Tree-Structured Indexes

15-415, Spring 2003, Lecture 6
R & G Chapter 10

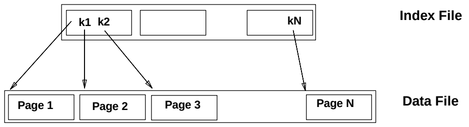
> "If I had eight hours to chop down a tree, I'd spend six sharpening my ax."
>
> *Abraham Lincoln*

Introduction

- ***Recall: 3 alternatives for data entries*** `k*`**:**
  - Data record with key value **k**
  - <**k**, rid of data record with search key value **k**>
  - <**k**, list of rids of data records with search key **k**>
- **Choice is orthogonal to the *indexing technique* used to locate data entries** `k*`**.**
- **Tree-structured indexing techniques support both *range searches* and *equality searches*.**
- ***ISAM*: static structure; *B+ tree*: dynamic, adjusts gracefully under inserts and deletes.**
- **ISAM =** Indexed Sequential Access Method

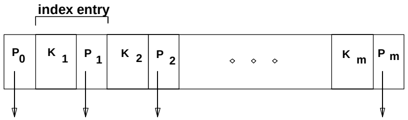
Range Searches

- ***``Find all students with gpa > 3.0''***
  - If data is in sorted file, do binary search to find first such student, then scan to find others.
  - Cost of binary search in a database can be quite high. Q: Why???
- **Simple idea: Create an `index' file.**

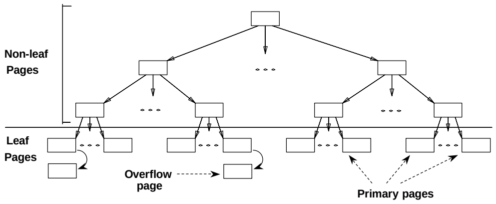


** Can do binary search on (smaller) index file!*

ISAM



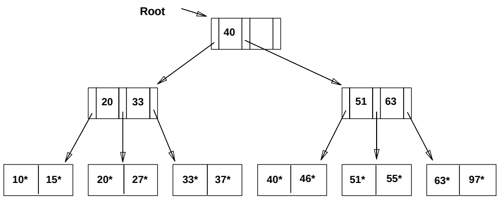
- **Index file may still be quite large. But we can apply the idea repeatedly!**



** Leaf pages contain data entries.*

Example ISAM Tree

- *Index entries*:<search key value, page id> they direct search for data entries *in leaves.*
- Example where each node can hold 2 entries;



ISAM is a STATIC Structure

Data Pages

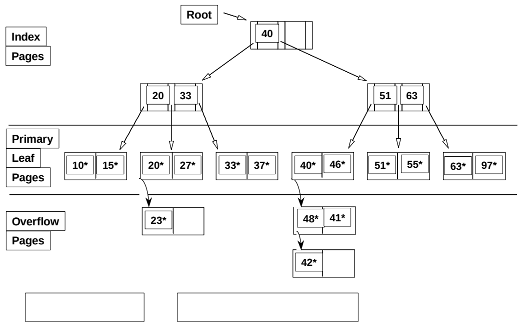
Index Pages

Overflow pages

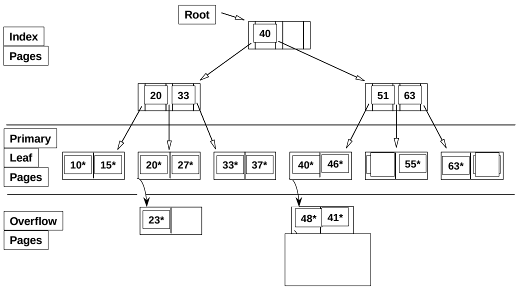
- *File creation*: Leaf (data) pages allocated sequentially, sorted by search key; then index pages allocated, then overflow pgs.
- *Search*: Start at root; use key comparisons to go to leaf. Cost = $\log_F N$; F = # entries/pg (i.e., fanout), N = # leaf pgs
  - no need for `next-leaf-page' pointers. (Why?)
- *Insert*: Find leaf that data entry belongs to, and put it there. Overflow page if necessary.
- *Delete*: Find and remove from leaf; if empty page, de-allocate.

