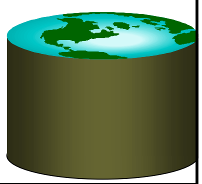
## Tree-Structured Indexes

**15-415, Spring 2003, Lecture 6**
**R & G Chapter 10**

"If I had eight hours to chop down a tree, I'd spend six sharpening my ax."

*Abraham Lincoln*

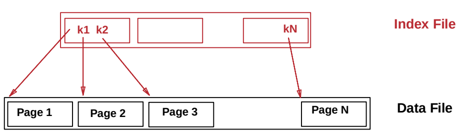


## Introduction

- ***Recall: 3 alternatives for data entries*** k*:
  - Data record with key value **k**
  - <**k**, rid of data record with search key value **k**>
  - <**k**, list of rids of data records with search key **k**>
- **Choice is orthogonal to the *indexing technique* used to locate data entries** k***.**
- **Tree-structured indexing techniques support both *range searches* and *equality searches*.**
- ***ISAM*: static structure; *B+ tree*: dynamic, adjusts gracefully under inserts and deletes.**
- **ISAM = Indexed Sequential Access Method**

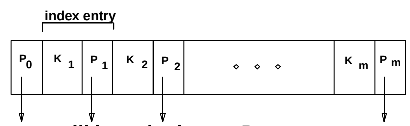
## Range Searches

- **``*Find all students with gpa > 3.0*''**
  - If data is in sorted file, do binary search to find first such student, then scan to find others.
  - Cost of binary search in a database can be quite high. Q: Why???
- **Simple idea: Create an `index' file.**

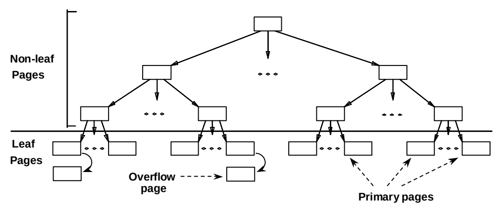



* *Can do binary search on (smaller) index file!*

## ISAM



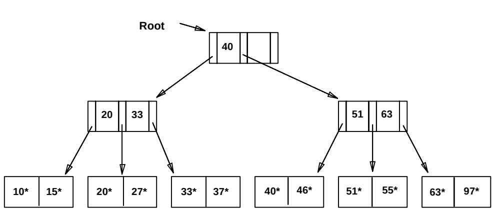

- **Index file may still be quite large. But we can apply the idea repeatedly!**




* *Leaf pages contain data entries.*

## Example ISAM Tree

- *Index entries*:<search key value, page id> they direct search for data entries *in leaves.*
- Example where each node can hold 2 entries;

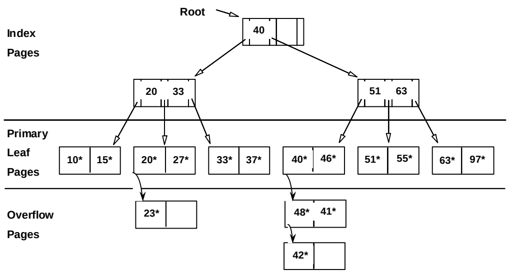



## ISAM is a STATIC Structure

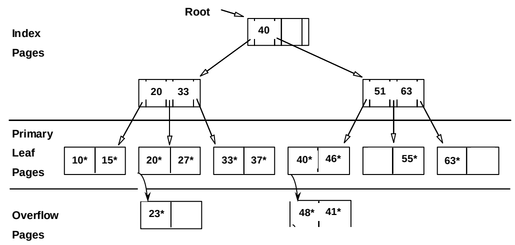

- *File creation*: Leaf (data) pages allocated sequentially, sorted by search key; then index pages allocated, then overflow pgs.
- *Search*: Start at root; use key comparisons to go to leaf. Cost = $\log_F N$ ; F = # entries/pg (i.e., fanout), N = # leaf pgs
  - no need for `next-leaf-page' pointers. (Why?)
- *Insert*: Find leaf that data entry belongs to, and put it there. Overflow page if necessary.
- *Delete*: Find and remove from leaf; if empty page, de-allocate.

