Functional Dependencies

15-415, Spring 2003, Lecture 17
R & G Chapter 19

Science is the knowledge of consequences, and dependence of one fact upon another.

Thomas Hobbes (1588-1679)

Functional Dependencies (FDs)

- **A <u>functional dependency</u> X → Y holds over relation schema R if, for every allowable instance *r* of R:**

  $t1 \in r,\ t2 \in r,\ \pi_X(t1) = \pi_X(t2)$
  implies $\pi_Y(t1) = \pi_Y(t2)$

  (where t1 and t2 are tuples;X and Y are sets of attributes)

- **In other words: X → Y means**
  Given any two tuples in *r*, if the X values are the same, then the Y values must also be the same. (but not vice versa)
- **Can read "→" as "determines"**

Review: Database Design

- Requirements Analysis
  - user needs; what must database do?
- Conceptual Design
  - high level descr (often done w/ER model)
- Logical Design
  - translate ER into DBMS data model
- Schema Refinement
  - **consistency,normalization**
- Physical Design - indexes, disk layout
- Security Design - who accesses what

FD's Continued

- **An FD is a statement about *all* allowable relations.**
  - Must be identified based on semantics of application.
  - Given some instance *r1* of R, we can check if *r1* violates some FD *f*, but we cannot determine if *f* holds over R.
- **Question: How related to keys?**
- **if "K → all attributes of R" then K is a *superkey* for R**
  (does not require K to be *minimal*.)
- **FDs are a generalization of keys.**

The Evils of Redundancy

- ***Redundancy* is at the root of several problems associated with relational schemas:**
  - *redundant storage, insert/delete/update anomalies*
- **Integrity constraints, in particular *functional dependencies*, can be used to identify schemas with such problems and to suggest refinements.**
- **Main refinement technique: *<u>decomposition</u>***
  - replacing ABCD with, say, AB and BCD, or ACD and ABD.
- **Decomposition should be used judiciously:**
  - Is there reason to decompose a relation?
  - What problems (if any) does the decomposition cause?

Example: Constraints on Entity Set

- **Consider relation obtained from** Hourly_Emps:
  Hourly_Emps (*<u>ssn</u>*, *name, lot, rating, wage_per_hr, hrs_per_wk*)
- We sometimes denote a relation schema by listing the attributes: e.g., SNLRWH
- This is really the *set* of attributes {S,N,L,R,W,H}.
- Sometimes, we refer to the set of *all attributes* of a relation by using the relation name. e.g., "Hourly_Emps" for SNLRWH

What are some FDs on Hourly_Emps?

ssn is the key: S → SNLRWH
rating determines *wage_per_hr*: R → W
lot determines *lot*: L → L ("trivial" dependnency)

Problems Due to R → W

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

Hourly_Emps

- ***Update anomaly***: **Can we modify W in only the 1st tuple of SNLRWH?**
- ***Insertion anomaly***: **What if we want to insert an employee and don't know the hourly wage for his or her rating? (or we get it wrong?)**
- ***Deletion anomaly***: **If we delete all employees with rating 5, we lose the information about the wage for rating 5!**

Decomposing a Relation

- **Redundancy can be removed by "chopping" the relation into pieces.**
- **FD's are used to drive this process.**

  R → W is causing the problems, so decompose SNLRWH into what relations?

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
231-31-5368	Smiley	22	8	30
131-24-3650	Smethurst	35	5	30
434-26-3751	Guldu	35	5	32
612-67-4134	Madayan	35	8	40

Hourly_Emps2

R	W
8	10
5	7

Wages

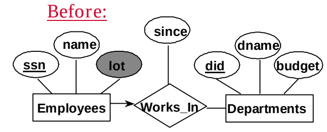
Null values

- **Can Null values help address anomalies?**
  - Clearly, not helpful for redundancy or update anomalies
  - Insertions?
    - Can insert employee with Null wages
    - Cannot insert a rating-to-wage correspondence (ssn cannot be null)
  - Same with deletions
    - Cannot store null in ssn to preserve a rating-to-wage correspondence