Static tree structure: *inserts/deletes affect only leaf pages.*

Example: Insert 23*, 48*, 41*, 42*



... then Deleting 42*, 51*, 97*



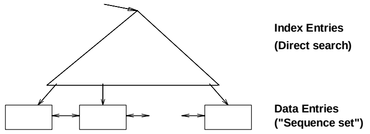
** Note that 51* appears in index levels, but not in leaf!*

ISAM ---- Issues?

- **Pros**
  - ????
- **Cons**
  - ????

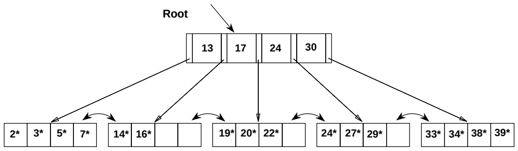
B+ Tree: The Most Widely-Used Index

- Insert/delete at log $_F$ N cost; keep tree *height-balanced*. (F = fanout, N = # leaf pages)
- Minimum 50% occupancy (except for root). Each node contains d <= *m* <= 2d entries. "d" is called the *order* of the tree.
- Supports equality and range-searches efficiently.
- As in ISAM, all searches go from root to leaves, but structure is dynamic.



Example B+ Tree

- **Search begins at root, and key comparisons direct it to a leaf (as in ISAM).**
- **Search for 5*, 15*, all data entries >= 24* ...**



** Based on the search for 15*, we know it is not in the tree!*

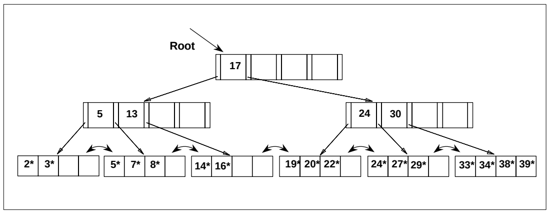
B+ Trees in Practice (cool facts!)

- **Typical order: 100. Typical fill-factor: 67%.**
  - average fanout = 133
- **Typical capacities:**
  - Height 4: 133^4 = 312,900,700 entries
  - Height 3: 133^3 = 2,352,637 entries
- **Can often hold top levels in buffer pool:**
  - Level 1 = 1 page = 8 KB
  - Level 2 = 133 pages = 1 MB
  - Level 3 = 17,689 pages = 133 MB

Inserting a Data Entry into a B+ Tree

- **Find correct leaf *L.***
- **Put data entry onto *L*.**
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.
- **This can happen recursively**
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- **Splits "grow" tree; root split increases height.**
  - Tree growth: gets *wider* or *one level taller at top.*

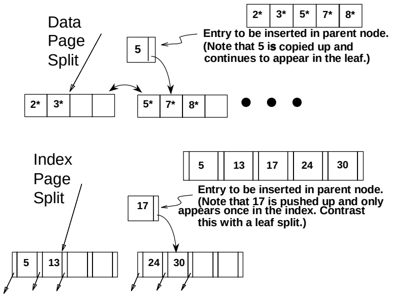
Example B+ Tree - Inserting 8*



- Notice that root was split, leading to increase in height.
- In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

Example: Data vs. Index Page Split

- **Observe how minimum occupancy is guaranteed in both leaf and index pg splits.**
- **Note difference between *copy-up* and *push-up*; be sure you understand the reasons for this.**



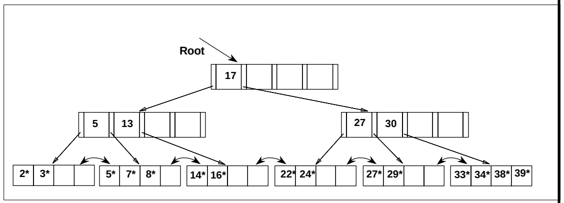
Entry to be inserted in parent node. (Note that 5 is copied up and continues to appear in the leaf.)

Entry to be inserted in parent node. (Note that 17 is pushed up and only appears once in the index. Contrast this with a leaf split.)

Deleting a Data Entry from a B+ Tree

- **Start at root, find leaf *L* where entry belongs.**
- **Remove the entry.**
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from *sibling (adjacent node with same parent as L).*
    - If re-distribution fails, *merge* *L* and sibling.
- **If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.**
- **Merge could propagate to root, decreasing height.**

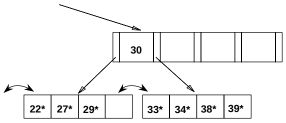
Example Tree (including 8*)
Delete 19* and 20* ...