**Static tree structure**: *inserts/deletes affect only leaf pages.*

## Example: Insert 23*, 48*, 41*, 42*




## ... then Deleting 42*, 51*, 97*



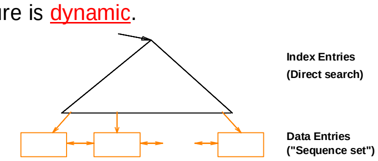

** Note that 51* appears in index levels, but not in leaf!*

## ISAM ---- Issues?

- **Pros**
  - ????
- **Cons**
  - ????

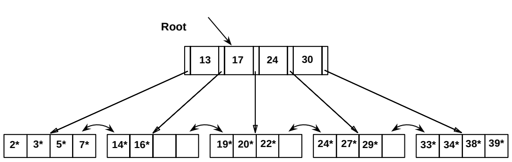
## B+ Tree: The Most Widely-Used Index

- Insert/delete at $\log_F N$ cost; keep tree *height-balanced.* (F = fanout, N = # leaf pages)
- Minimum 50% occupancy (except for root). Each node contains d <= *m* <= 2d entries. "d" is called the *order* of the tree.
- Supports equality and range-searches efficiently.
- As in ISAM, all searches go from root to leaves, but structure is dynamic.




## Example B+ Tree

- **Search begins at root, and key comparisons direct it to a leaf (as in ISAM).**
- **Search for 5*, 15*, all data entries >= 24* ...**




** Based on the search for 15*, we know it is not in the tree!*

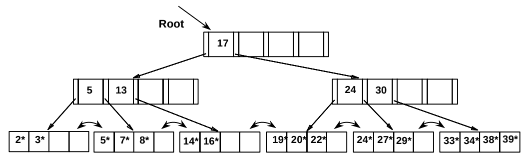
## B+ Trees in Practice (cool facts!)

- **Typical order: 100. Typical fill-factor: 67%.**
  - average fanout = 133
- **Typical capacities:**
  - Height 4: $133^4$ = 312,900,700 entries
  - Height 3: $133^3$ = 2,352,637 entries
- **Can often hold top levels in buffer pool:**
  - Level 1 = 1 page = 8 KB
  - Level 2 = 133 pages = 1 MB
  - Level 3 = 17,689 pages = 133 MB

## Inserting a Data Entry into a B+ Tree

- **Find correct leaf *L*.**
- **Put data entry onto *L*.**
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.
- **This can happen recursively**
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- **Splits "grow" tree; root split increases height.**
  - Tree growth: gets *wider* or *one level taller at top.*

## Example B+ Tree - Inserting 8*

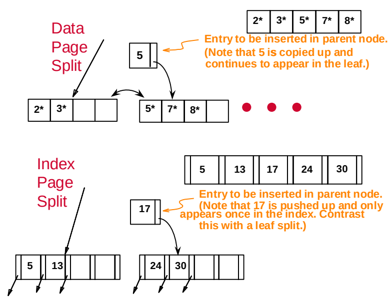



v Notice that root was split, leading to increase in height.

v In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

## Example: Data vs. Index Page Split

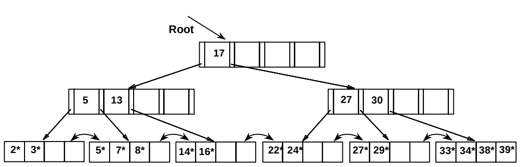
- **Observe how minimum occupancy is guaranteed in both leaf and index pg splits.**
- **Note difference between *copy-up* and *push-up*; be sure you understand the reasons for this.**




## Deleting a Data Entry from a B+ Tree

- **Start at root, find leaf *L* where entry belongs.**
- **Remove the entry.**
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from *sibling (adjacent node with same parent as L).*
    - If re-distribution fails, *merge L* and sibling.
- **If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.**
- **Merge could propagate to root, decreasing height.**

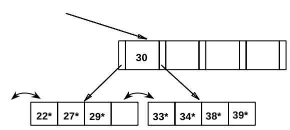
## Example Tree (including 8*) Delete 19* and 20* ...

