



# Puzzles with SMV

**Bug Catching** in 2006 Fall

Nov. 09, 2006

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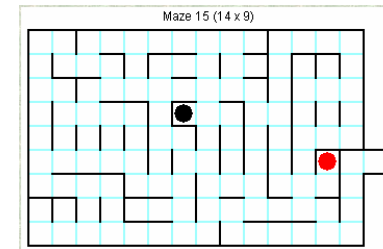
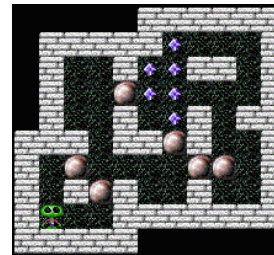
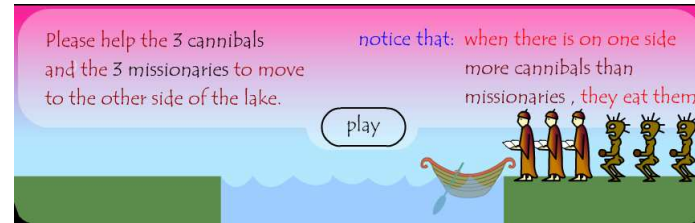
# Agenda

- Puzzles as Model Checking Problem
  - Missionary cannibal problem
  - Bridge crossing problem
  - Maze game
  - Push-Push game
  - Sokomind-PLUS game
  - Sokomind Game
- Challenge works
  - Puzzle as Bounded Model Checking Problem
  - Puzzle as Directed Model Checking Problem

# Puzzle Worlds

	6	1	2	5	
3	9			1	4
		4			
9	2	3	4		1
	8				7
1	3	6	8		9
		1			
5	4			9	1
	7	5	3	2	

6	1	8
7	5	3
2	9	4



solvable in one time  
no temporal properties

solvable in several times  
temporal properties

# Missionary Cannibal Problem

Please help the 2 cannibals and the 2 missionaries to move to the other side of the lake.

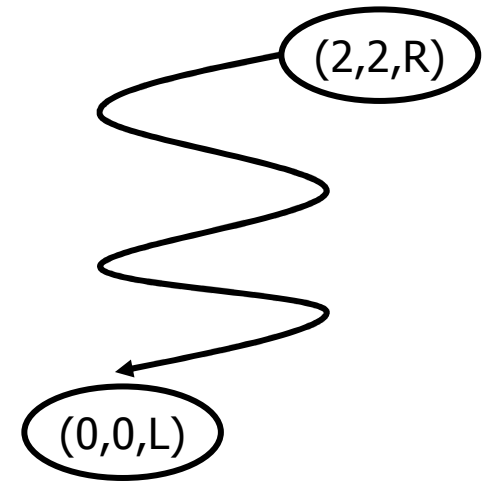
notice that: when there is on one side more cannibals than missionaries, they eat them.

play



# Missionary Cannibal Problem

- State is 3-tuple  $(M,C,B)$ 
  - $M = \{0,1,2\}$  on the right side
  - $C = \{0,1,2\}$  on the right side
  - $B = \{L,R\}$
  - initial state:  $(2,2,R)$
  - final state:  $(0,0,L)$
  - unsafe state:  $(m,c,?)$ , where  $m < c$
- Transitions
  - $(0,1)$  : 1 cannibal on board
  - $(0,2)$  : 2 cannibals on board
  - $(1,0)$  : 1 missionary on board
  - $(1,1)$  : 1 missionary and 1 cannibal on board
  - $(2,0)$  : 2 missionaries



# Missionary Cannibal Problem

State	Transition				
	(0,1)	(0,2)	(1,0)	(1,1)	(2,0)
(2,2,R)	(2,1,L)	(2,0,L)	(unsafe)	(1,1,L)	(0,2,L)
(2,1,R)	(2,0,L)	(unsafe)	(1,1,L)	(1,0,L)	(0,1,L)
(2,0,R)	(unsafe)	(unsafe)	(1,0,L)	(unsafe)	(0,0,L)
(1,1,R)	(1,0,L)	(unsafe)	(0,1,L)	(0,0,L)	(unsafe)
(0,2,R)	(0,1,L)	(0,0,L)	(unsafe)	(unsafe)	(unsafe)
(0,1,R)	(0,0,L)	(unsafe)	(unsafe)	(unsafe)	(unsafe)
(2,1,L)	(2,2,R)	(unsafe)	(unsafe)	(unsafe)	(unsafe)
(2,0,L)	(2,1,R)	(2,2,R)	(unsafe)	(unsafe)	(unsafe)
(1,1,L)	(unsafe)	(unsafe)	(2,1,R)	(2,2,R)	(unsafe)
(0,2,L)	(unsafe)	(2,2,R)	(unsafe)	(unsafe)	(2,2,R)
(0,1,L)	(0,2,R)	(unsafe)	(1,1,R)	(unsafe)	(2,1,R)
(0,0,L)	(0,1,R)	(0,2,R)	(1,0,R)	(1,1,R)	(2,0,R)
(unsafe)	(unsafe)	(unsafe)	(unsafe)	(unsafe)	(unsafe)

EF (0,0,L)

$\neg$ EF (0,0,L)

# Missionary Cannibal Problem

7% cmp.smv

File Prop View Goto History Abstraction Help

Browser Properties Results Cone Using Groups

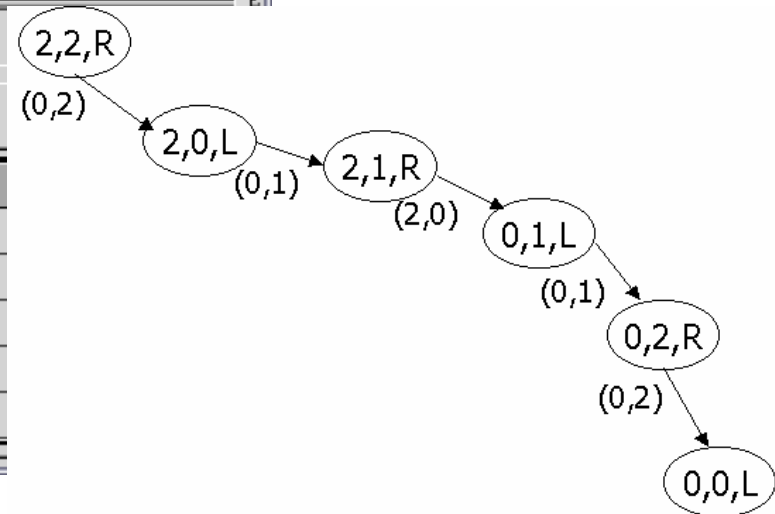
All results ↓

Property	Result	Time
$\sim(EF(((M=0)\&(C=0))\&(B=L)))$	false	Thu Nov 09 22:48:32 2006

Source Trace Log

File Edit Run View

	1	2	3	4	5	6
B	R	L	R	L	R	L
C	2	0	1	1	2	0
M	2	2	2	0	0	0



# Quiz #1

Please help the 3 cannibals  
and the 3 missionaries to move  
to the other side of the lake.

notice that: when there is on one side  
more cannibals than  
missionaries, they eat them.

play





# Solution

The screenshot shows a software window titled "cmp3.smv" with a menu bar (File, Prop, View, Goto, History, Abstraction, Help) and a toolbar (Browser, Properties, Results, Cone, Using, Groups). Below the toolbar is a dropdown menu set to "All results" with a download icon. A table displays the results of a property check.

Property	Result	Time
$\sim(EF (((M=0)\&(C=0))\&(B=L)))$	false	Thu Nov 09 22:51:59 <i>ë¸ í¸ìì í¸ì 2006</i>

Below the results table are tabs for "Source", "Trace", and "Log". A sub-window titled "File Edit Run View" contains a state transition matrix:

	1	2	3	4	5	6	7	8	9	10	11	12	
B	R	L	R	L	R	L	R	L	R	L	R	L	
C	3	2	2	0	1	1	2	2	3	1	2	0	
M	3	2	3	3	3	1	2	0	0	0	0	0	

# Crossing Bridge Problem

Please help this family to cross to the other side of the bridge.

Notice that: It is night . There is 1 lamp .


A maximum of 2 persons can cross at one time, and they must have the lamp with them.

Each person walks at a different speed: 1sec 3sec 6sec 8sec 12sec.

A pair must walk together at the rate of the slower person.

The lamp enough for 30 sec only!!!

play



# Quiz #2



¬EF (time ≤ 30 & all family on the left side & lamp also on the left side)

# Solution

lamp.smv

File Prop View Goto History Abstraction Help

Browser Properties Results Cone Using Groups

All results

Property	Result	Time
$\sim(EF ((MOT \geq 0) \& (((((J=0) \& (B=0)) \& (M=0)) \& (F=0)) \& (G=0)) \& (LA=L))))))$	false	Thu Nov 09 22:58:14

Source Trace Log

File Edit Run View

	1	2	3	4	5	6	7	8		
B	1	1	1	0	0	0	1	0		
F	1	1	1	1	1	0	0	0		
G	1	1	1	1	1	0	0	0		
J	1	0	1	0	1	1	1	0		
LA	R	L	R	L	R	L	R	L		
M	1	0	0	0	0	0	0	0		
MO	2	11	1	11	10	12	1	-		
MOT	30	24	23	20	19	7	4	1		

# Maze Games

## Theseus and the Minotaur Mazes

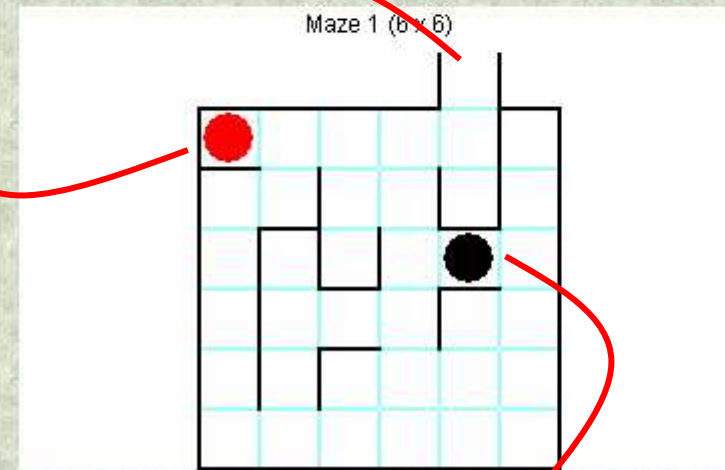
Robert Abbott invented the rules for this intriguing logic puzzle, and designed the first maze ("Maze 14" in the applet below). Take a look at his excellent [Logic Mazes](#) site for lots more information on this and many other puzzles. The other maze designs are my own inventions.