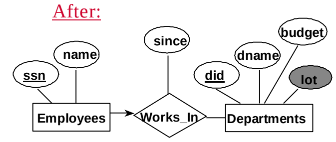
Refining an ER Diagram

- **1st diagram becomes:**
  Workers(S,N,L,D,Si)
  Departments(D,M,B)
  - Lots associated with workers.
- **Suppose all workers in a dept are assigned the same lot: D → L**
- **Redundancy; fixed by:**
  Workers2(S,N,D,Si)
  Dept_Lots(D,L)
  Departments(D,M,B)
- **Can fine-tune this:**
  Workers2(S,N,D,Si)
  Departments(D,M,B,L)

Before:



After:



Detecting Redundancy

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
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Hourly_Emps

Q: Why was R → W problematic, but S→W not?

Reasoning About FDs

- **Given some FDs, we can usually infer additional FDs:**

  title → studio, star implies *title → studio* and *title → star*

  title → studio and *title → star* implies *title → studio, star*

  title → studio, *studio → star* implies *title → star*

  But,

  title, star → studio does NOT necessarily imply that *title → studio* or that *star → studio*
- **An FD *f* is *implied by* a set of FDs *F* if *f* holds whenever all FDs in *F* hold.**
- **F^+ = *closure of F* is the set of all FDs that are implied by *F*. (includes "trivial dependencies")**

Rules of Inference

- **Armstrong's Axioms (X, Y, Z are <u>sets</u> of attributes):**
  - *Reflexivity*: If $X \supseteq Y$, then $X \rightarrow Y$
  - *Augmentation*: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
  - *Transitivity*: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- **These are *sound* and *complete* inference rules for FDs!**
  - i.e., using AA you can compute all the FDs in F+ and only these FDs.
- **Some additional rules (that follow from AA):**
  - *Union*: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - *Decomposition*: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

Attribute Closure (example)

- **R = {A, B, C, D, E}**
- **F = { B → CD, D → E, B → A, E → C, AD → B }**
- **Is B → E in F^+ ?**

  $B^+ = B$

  $B^+ = BCD$

  $B^+ = BCDA$

  $B^+ = BCDAE$ ... Yes!

  and B is a key for R too!
- **Is D a key for R?**

  $D^+ = D$

  $D^+ = DE$

  $D^+ = DEC$

  ... Nope!
- **Is AD a key for R?**

  $AD^+ = AD$

  $AD^+ = ABD$ and B is a key, so Yes!
- **Is AD a *candidate* key for R?**

  $A^+ = A$

  ... A not a key, so Yes!
- **Is ADE a *candidate* key for R?**

  ... No! AD is a key, so ADE is a superkey, but not a cand. key

Example

- **Contracts(*<u>cid</u>,sid,jid,did,pid,qty,value*), and:**
  - C is the key: C → CSJDPQV
  - Proj purchases each part using single contract: JP → C
  - Dept purchases at most 1 part from a supplier: SD → P
- **Problem: Prove that SDJ is a key for Contracts**
- **JP → C, C → CSJDPQV imply JP → CSJDPQV**

  (by transitivity) (shows that JP is a key)
- **SD → P implies SDJ → JP** (by augmentation)
- **SDJ → JP, JP → CSJDPQV imply SDJ → CSJDPQV**

  (by transitivity) thus SDJ is a key.

Q: can you now infer that SD → CSDPQV (i.e., drop J on both sides)?

No! FD inference is not like arithmetic multiplication.

Next Class...

- **Normal forms and normalization**
- **Table decompositions**

Attribute Closure

- **Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in # attrs!)**
- **Typically, we just want to check if a given FD *X* → *Y* is in the closure of a set of FDs *F*. An efficient check:**
  - Compute *attribute closure* of X (denoted X^+) wrt *F*. X^+ = Set of all attributes A such that X → A is in F^+
    - $X^+ := X$
    - Repeat until no change: if there is in fd U → V in *F* such that U is in X^+, then add V to X^+
  - Check if Y is in X^+
  - Approach can also be used to find the keys of a relation.
    - If all attributes of R are in the closure of X then X is a superkey for R.
    - Q: How to check if X is a "candidate key"?