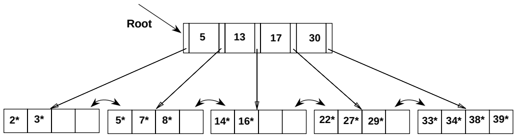


- **Deleting 19* is easy.**
- **Deleting 20* is done with re-distribution. Notice how middle key is *copied up*.**

... And Then Deleting 24*

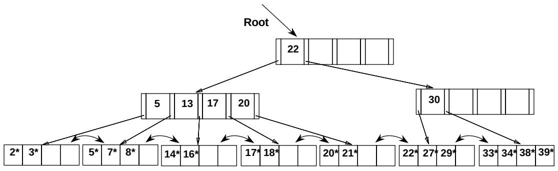
- **Must merge.**
- **Observe `*toss*' of index entry (on right), and `*pull down*' of index entry (below).**





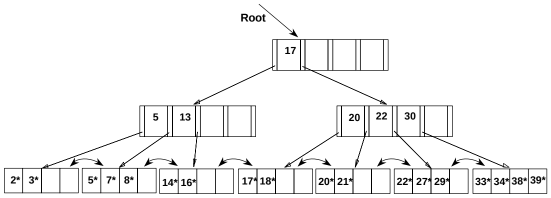
Example of Non-leaf Re-distribution

- **Tree is shown below *during deletion* of 24*. (What could be a possible initial tree?)**
- **In contrast to previous example, can re-distribute entry from left child of root to right child.**



After Re-distribution

- **Intuitively, entries are re-distributed by `*pushing through*' the splitting entry in the parent node.**
- **It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well for illustration.**

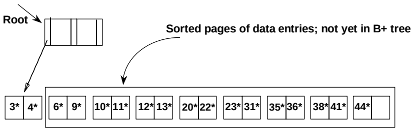


Prefix Key Compression

- **Important to increase fan-out. (Why?)**
- **Key values in index entries only `direct traffic'; can often compress them.**
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt*, *David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- **Insert/delete must be suitably modified.**

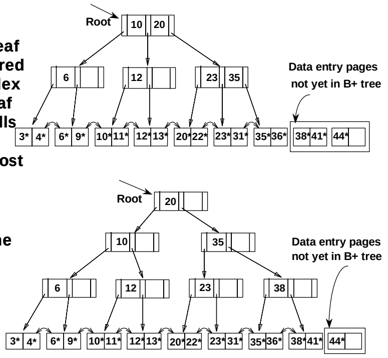
Bulk Loading of a B+ Tree

- **If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.**
  - Also leads to minimal leaf utilization --- why?
- ***Bulk Loading* can be done much more efficiently.**
- ***Initialization*: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.**



Bulk Loading (Contd.)

- **Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)**
- **Much faster than repeated inserts, especially when one considers locking!**



Summary of Bulk Loading

- **Option 1: multiple inserts.**
  - Slow.
  - Does not give sequential storage of leaves.
- **Option 2: *Bulk Loading***
  - Has advantages for concurrency control.
  - Fewer I/Os during build.
  - Leaves will be stored sequentially (and linked, of course).
  - Can control "fill factor" on pages.

A Note on `Order’

- ***Order*** **(**d**) concept replaced by physical space criterion in practice (`*at least half-full*').**
  - Index pages can typically hold many more entries than leaf pages.
  - Variable sized records and search keys mean different nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).
- **Many real systems are even sloppier than this --- only reclaim space when a page is *completely* empty.**

Summary

- **Tree-structured indexes are ideal for range-searches, also good for equality searches.**
- **ISAM is a static structure.**
  - Only leaf pages modified; overflow pages needed.
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant.
- **B+ tree is a dynamic structure.**
  - Inserts/deletes leave tree height-balanced; $\log_F N$ cost.
  - High fanout (**F**) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.

Summary (Contd.)

  - Typically, 67% occupancy on average.
  - Usually preferable to ISAM, modulo *locking* considerations; adjusts to growth gracefully.
  - If data entries are data records, splits can change rids!
- **Key compression increases fanout, reduces height.**
- **Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.**
- **Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.**