### How To Play

- Guide Theseus (the red dot) through the maze to the exit, but avoid the Minotaur (the black dot).
- The Minotaur always moves two squares towards you, if possible.
- He always chooses left / right moves when they are available.

exit

you



attacker

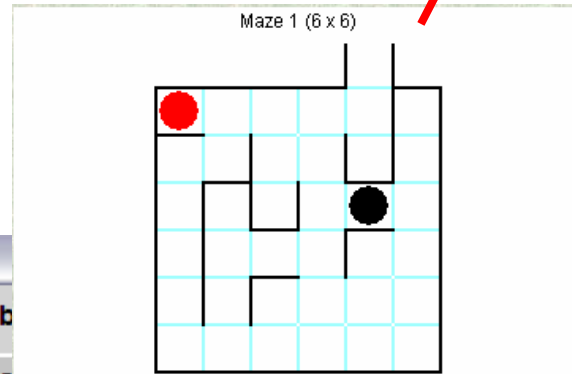
### Keys

- Cursor keys = Move Theseus.
- D = Delay (choose not to move).
- R = Reset maze.
- N = Next maze.
- P = Previous maze.
- S = Toggle sound on/off.

delay

# Maze Games

$\neg E[ \neg \text{unsafe} U \text{exit} ]$



76 maze-01.smv

File Prop View Goto History Ab Help

Browser Properties Results Cone Using Groups

All results


Property	Result	Time
$(\neg(E((\neg(v1.states=v2.states)) U (v1.states=5))))$	false	Fri Nov 10 00:13:37 06년 11월 10일 2006

Source Trace Log

File Edit Run View

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
v1.states	11	12	12	22	22	21	21	31	31	41	41	51	51	61	61	62	62	52	52	42	42	43	43	44	44	34	34	35	35	36
v2.notUp	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v2.states	35	34	24	23	23	23	23	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	



All results 

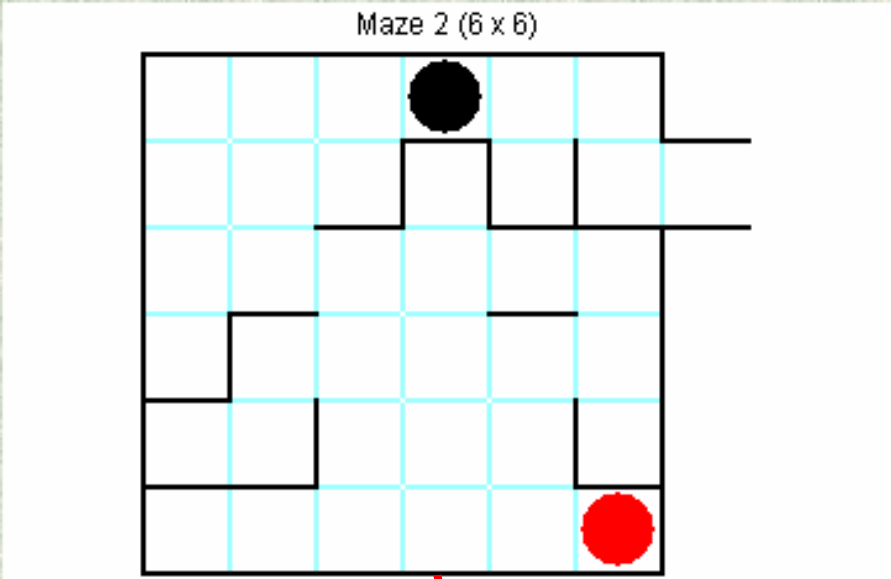
Property	Result	Time
$(\sim(E ((\sim(v1.states=v2.states)) \cup (v1.states=5))))$	false	Fri Nov 10 00:13:37 EDT 2006

	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
v1.states	36	26	26	16	16	26	26	36	36	46	46	56	56	66	66	65	65	64	64	63	63	62	62	61	61	51	51	41	41	31
v2.notUp	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v2.states	33	23	24	14	15	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	

	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
v1.states	31	21	21	22	22	12	12	22	22	21	21	31	31	41	41	51	51	61	61	62	62	61	61	62	62	52	52	42	42	43
v2.notUp	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	
v2.states	25	25	25	25	25	15	14	13	12	11	11	11	11	11	11	11	11	11	11	12	22	21	31	41	51	51	51	41	41	41

	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118
v1.states	42	43	43	44	44	54	54	55	55	56	56	66	66	56	56	46	46	36	36	35	35	34	34	24	24	14	14	15	15	5
v2.notUp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	
v2.states	41	41	41	41	41	51	51	51	51	51	51	61	62	63	64	65	66	56	46	45	45	45	45	45	45	45	45	45	45	45

# Quiz #3



$\neg E[ \neg \text{unsafe } U \text{ exit}]$



# Solution

76 maze - Solution

File Prop View Goto History Abstraction Help

Browser Properties Results Cone Using Groups

All results

Property	Result	Time
$\sim(E((\sim(v1.states=v2.states)) \cup (v1.states=27)))$	false	Fri Nov 10 00:13:05 EDT 2006

Source Trace Log

File Edit Run View

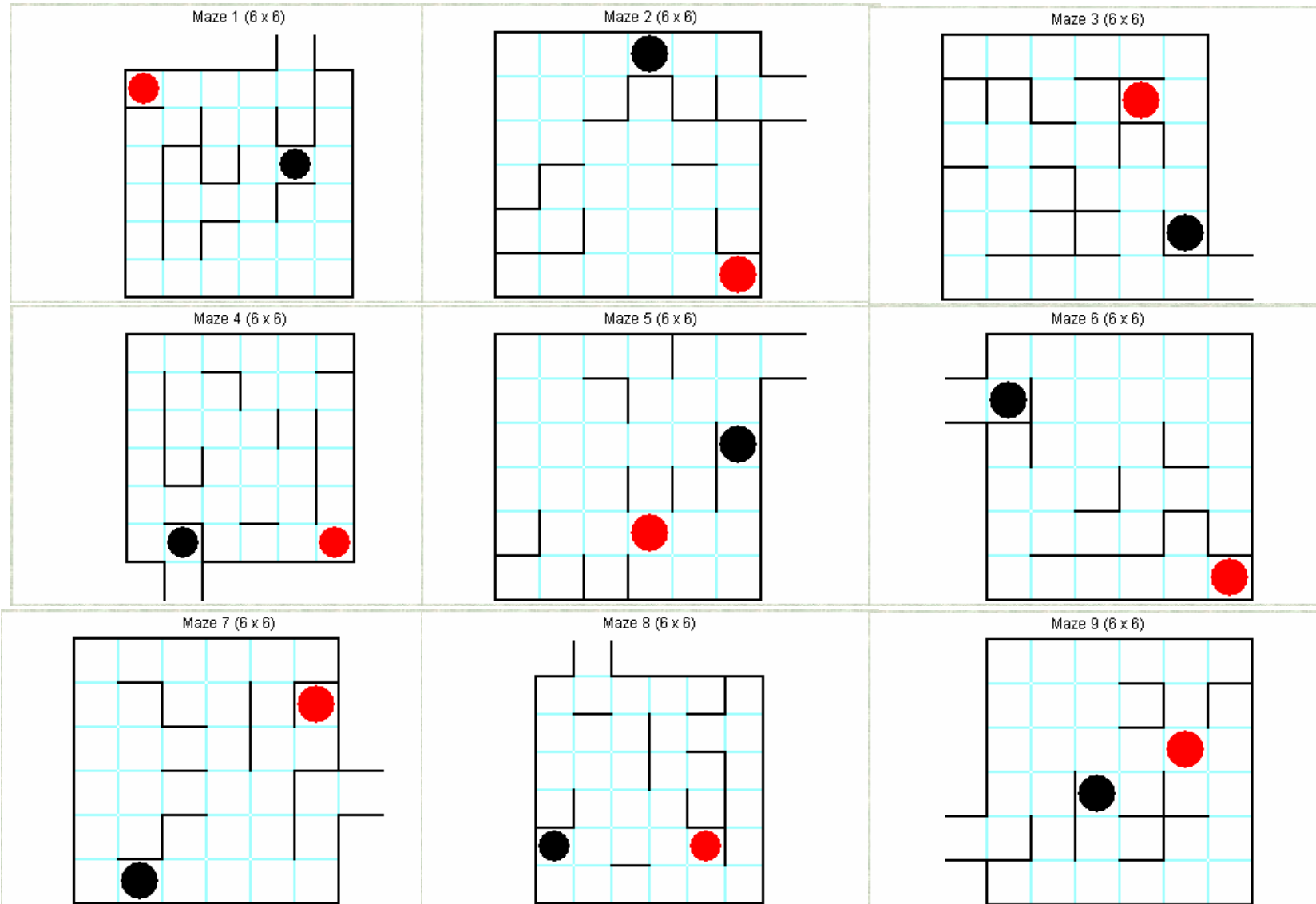
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
v1.states	66	66	66	66	66	65	65	64	64	54	54	44	44	34	34	33	33	32	32	22	22	12	12	22	22	32	32	33	33	43
v2.notUp	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
v2.states	14	15	16	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	16	15	14	13	12	22	23	23	23

	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
v1.states	43	42	42	52	52	51	51	52	52	42	42	43	43	44	44	34	34	44	44	45	45	55	55	65	65	66	66	66	66	65
v2.notUp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	0	0	0	0
v2.states	23	22	32	32	32	31	41	41	41	41	41	41	41	41	41	31	32	33	34	35	35	35	35	35	35	36	46	56	56	56

