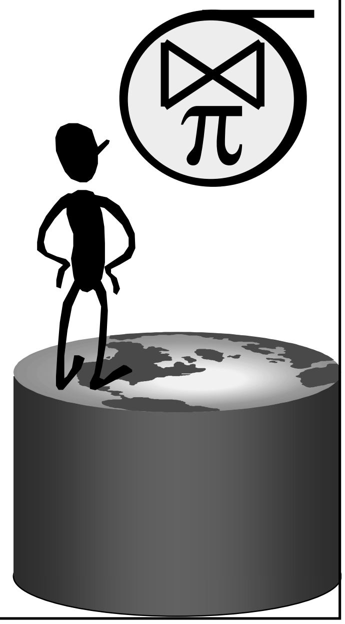
Relational Algebra

15-415 Spring 2003, Lecture 9

R & G, Chapter 4



By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, *increases the mental power of the race*.

-- **Alfred North Whitehead (1861 - 1947)**

Relational Query Languages

- ***Query languages:*** **Allow manipulation and retrieval of data from a database.**
- **Relational model supports simple, powerful QLs:**
  - Strong formal foundation based on logic.
  - Allows for much optimization.
- **Query Languages** != **programming languages!**
  - QLs not expected to be “Turing complete”.
  - QLs not intended to be used for complex calculations.
  - QLs support easy, efficient access to large data sets.

Formal Relational Query Languages

Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:

***Relational Algebra*: More operational, very useful for representing execution plans.**

***Relational Calculus*: Lets users describe what they want, rather than how to compute it. (Non-procedural, *declarative*.)**

☒ *Understanding Algebra & Calculus is key to understanding SQL, query processing!*

Preliminaries

- **A query is applied to *relation instances*, and the result of a query is also a relation instance.**
  - *Schemas* of input relations for a query are fixed (but query will run over any legal instance)
  - The schema for the *result* of a given query is also fixed. It is determined by the definitions of the query language constructs.
- **Positional vs. named-field notation:**
  - Positional notation easier for formal definitions, named-field notation more readable.
  - Both used in SQL

Relational Algebra: 5 Basic Operations

- *Selection* (σ) Selects a subset of ***rows*** from relation (horizontal).
- *Projection* (π) Retains only wanted ***columns*** from relation (vertical).
- *Cross-product* (x) Allows us to combine two relations.
- *Set-difference* (–) Tuples in r1, but not in r2.
- *Union* ($\cup$) Tuples in r1 and/or in r2.

Since each operation returns a relation, operations can be *composed!* (Algebra is "closed".)

Example Instances

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Boats

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Projection

- Examples: $\pi_{age}(S2)$; $\pi_{sname,rating}(S2)$
- Retains only attributes that are in the "*projection list*".
- *Schema* of result:
  - exactly the fields in the projection list, with the same names that they had in the input relation.
- Projection operator has to *eliminate duplicates* (How do they arise? Why remove them?)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)

Projection

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$\pi_{sname,rating}(S2)$

age
35.0
55.5

$\pi_{age}(S2)$

Selection (σ)

- Selects rows that satisfy *selection condition*.
- Result is a relation.

  Schema of result is same as that of the input relation.
- Do we need to do duplicate elimination?

sid	sname	rating	age
28	yuppy	9	35.0
~~31~~	~~lubber~~	~~8~~	~~55.5~~
~~44~~	~~guppy~~	~~5~~	~~35.0~~
58	rusty	10	35.0

$\sigma_{rating>8}(S2)$

sname	rating
yuppy	9
rusty	10

$\pi_{sname,rating}(\sigma_{rating>8}(S2))$

Union and Set-Difference

- All of these operations take two input relations, which must be ***union-compatible***:
  - Same number of fields.
  - \`Corresponding' fields have the same type.
- For which, if any, is duplicate elimination required?

Union

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$S1 \cup S2$

Set Difference

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
22	dustin	7	45.0

$S1 - S2$

sid	sname	rating	age
28	yuppy	9	35.0
44	guppy	5	35.0

$S2 - S1$

Cross-Product

- S1 x R1: Each row of S1 paired with each row of R1.
- Q: How many rows in the result?
- *Result schema* has one field per field of S1 and R1, with field names `inherited' if possible.
  - *May have a naming conflict*: Both S1 and R1 have a field with the same name.
  - In this case, can use the *renaming operator*:

$$\rho\,(C(1 \rightarrow sid1, 5 \rightarrow sid2),\ S1 \times R1)$$

Cross Product Example

sid	bid	day
22	101	10/10/96
58	103	11/12/96

R1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

R1 X S1 =

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

Compound Operator: Intersection

- In addition to the 5 basic operators, there are several additional “Compound Operators”
  - These add no computational power to the language, but are useful shorthands.
  - Can be expressed solely with the basic ops.