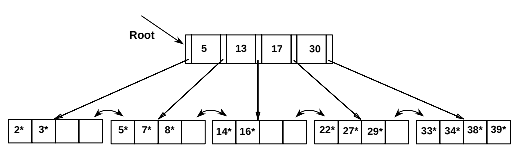



- **Deleting 19* is easy.**
- **Deleting 20* is done with re-distribution. Notice how middle key is *copied up*.**

## ... And Then Deleting 24*

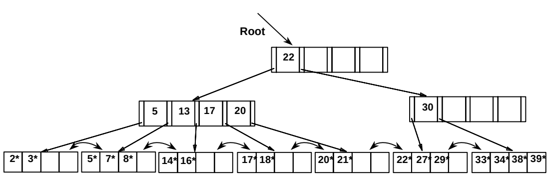
- **Must merge.**
- **Observe `*toss*' of index entry (on right), and `*pull down*' of index entry (below).**



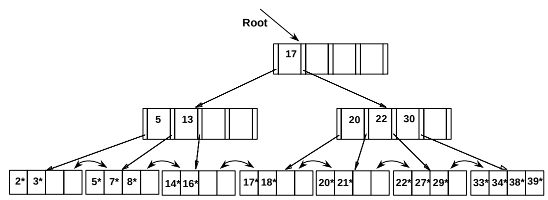
## Example of Non-leaf Re-distribution

- Tree is shown below ***during deletion*** of 24*. (What could be a possible initial tree?)
- In contrast to previous example, can re-distribute entry from left child of root to right child.




## After Re-distribution

- Intuitively, entries are re-distributed by `***pushing through***' the splitting entry in the parent node.
- It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well for illustration.

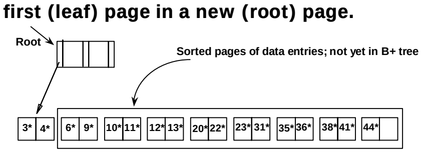



## Prefix Key Compression

- **Important to increase fan-out. (Why?)**
- **Key values in index entries only `direct traffic'; can often compress them.**
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt*, *David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- **Insert/delete must be suitably modified.**

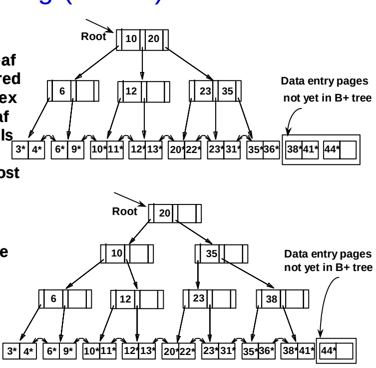
## Bulk Loading of a B+ Tree

- **If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.**
  - Also leads to minimal leaf utilization --- why?
- ***Bulk Loading*** **can be done much more efficiently.**
- ***Initialization*****: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.**




## Bulk Loading (Contd.)

- **Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)**
- **Much faster than repeated inserts, especially when one considers locking!**




## Summary of Bulk Loading

- **Option 1: multiple inserts.**
  - Slow.
  - Does not give sequential storage of leaves.
- **Option 2: *Bulk Loading***
  - Has advantages for concurrency control.
  - Fewer I/Os during build.
  - Leaves will be stored sequentially (and linked, of course).
  - Can control "fill factor" on pages.

## A Note on `Order'

- ***Order*** **(d) concept replaced by physical space criterion in practice (`*at least half-full*').**
  - Index pages can typically hold many more entries than leaf pages.
  - Variable sized records and search keys mean different nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).
- **Many real systems are even sloppier than this --- only reclaim space when a page is *completely* empty.**

## Summary

- **Tree-structured indexes are ideal for range-searches, also good for equality searches.**
- **ISAM is a static structure.**
  - Only leaf pages modified; overflow pages needed.
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant.
- **B+ tree is a dynamic structure.**
  - Inserts/deletes leave tree height-balanced; $\log_F N$ cost.
  - High fanout (**F**) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.

## Summary (Contd.)

  - Typically, 67% occupancy on average.
  - Usually preferable to ISAM, modulo *locking* considerations; adjusts to growth gracefully.
  - If data entries are data records, splits can change rids!
- **Key compression increases fanout, reduces height.**
- **Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.**
- **Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.**