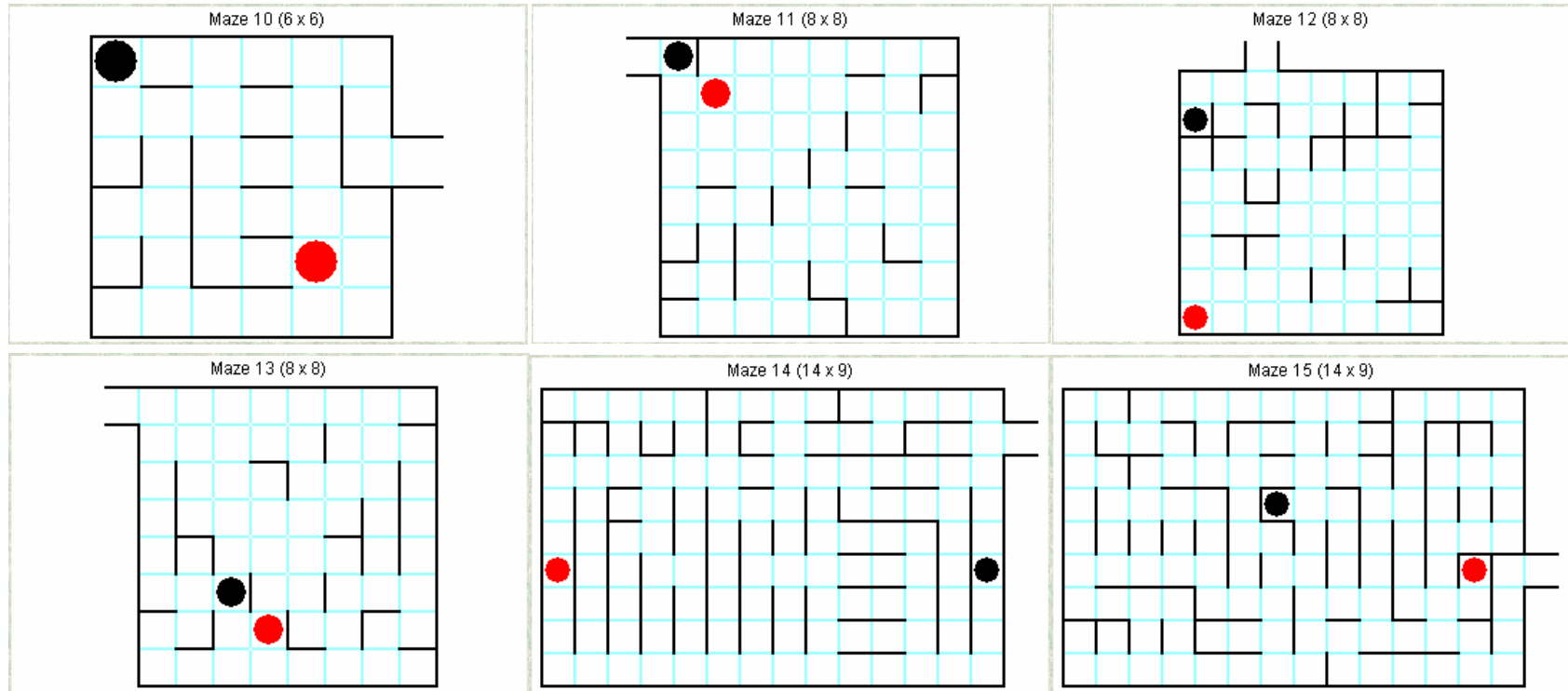
	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88
v1.states	66	65	65	64	64	54	54	53	53	43	43	33	33	32	32	22	22	23	23	13	13	14	14	15	15	16	16	26	26	27
v2.notUp	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1
v2.states	56	56	56	56	56	56	56	56	56	46	45	44	43	42	42	42	42	43	33	33	33	34	24	24	24	24	24	24	24	24

# Maze Games

15 levels



# Maze Games

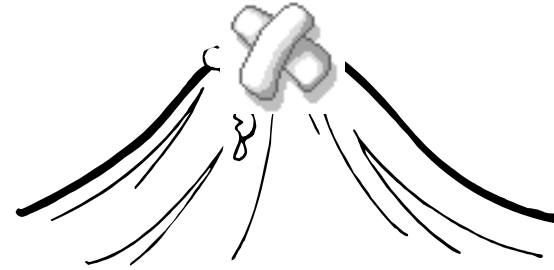


# Model Checking Results

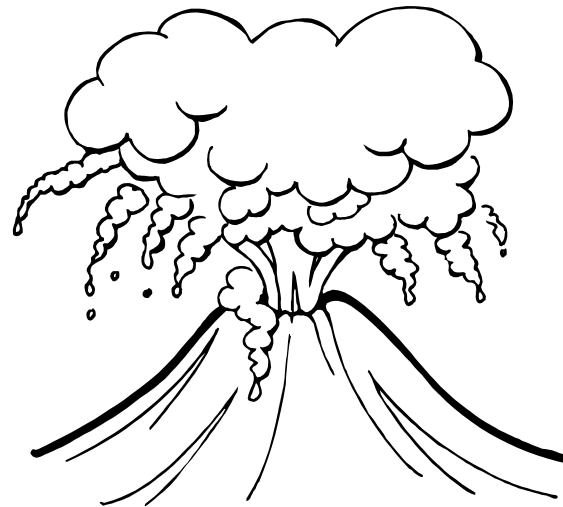
Level	Size	Time(sec)	Memory(KB)	BDD	Steps
1	6x6	0.16	2440	35922	59
2	6x6	0.18	2384	33563	44
3	6x6	0.17	2312	28107	48
4	6x6	0.18	2420	33894	37
5	6x6	0.13	2300	28401	25
6	6x6	0.20	2348	31277	49
7	6x6	0.18	2468	37450	43
8	6x6	0.22	2564	42667	63
9	6x6	0.22	2484	38182	52
10	6x6	0.20	2392	33568	27
11	8x8	0.46	3188	76334	66
12	8x8	0.47	3156	74311	79
13	8x8	0.44	3308	83440	72
14	14x9	1.57	5288	39799	94
15	14x9	2.06	6320	237181	184

# So Far

😊 Are you happy ?



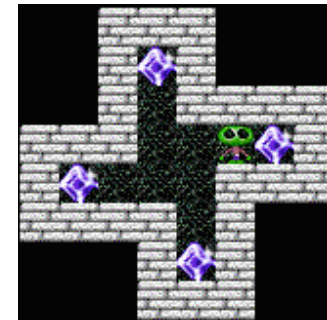
☹️ When model checking people sad ?



# Push Push Games



box  
goal position  
agent



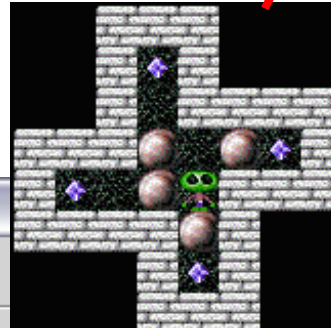
goal state

## Two questions

Is there a feasible path to a goal state ?  
Which one is optimal ?

# Push Push Games

¬EF (goal state)



76 pushpush01\_cell12.smv

File Prop View Goto History Help

Browser Properties Results Cone Using Groups

All results

Property	Result	Time
(AG (~(((ball1.s13&ball1.s36)&ball1.s41)&ball1.s64)))	false	Fri Nov 10 01:47:30 0000

Source Trace Log

File Edit Run View

	1	2	3	4	5	6	7	8	9	10	11
P.Direction	down	up	left	left	right	up	up	down	right	right	-
P.Down	0	0	0	1	1	1	0	0	0	0	1
P.Left	0	1	0	0	0	0	1	1	1	0	0

# Quiz #4



¬EF (13 & 23 & 33 & 43 & 53 & 63)





# Push Push Games

50 levels



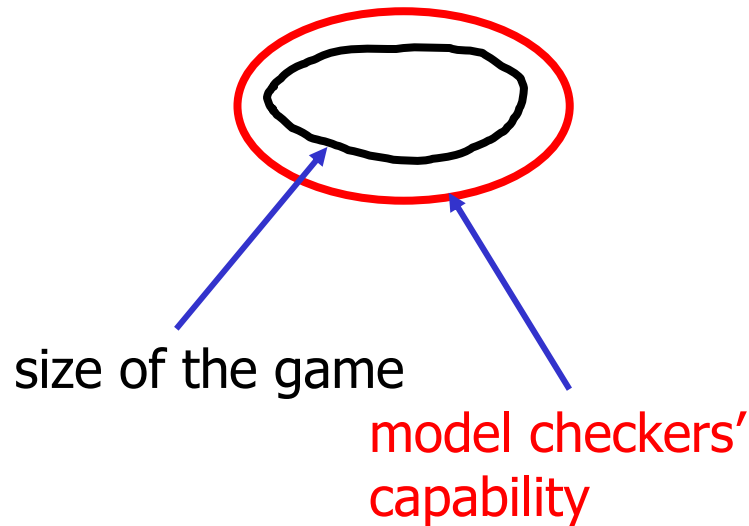
# Model Checking Results

Level	Time(sec)	Memory(MB)	BDD	Level	Time(sec)	Memory(MB)	BDD
1	0.6	2	10190	26	60.0	34	1352886
2	3.6	6	162264	27	30.0	20	753887
3	33.8	32	1179417	28	420.0	210	8046131
4	0.7	3	43314	29	16.0	13	493167
5	1.0	3	45487	30	> 24 hours		
6	123.3	93	3293300	31	1020.0	500	20533835
7	53.6	37	1493151	32	> 24 hours		
8	> 24 hours			33	> 24 hours		
9	11.0	11	352558	34	> 24 hours		
10	5.0	8	260339	35	> 24 hours		
11	6.2	8	260339	36	> 24 hours		
12	10.7	12	409438	37	2200.0	854	34330495
13	355.3	240	9794593	38	28.0	18	635604
14	125.8	80	3490931	39	960.0	336	15431568
15	> 24 hours			40	> 24 hours		
16	16.5	16	514515	41	120.0	58	2245640
17	> 24 hours			42	1260.0	532	23060315
18	1200.0	508	22391859	43	480.0	299	13302604
19	1200.0	613	27678695	44	1860.0	539	23143060
20	> 24 hours			45	300.0	109	4242756
21	2880.0	1109	48330206	46	780.0	413	15742937
22	420.0	219	8571816	47	35.0	25	1019209
23	3000.0	870	34901698	48	1980.0	841	33009636
24	600.0	317	14238083	49	> 24 hours		
25	10.0	305	13229503	50	> 24 hours		



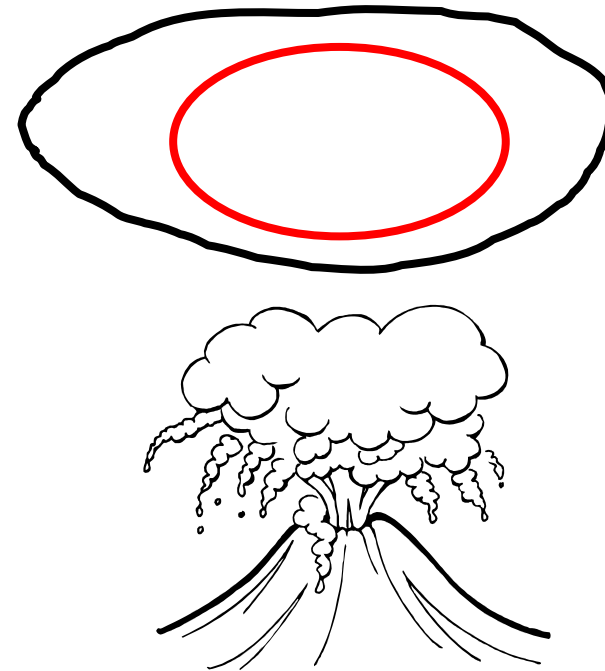
# Naïve Model Checking Results

**37 levels are solved**



no state explosion 😊

**13 levels are failed**

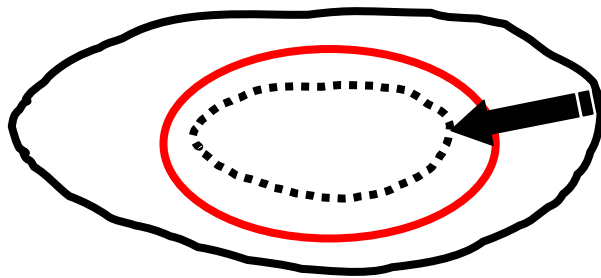


state explosion occurs ☹️

**How to avoid the state explosion ?**

# State Explosion Avoidance

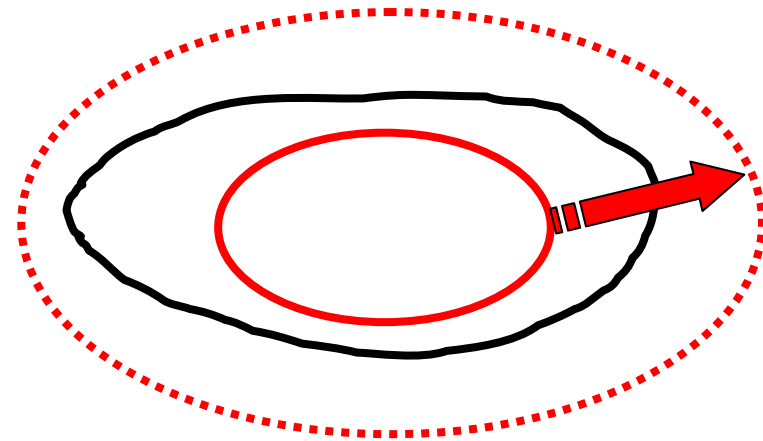
## Defensive approach



- Abstraction
- Symmetry reduction
- Partial order reduction

.....

