15-414 — Bug Catching — Fall 2006

Handout Sep 12, 2006

1 Principle of Structural Induction

Definition 1: A (propositional) *atomic formula* is a propositional letter, $\top$ (true), $\bot$ (false).

Definition 2: The set of *propositional formulas* is the smallest set $\mathbf{P}$ such that
1. if A is an atomic formula, $A \in \mathbf{P}$,
2. $X \in \mathbf{P} \Rightarrow \neg X \in \mathbf{P}$,
3. if $\circ$ is a binary symbol, then $X, Y \in \mathbf{P} \Rightarrow (X \circ Y) \in \mathbf{P}$.

Using the above definition one can derive the following principle:

Principle of Structural Induction Every formula of propositional logic has a property, $\mathbf{Q}$, provided:

basic step *every atomic formula has property* $\mathbf{Q}$
induction steps
1. *if* X *has property* $\mathbf{Q}$ *do does* $\neg X$*;*
2. *if* X *and* Y *have property* $\mathbf{Q}$ *so does* $X \circ Y$*, where* $\circ$ *is a binary symbol.*

(a) Using structural induction show that if v_1 and v_2 are two Boolean valuations that agree on a set S of propositional letters (which may not include all propositional letters), then v_1 and v_2 agree on all propositional formulas that contain only propositional letters from S.