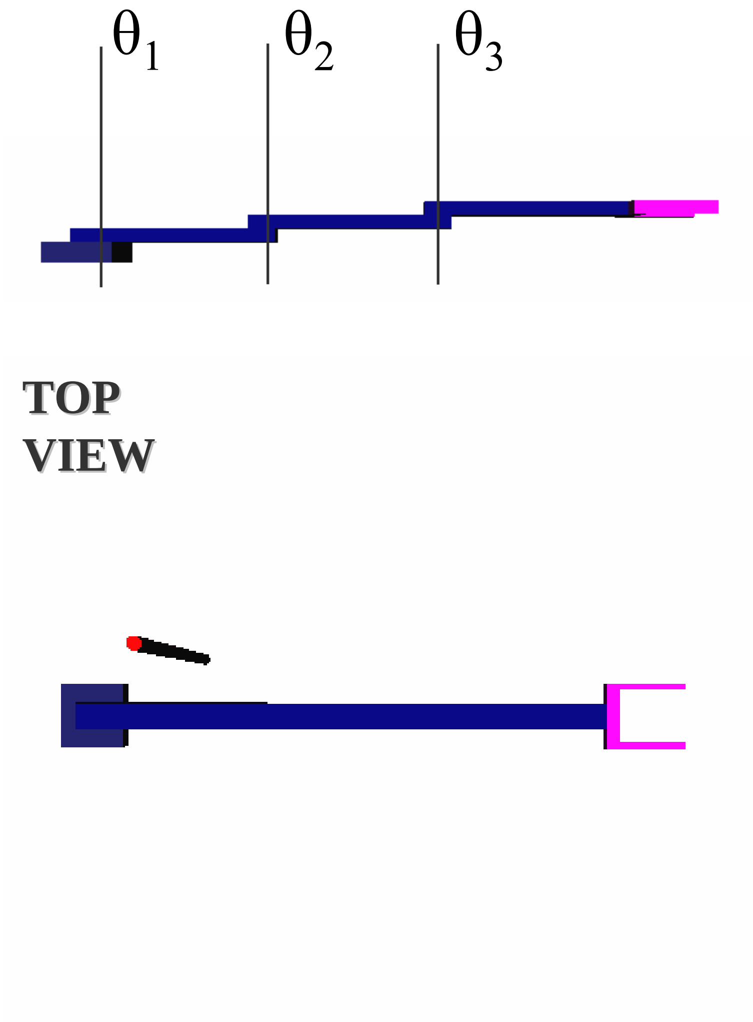


this is twisted...

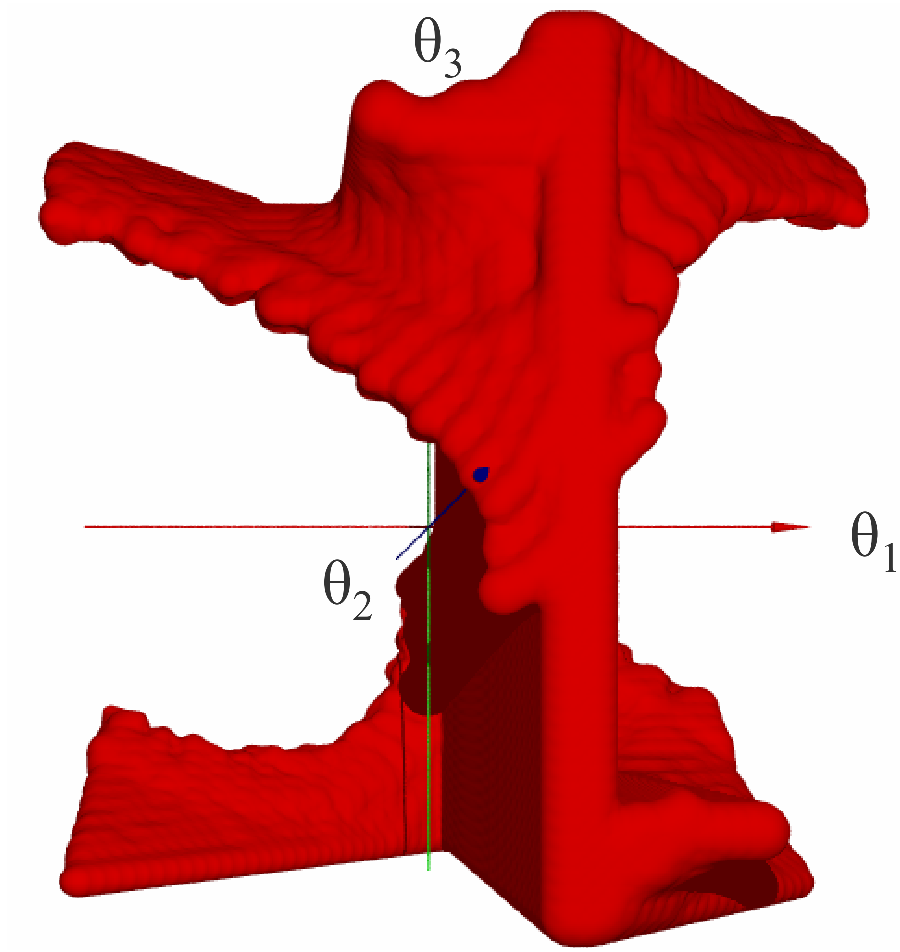
2D Rigid Object



The Configuration Space (*C*-space)