## Offensive approach

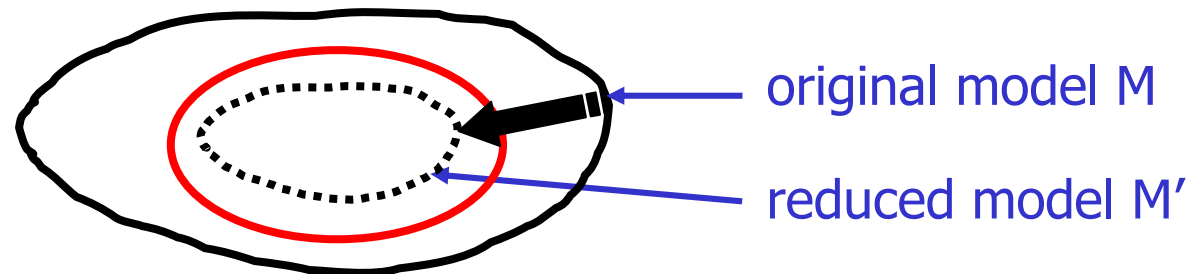


- Efficient data structures
- Distributed and parallelized
- Buy an expensive computer

.....

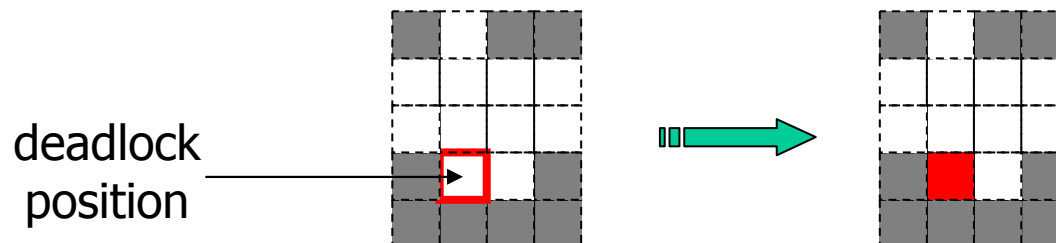
# Abstraction

**Reduction of the original model into a smaller one**

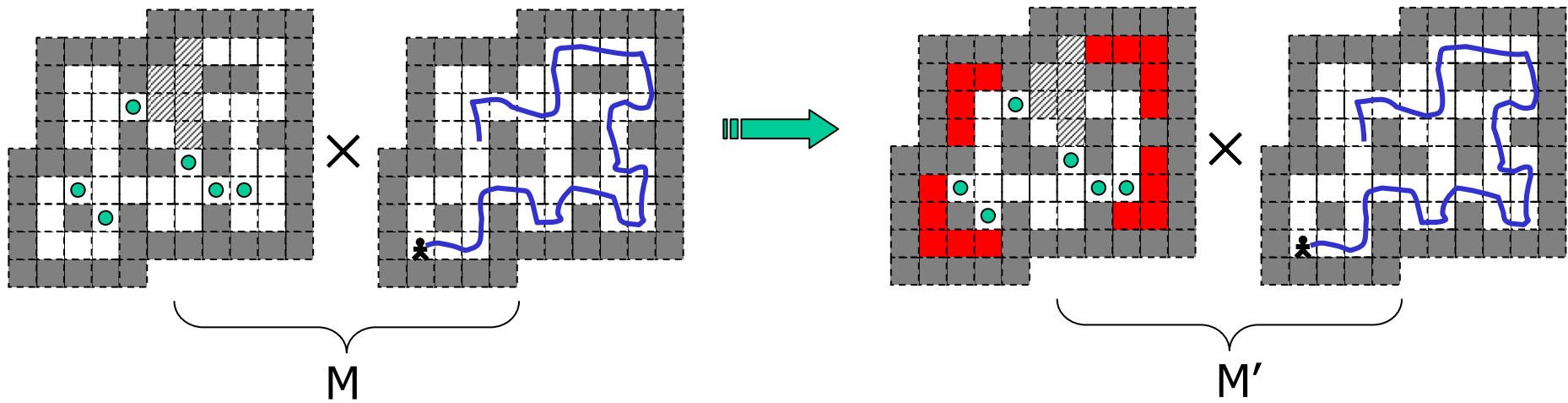


- Removing irrelevant parts w.r.t. properties to be verified
- Reduced model M' still preserves properties of interests

**Which parts abstracted away ?**



# Abstraction



$M \equiv M'$  w.r.t. witness

$M' \models \text{witness} \Rightarrow M \models \text{witness}$

# Abstraction Results

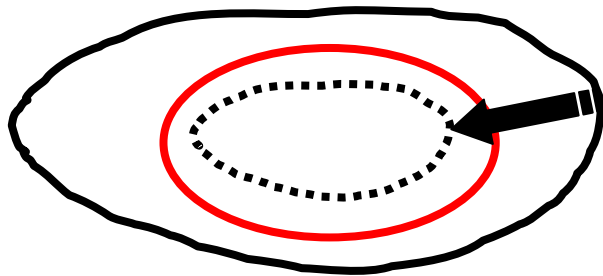
Level	Time(sec)	Memory(MB)	BDD	Level	Time(sec)	Memory(MB)	BDD
1	0.1	< 1	6523	26	5.4	8	269886
2	0.5	5	53669	27	3.0	7	233811
3	2.6	7	181424	28	4.8	13	376884
4	0.5	< 1	51241	29	1.4	5	118553
5	0.3	< 1	23324	30	248.1	< 1	28265959
6	7.5	11	399445	31	8.3	16	655700
7	8.1	13	361845	32	9.9	20	759229
8	53.9	105	4037813	33	168.2	298	9446378
9	0.3	< 1	23287	34	101.8	113	4510854
10	0.4	< 1	34465	35	98.6	133	5571438
11	0.3	< 1	32802	36	73.6	122	5004938
12	1.5	6	135401	37	7.7	18	647170
13	0.4	< 1	29385	38	3.0	8	237044
14	2.3	7	172012	39	10.4	11	399459
15	167.8	190	8495225	40	4.3	13	357824
16	0.5	< 1	45514	41	3.4	10	310274
17	415.0	470	13184027	42	25.0	40	1195649
18	8.1	21	586773	43	223.3	272	11769524
19	209.0	325	11037872	44	12.9	18	659379
20	224.0	358	13007571	45	6.5	13	336695
21	14.8	26	893813	46	10.2	17	595338
22	52.5	59	2337064	47	1.8	5	134084
23	79.0	67	4278917	48	16.6	35	1390943
24	7.1	17	611142	49	234.2	321	10880846
25	13.5	25	697371	50	> 24 hours		





# Abstraction Results

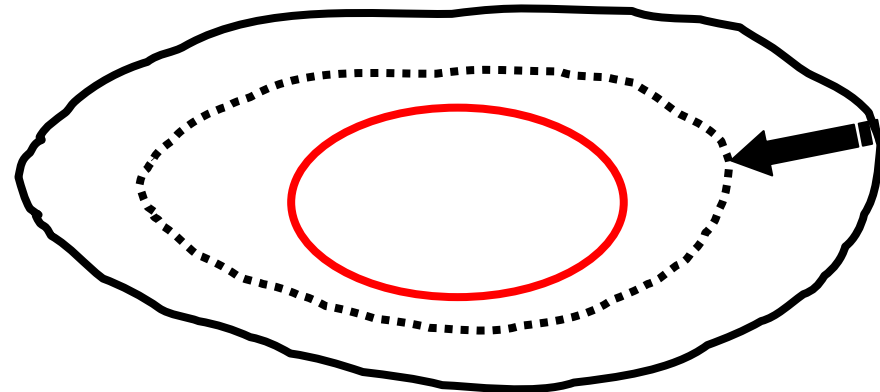
49 levels are solved



no state explosion



Last level is failed



state explosion occurs

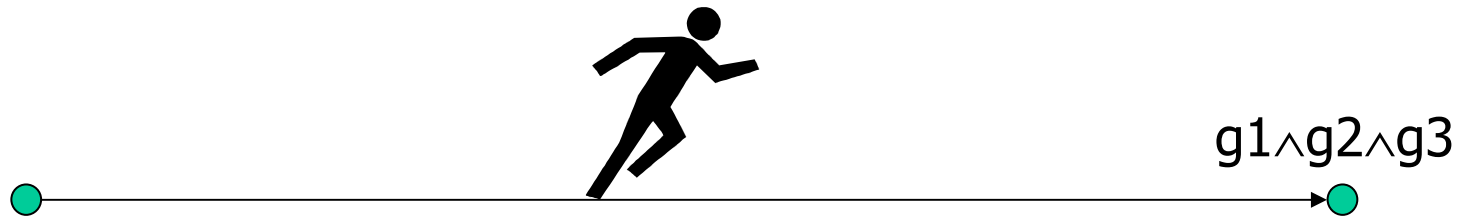


A reduced model is still too large

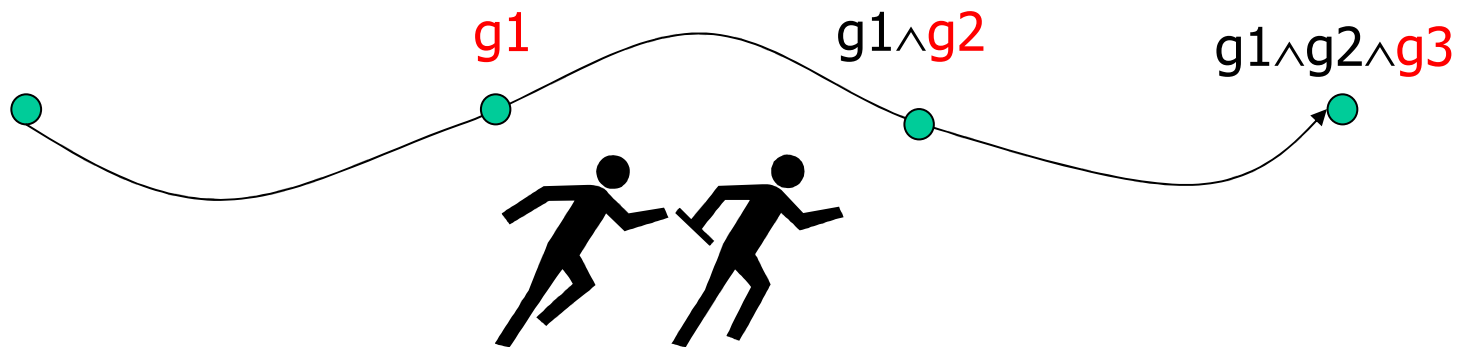
**What have to do when abstraction is failed ?**