- Intersection takes two input relations, which must be ***union-compatible***.
- Q: How to express it using basic operators?

$$R \cap S = R - (R - S)$$

Intersection

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$$S1 \cap S2$$

Compound Operator: Join

- Joins are compound operators involving cross product, selection, and (sometimes) projection.
- Most common type of join is a "*natural join*" (often just called "join"). R $\bowtie$ S conceptually is:
  - Compute R X S
  - Select rows where attributes that appear in both relations have equal values
  - Project all unique atttributes and one copy of each of the common ones.
- Note: Usually done much more efficiently than this.
- Useful for putting "normalized" relations back together.

Natural Join Example

sid	bid	day
22	101	10/10/96
58	103	11/12/96

R1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

R1 $\bowtie$ S1 =

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

Other Types of Joins

- ***Condition Join (or "theta-join")***:

$$R \bowtie_c S = \sigma_c (R \times S)$$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

- *Result schema* same as that of cross-product.
- May have fewer tuples than cross-product.
- ***Equi-Join***: **Special case: condition *c* contains only conjunction of** *equalities*.

Compound Operator: Division

- Useful for expressing "for all" queries like: *Find sids of sailors who have reserved <u>all</u> boats*.
- For A/B attributes of B are subset of attrs of A.
  - May need to "project" to make this happen.
- E.g., let *A* have 2 fields, *x* and *y*; *B* have only field *y*:

$$A/B = \left\{ \langle x \rangle \middle| \forall \langle y \rangle \in B (\exists \langle x, y \rangle \in A) \right\}$$

***A/B* contains all *x* tuples such that for *every* *y* tuple in *B*, there is an *xy* tuple in *A*.**

Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

A

pno
p2

B1

pno
p2
p4

B2

pno
p1
p2
p4

B3

sno
s1
s2
s3
s4

A/B1

sno
s1
s4

A/B2

sno
s1

A/B3

Expressing A/B Using Basic Operators

- **Division is not essential op; just a useful shorthand.**
  - (Also true of joins, but joins are so common that systems implement joins specially.)
- ***Idea*: For *A/B*, compute all *x* values that are not \`disqualified' by some *y* value in *B*.**
  - *x* value is *disqualified* if by attaching *y* value from *B*, we obtain an *xy* tuple that is not in *A*.

Disqualified *x* values: $\pi_x((\pi_x(A) \times B) - A)$

A/B: $\pi_x(A) -$ Disqualified *x* values

Examples

Reserves

sid	bid	day
22	101	10/10/96
58	103	11/12/96

Sailors

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Boats

bid	bname	color	
101	Interlake	Blue	
102	Interlake	Red	
103	Clipper	Green	
104	Marine	Red	

Find names of sailors who've reserved boat #103

- **Solution 1:** $\pi_{sname}((\sigma_{bid=103}\text{Re}serves) \bowtie Sailors)$

- **Solution 2:** $\pi_{sname}(\sigma_{bid=103}(\text{Re}serves \bowtie Sailors))$

Find names of sailors who've reserved a red boat

- **Information about boat color only available in Boats; so need an extra join:**

$$\pi_{sname}((\sigma_{color='red'} Boats) \bowtie \text{Reserves} \bowtie Sailors)$$

❖ A more efficient solution:

$$\pi_{sname}(\pi_{sid}((\pi_{bid} \sigma_{color='red'} Boats) \bowtie \text{Res}) \bowtie Sailors)$$

☒ *A query optimizer can find this given the first solution!*

Find sailors who've reserved a red or a green boat

- **Can identify all red or green boats, then find sailors who've reserved one of these boats:**

$$\rho\ (Tempboats, (\sigma_{color='red' \vee color='green'} Boats))$$

$$\pi_{sname}(Tempboats \bowtie \text{Reserves} \bowtie Sailors)$$

Find sailors who’ve reserved a red and a green boat

- **Previous approach won’t work! Must identify sailors who’ve reserved red boats, sailors who’ve reserved green boats, then find the intersection (note that *sid* is a key for Sailors):**

$$\rho\ (Tempred,\ \pi_{sid}((\sigma_{color='red'}\ Boats) \bowtie \mathrm{Re}serves))$$

$$\rho\ (Tempgreen,\ \pi_{sid}((\sigma_{color='green'}\ Boats) \bowtie \mathrm{Re}serves))$$

$$\pi_{sname}((Tempred \cap Tempgreen) \bowtie Sailors)$$

Find the names of sailors who’ve reserved all boats

- **Uses division; schemas of the input relations to / must be carefully chosen:**

$$\rho\ (Tempsids,\ (\pi_{sid,bid}\mathrm{Re}serves)\ /\ (\pi_{bid}\ Boats))$$

$$\pi_{sname}\ (Tempsids \bowtie Sailors)$$

- To find sailors who’ve reserved all ‘Interlake’ boats:

$$.....\quad /\ \pi_{bid}(\sigma_{bname='Interlake'}\ Boats)$$