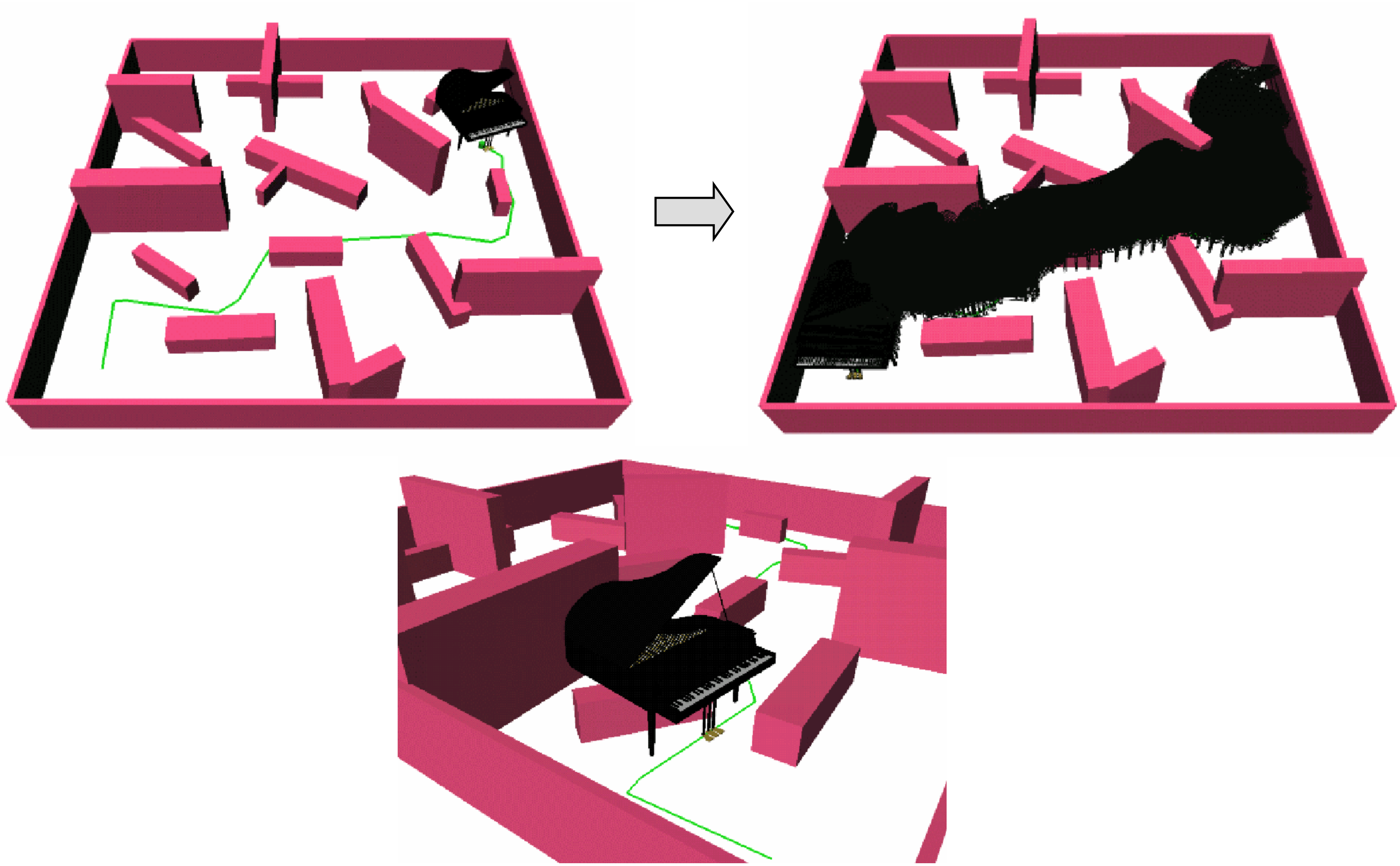


workspace



C-space

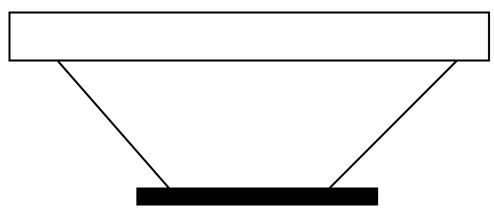
Moving a Piano



Open vs. Closed Chains

- Serial (or open) chain mechanisms can usually be understood simply by looking at how they are put together (like our 2-link manipulator)
- Closed chain mechanisms have additional internal constraints --- the links form closed loops, e.g.

Suppose 4 revolute, 2 prismatic, 6 links



Gruebler's formula: $N(k-n-1) + \sum f_i$

N = DOF of space (here 3)   f = dof of joints (here 1)   n=# of joints; k # of links