# Relay Model Checking

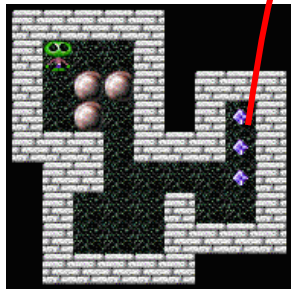
## Conventional model checking



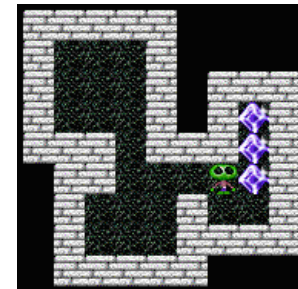
## Divide and conquer



# Relay Model Checking



-EF (73 & 74 & 75)



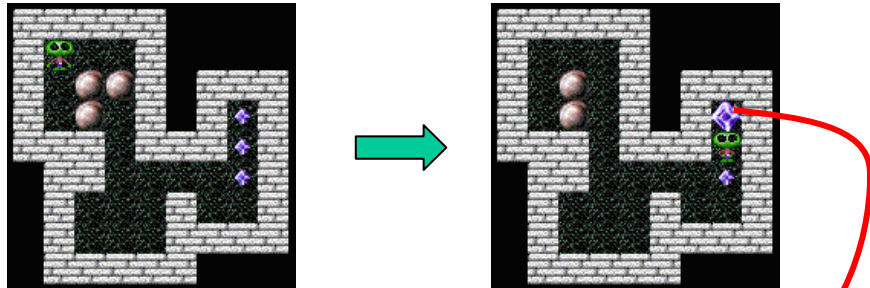
(~(EF ((ball1.s75&ball1.s74)&ball1.s73))) false      Fri Dec 01 01:23:32 오후 1월 1일 2006

Source	Trace	Log																												
File Edit Run View																														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
P.Direction	rig	rig	dow	dow	dow	dow	rig	dow	dow	lef	lef	up	rig	dow	rig	up	lef	up	up	up	up	up	lef	lef	dow	rig	up	rig	dow	dow

31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	
dow	dow	rig	rig	rig	dow	rig	up	up	dow	lef	lef	lef	dow	dow	lef	lef	up	rig	dow	rig	up	lef	up	up	up	up	up	lef	lef	dow

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
rig	up	rig	dow	dow	dow	rig	rig	rig	dow	rig	up	lef	lef	lef	dow	dow	lef	lef	up	rig	dow	rig	up	lef	up	rig	rig	rig	-

# Relay Model Checking



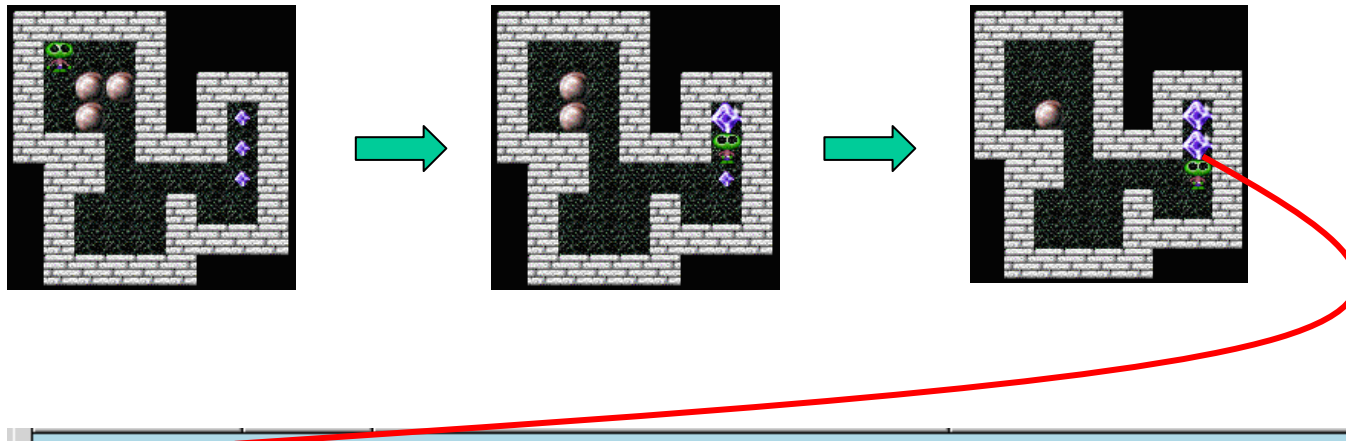
(~(EF ball1.s75)) false    Fri Dec 01 01:29:41 2006

Source    Trace    Log

File    Edit    Run    View

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
P.Direction	rig	rig	dow	dow	dow	dow	rig	dow	dow	lef	lef	up	rig	dow	rig	up	lef	up	rig	rig	rig	dow	rig	up	up	-	
P.Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	
P.s	17	27	37	36	35	34	33	43	42	41	31	21	22	32	31	41	42	32	33	43	53	63	62	72	73	74	

# Relay Model Checking



```
(~(EF ball1.s74)) false      Fri Dec 01 01:43:13 06 1월 1일 2006
```

---

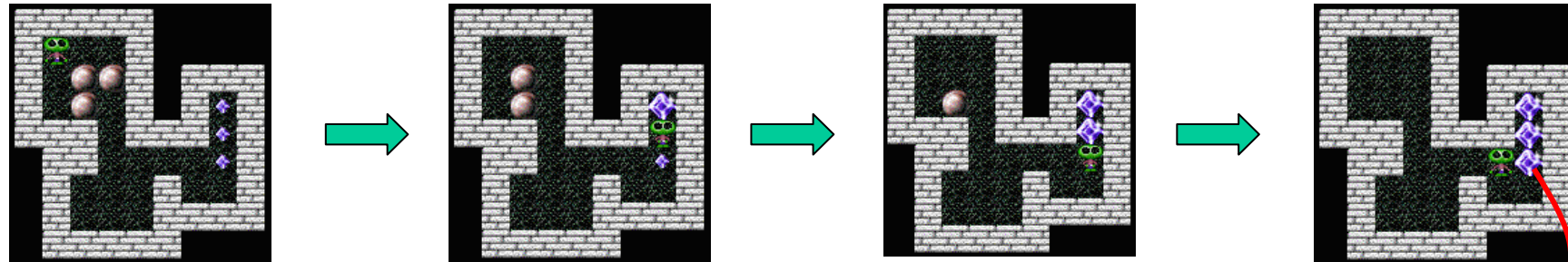
**Source** | **Trace** | **Log**

---

**File** | **Edit** | **Run** | **View**

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
P.Direction	dc	le	le	le	le	up	up	up	up	le	le	dc	ri	up	ri	dc	dc	dc	dc	ri	dc	dc	le	le	up	ri	dc	ri	up	le	up	ri	ri	ri	dc	ri	up	-
P.canright	1	1	1	1	1	1	0	0	0	0	1	1	1	0	1	0	0	0	0	1	1	0	0	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1
P.s	74	73	63	53	43	33	34	35	36	37	27	17	16	26	27	37	36	35	34	33	43	42	41	31	21	22	32	31	41	42	32	33	43	53	63	62	72	73

# Relay Model Checking



(~(EF ball1.s73)) - false    Fri Dec 01 01:57:46 EDT 2006

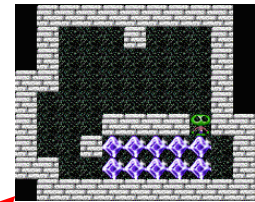
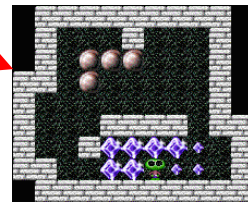
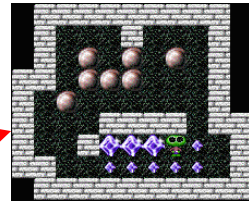
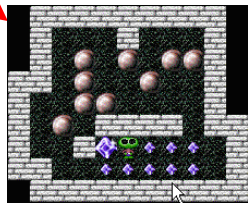
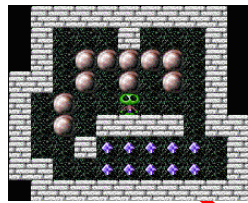
Source    Trace    Log

File    Edit    Run    View

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
P.Direction	le	le	le	le	up	up	up	le	le	dc	ri	up	ri	dc	dc	dc	ri	dc	dc	le	le	up	ri	dc	ri	up	le	up	ri	ri	ri	-
P.canup	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	0	1	1	0	0	0	
P.s	73	63	53	43	33	34	35	36	26	16	15	25	26	36	35	34	33	43	42	41	31	21	22	32	31	41	42	32	33	43	53	63

# Results

## Good news



338 steps

## Bad news

338 steps is not optimal

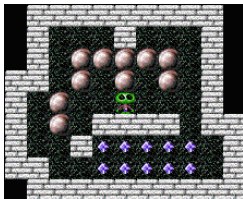
**How to get the optimal solution ?**

# Observations

## #1. Conservative, not complete

- Not always working.
- But if it works, then the path is a witness in an original model.

## #2. Goal ordering is important



→ Linear ordering of subgoals

## #3. No guarantees an optimal solution

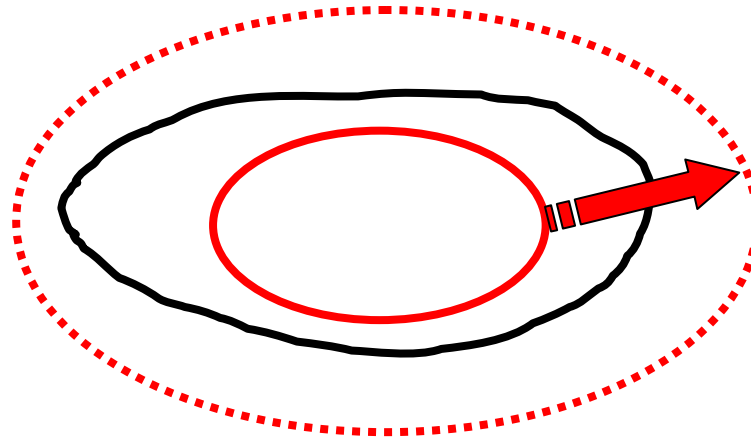
- Optimization is needed

## #4. Sometimes, we get lost on the way to a goal

- Guidance is needed

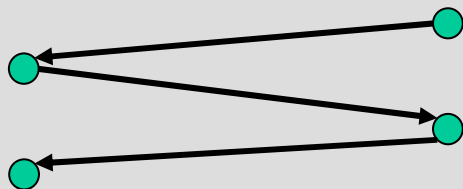


# Modified Model Checker

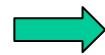


## Current NuSMV

- 3 times traversals

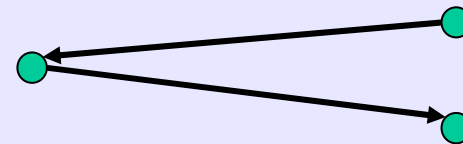


- BDD storage on RAM

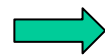


## Modified NuSMV

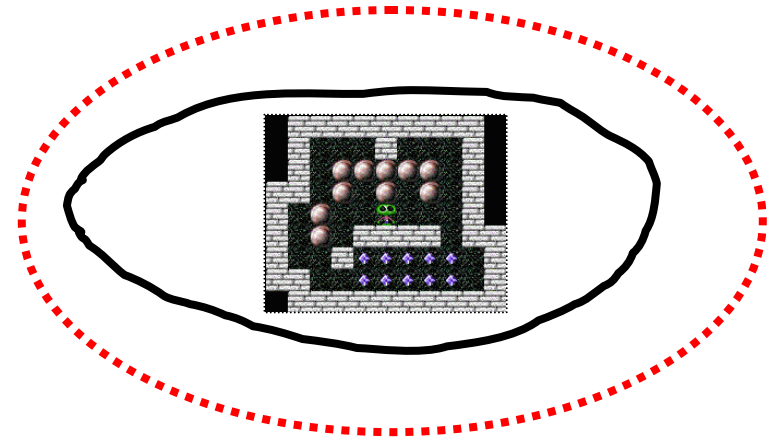
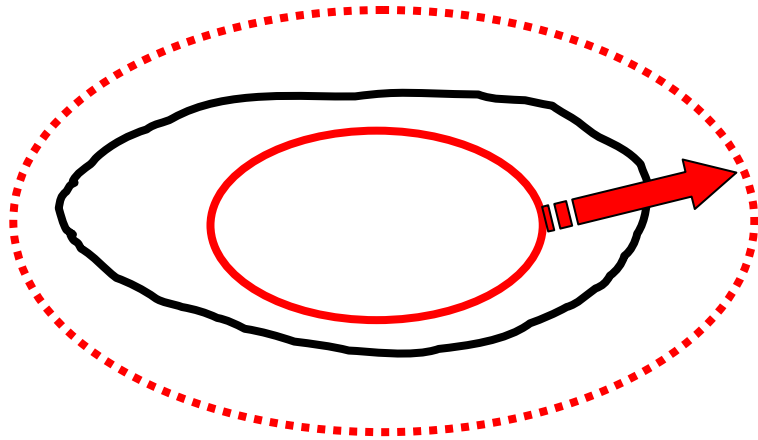
- 2 times traversals



- BDD storage on HD



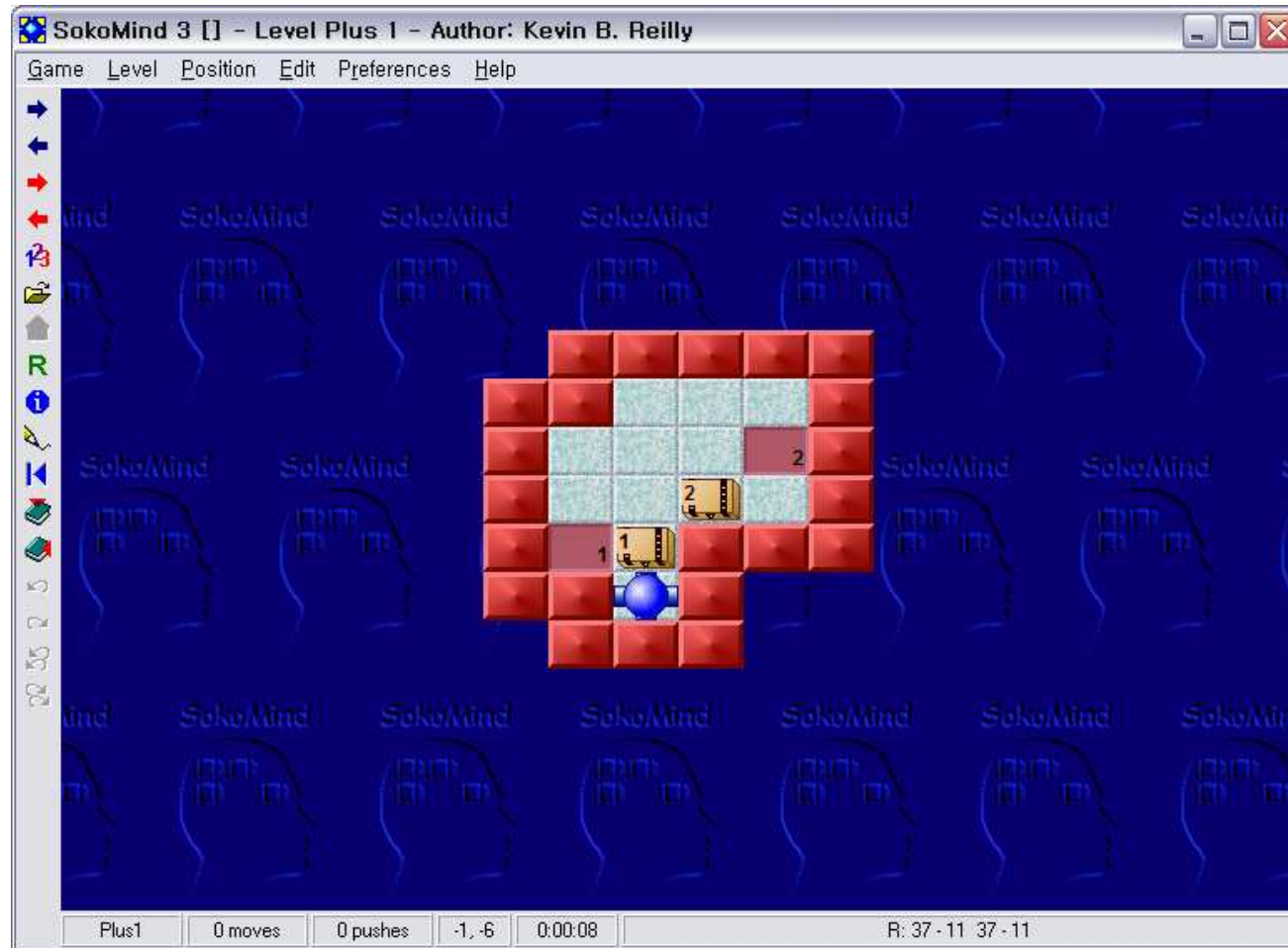
# Modified Model Checker



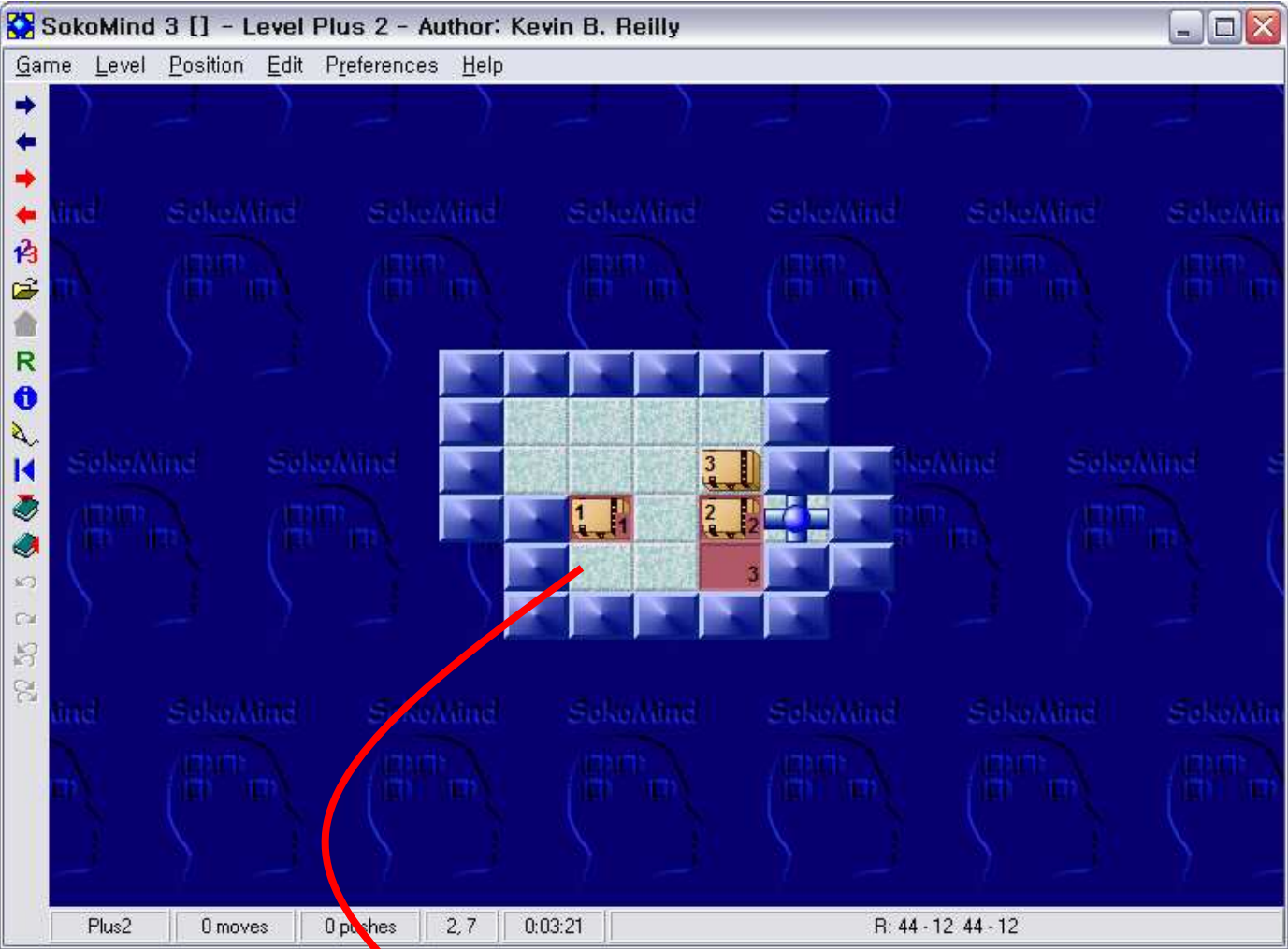
322 steps

**Eventually we get an optimal path !!**

# SokoMind-PLUS Game



# Quiz #4



→EF (ball1=goal1 & ball2=goal2 & ball3=goal3)

# Solution

76 test-plus-02-AG.smv

File Prop View Goto History Abstraction Help

Browser Properties Results Cone Using Groups

All results

Property	Result	Time
(AG (~(((B1.s=22)&(B2.s=42))&(B3.s=41))))	false	Mon May 08 23:49:10 EDT 2006

Source Trace Log

File Edit Run View

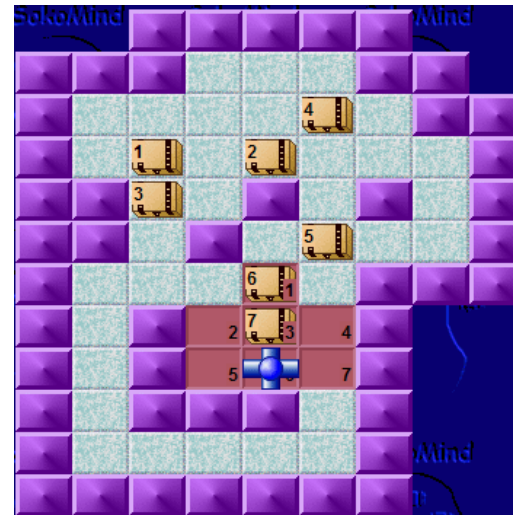
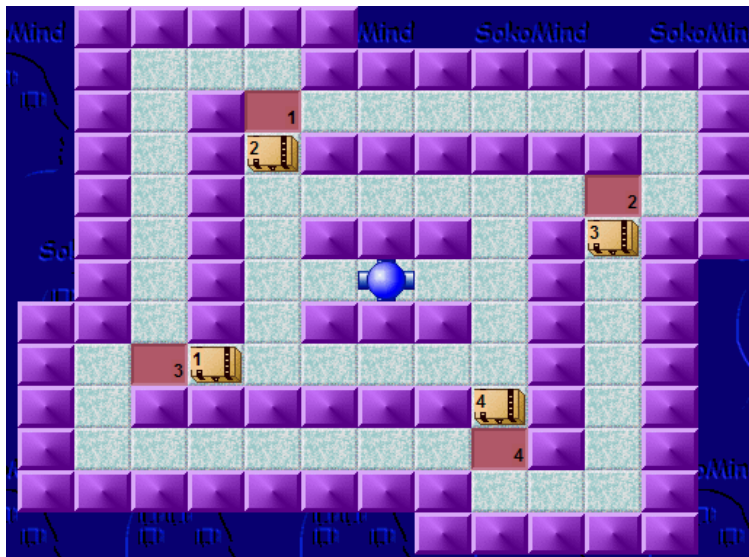
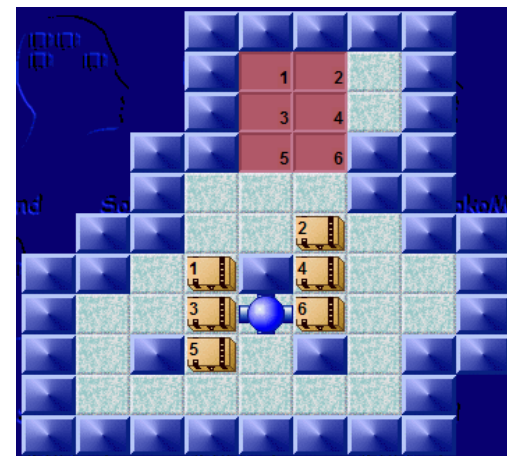
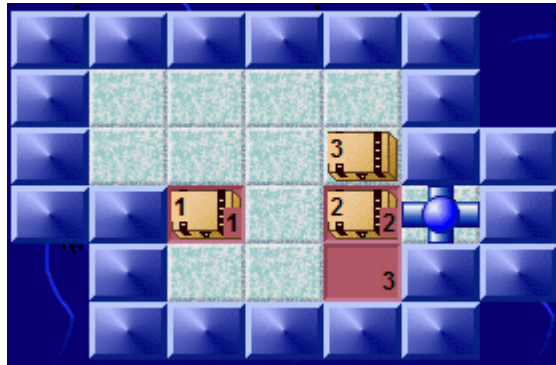
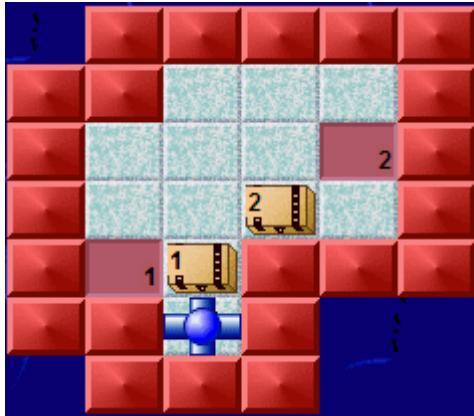
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
P.BlockOnUp	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0
P.Direction	lef	dow	lef	lef	up	dow	rig	rig	up	lef	up	up	lef	lef	dow	rig	up	rig	rig	dow	dow	lef	dow	lef	up
P.s	52	42	41	31	21	22	21	31	41	42	32	33	34	24	14	13	23	24	34	44	43	42	32	31	21

File Edit Run View

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
P.BlockOnUp	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	1	1	0
P.Direction	dow	lef	dow	lef	up	rig	rig	up	up	lef	dow	rig	dow	lef	up	up	lef	lef	dow	rig	rig	up	rig	dow	-
P.s	43	42	32	31	21	22	32	42	43	44	34	33	43	42	32	33	34	24	14	13	23	33	34	44	43

# Sokomind-PLUS Game

20 levels



# Model Checking Results



Game	Time (sec)	Space (KB)	Best Moves by human	Best Moves by MC	Remarks
1	0.88	4,245	37	37	–
2	0.08	2,217	44	44	–
3	0.55	3,381	60	60	–
4	0.35	3,055	53	53	–
5	0.15	2,515	77	77	–
6	0.22	2,771	129	129	–
7	6.69	12,009	117	117	–
8	2,739.91	353,037	154	154	–
9	291.80	38,937	97	97	–
10	795.58	93,493	99	99	–
11	1,070.81	215,445	234	234	–
12	15,659.66	916,689	336	328	↓8
13	47.18	13,181	154	152	↓2
14	40.68	13,477	199	199	–
15	681.81	66,369	339	337	↓2

(↓ $n$  means the game is solved in  $n$  fewer steps)

**But 16<sup>th</sup> ~ 20<sup>th</sup> levels are failed !!**



# SokoMind Games

60 levels



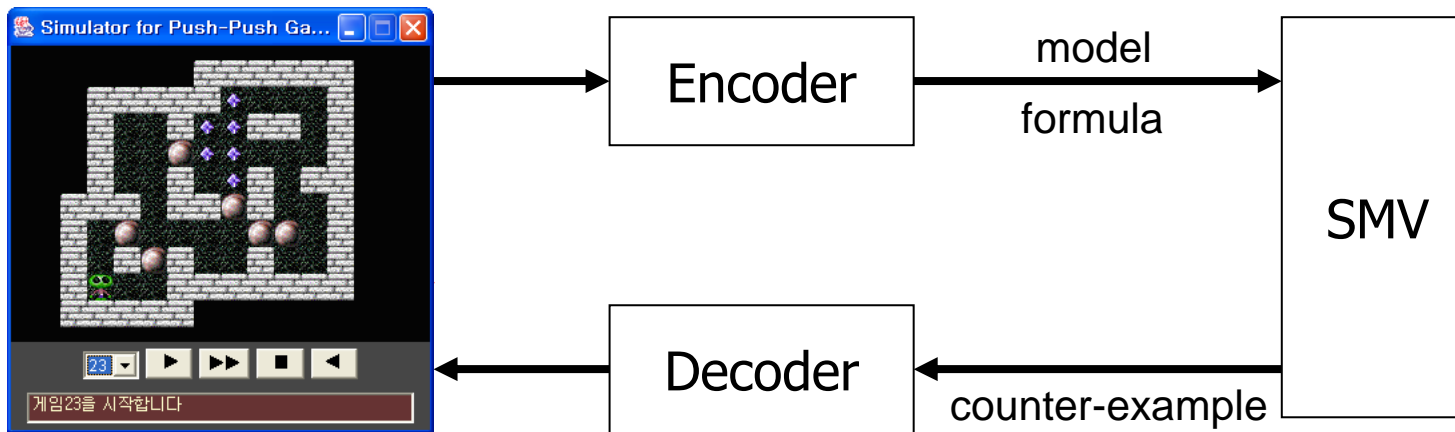
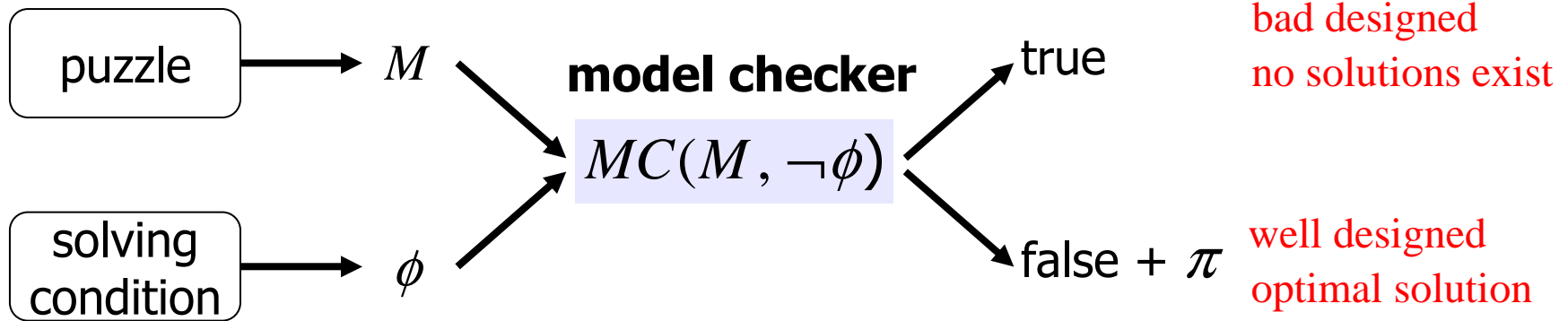


# Model Checking Results

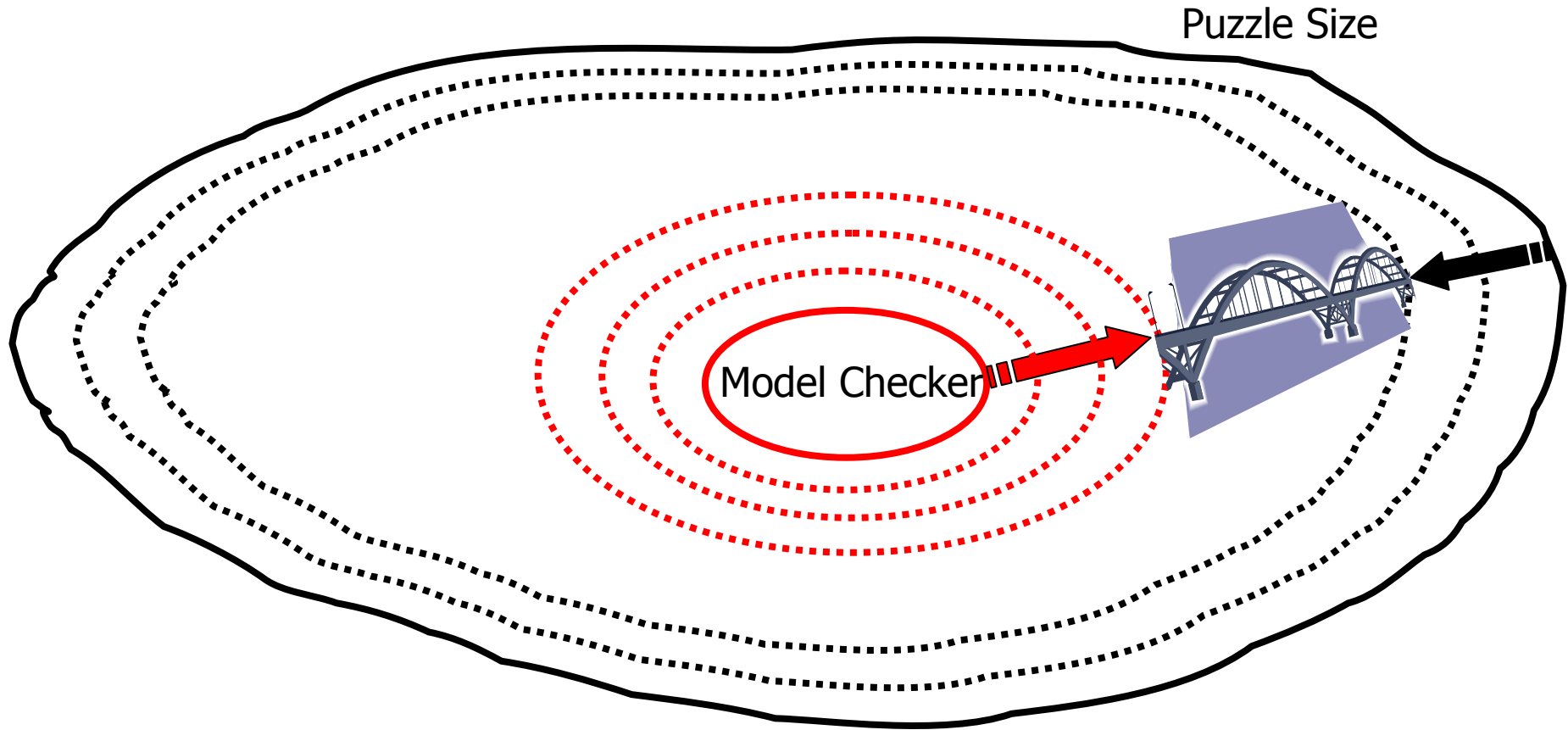
Level	Number of positions	Number of steps	World records	Remark
1	37	146	146	solved
2	25	81	81	solved
3	24	70	70	solved
4	47	229	229	solved
5	54		313	failed
6	49	165	165	solved
7	51	229	229	solved
8	39	187	187	solved
9	62		307	failed
10	20	58	58	solved

Level	Number of positions	Number of steps	World records	Remark
11	71		456	failed
12	77		365	failed
13	71		433	failed
14	66		369	failed
				failed
				failed
				failed
				failed
59	.....		.....	failed
60	.....		.....	failed

# Puzzles as Model Checking Problem



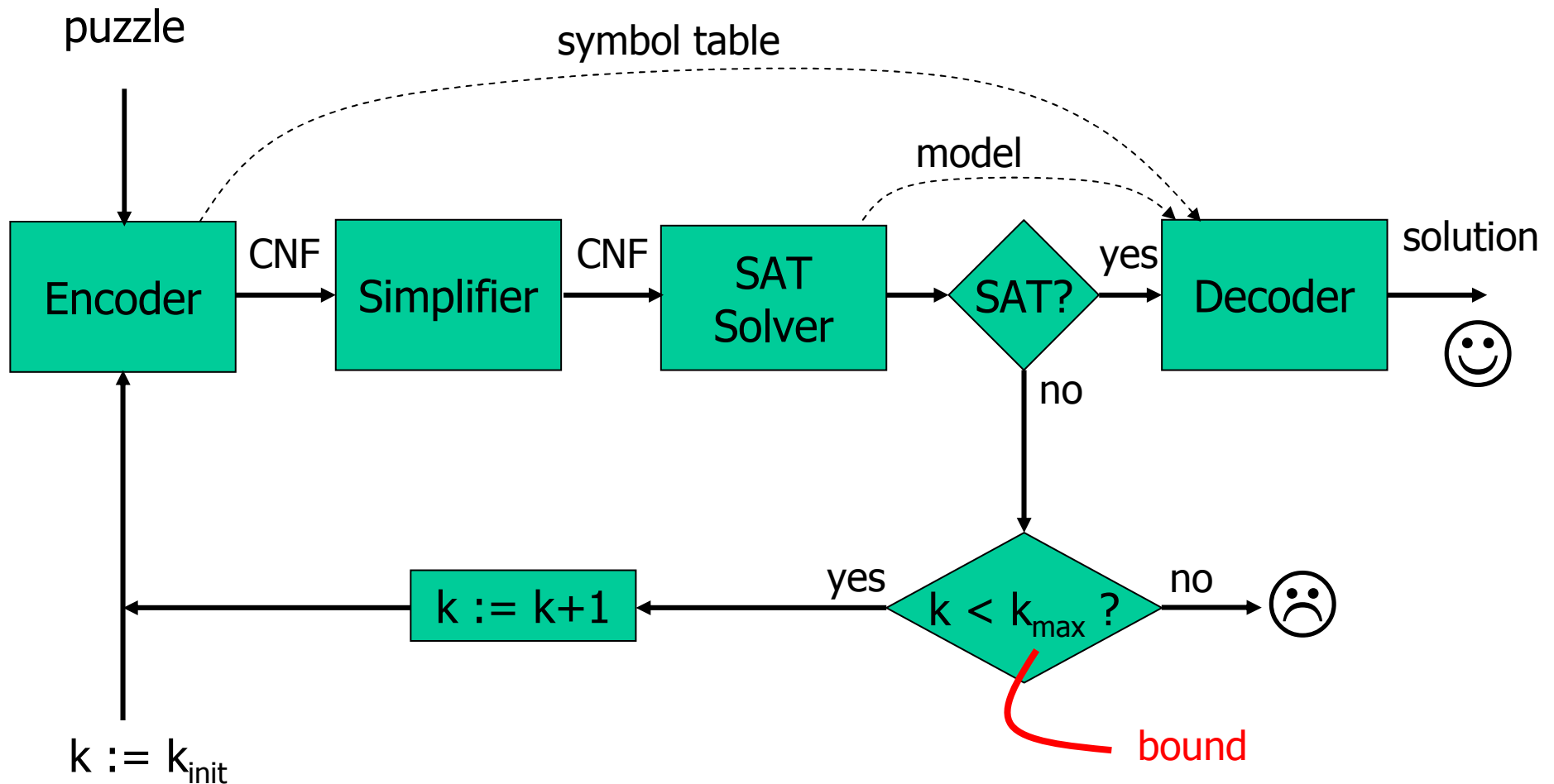
# However



**How to bridge the gap ?**

# Challenge Works

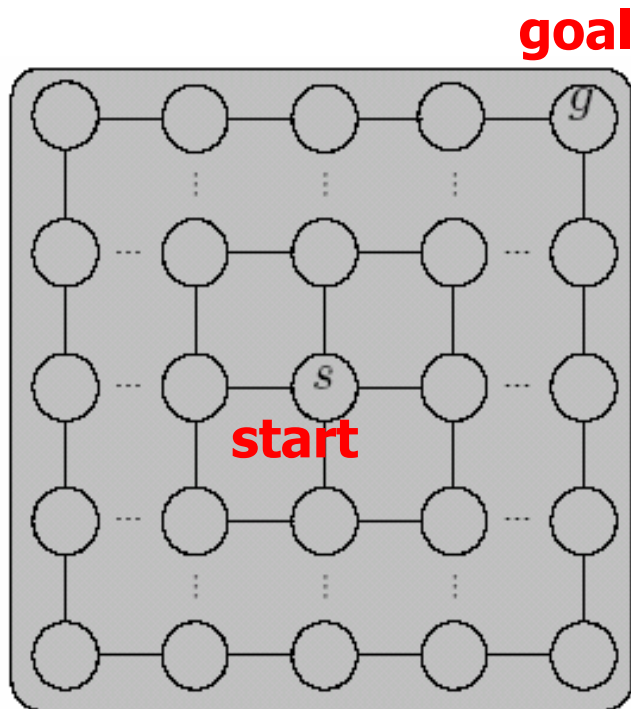
## Puzzles as Bounded Model Checking Problem



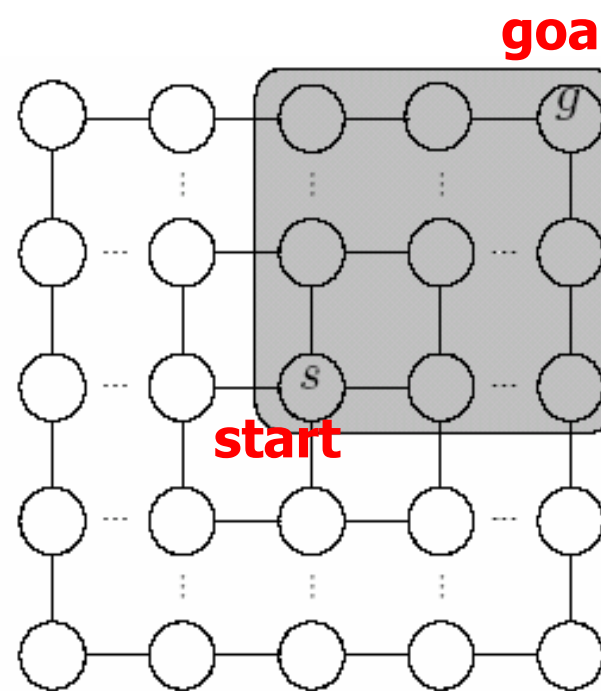
# Challenge Works

## Puzzles as Directed Model Checking Problem

Traditional  
Model Checking



Directed  
Model Checking



explicit MC  
+  
guided search

# Challenge Works

**Puzzles are waiting for **Your Breakthrough Technique !!****

**Thank You!!**