

Carnegie Mellon Univ.  
 Dept. of Computer Science  
 15-415 - Database Applications  
  
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 Relational model

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## Overview

- history
- concepts
- Formal query languages
  - relational algebra
  - rel. tuple calculus
  - rel. domain calculus

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## History

- before: records, pointers, sets etc
- introduced by E.F. Codd in 1970
- revolutionary!
- first systems: 1977-8 (System R; Ingres)
- Turing award in 1981

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## Concepts

- Database: a set of relations (= tables)
- rows: tuples
- columns: attributes (or keys)
- superkey, candidate key, primary key

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## Example

Database:

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

| SSN | c-id   | grade |
|-----|--------|-------|
| 123 | 15-413 | A     |
| 234 | 15-413 | B     |

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## Example: cont'd

Database:

k-th attribute  
(D<sub>k</sub> domain)

↓

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

rel. schema (attr+domains)

← tuple

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### Example: cont'd

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

rel. schema (attr+domains)  
↑ instance

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### Example: cont'd

- Di: the domain of the I-th attribute (eg., char(10))
- Formally: an instance is a subset of (D1 x D2 x ...x Dn)

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

rel. schema (attr+domains)  
↑ instance

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### Example: cont'd

- superkey (eg., 'ssn , name'): determines record
- cand. key (eg., 'ssn', or 'st#'): minimal superkey
- primary key: one of the cand. keys

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### Overview

- history
- concepts
- **Formal query languages**
  - relational algebra
  - rel. tuple calculus
  - rel. domain calculus

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### Formal query languages

- How do we collect information?
- Eg., find ssn's of people in 415
- (recall: everything is a set!)
- One solution: Rel. algebra, ie., set operators
- Q1: Which ones??
- Q2: what is a minimal set of operators?

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### Relational operators

- .
- .
- .
- set union      U
- set difference      '-'

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### Example:

- Q: find all students (part or full time)
- A: PT-STUDENT union FT-STUDENT

| FT-STUDENT |        |          | PT-STUDENT |       |            |
|------------|--------|----------|------------|-------|------------|
| Ssn        | Name   | Address  | Ssn        | Name  | Address    |
| 129        | peters | main str | 123        | smith | main str   |
| 239        | lee    | 5th ave  | 234        | jones | forbes ave |

### Observations:

- two tables are 'union compatible' if they have the same attributes ('domains')
- Q: how about intersection  $\cap$

### Observations:

- A: redundant:
- $STUDENT \cap STAFF = STUDENT - (STUDENT - STAFF)$



### Relational operators

- .
- .
- .
- set union  $\cup$
- set difference  $'-'$

### Other operators?

- eg, find all students on 'Main street'
- A: 'selection'

$$\sigma_{address='main str'} (STUDENT)$$

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

### Other operators?

- Notice: selection (and rest of operators) expect tables, and produce tables (-> can be cascaded!!)
- For selection, in general:

$$\sigma_{condition} (RELATION)$$

### Selection - examples

- Find all 'Smiths' on 'Forbes Ave'

$$\sigma_{name='Smith' \wedge address='Forbes\ ave'}(STUDENT)$$

'condition' can be any boolean combination of '=', '>', '>=', ...

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### Relational operators

- selection  $\sigma_{condition} (R)$
- .
- .
- set union R U S
- set difference R - S

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### Relational operators

- selection picks rows - how about columns?
- A: 'projection' - eg.:  $\pi_{ssn}(STUDENT)$

finds all the 'ssn' - removing duplicates

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

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### Relational operators

Cascading: 'find ssn of students on 'forbes ave'

$$\pi_{ssn}(\sigma_{address='forbes\ ave'}(STUDENT))$$

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

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### Relational operators

- selection  $\sigma_{condition} (R)$
- projection  $\pi_{att-list} (R)$
- .
- set union R U S
- set difference R - S

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### Relational operators

Are we done yet?  
Q: Give a query we can not answer yet!

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### Relational operators

A: any query across two or more tables,  
eg., 'find names of students in 15-415'

Q: what extra operator do we need??

A: surprisingly, cartesian product is enough!

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

| TAKES |        |       |
|-------|--------|-------|
| SSN   | c-id   | grade |
| 123   | 15-413 | A     |
| 234   | 15-413 | B     |

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### Cartesian product

- eg., dog-breeding: MALE x FEMALE
- gives all possible couples

| MALE  |  |
|-------|--|
| name  |  |
| spike |  |
| spot  |  |

x

| FEMALE |  |
|--------|--|
| name   |  |
| lassie |  |
| shiba  |  |

=

| M.name | F.name |
|--------|--------|
| spike  | lassie |
| spike  | shiba  |
| spot   | lassie |
| spot   | shiba  |

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### so what?

- Eg., how do we find names of students taking 415?

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

| TAKES |        |       |
|-------|--------|-------|
| SSN   | c-id   | grade |
| 123   | 15-413 | A     |
| 234   | 15-413 | B     |

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### Cartesian product

- A:  $\sigma_{STUDENT.ssn=TAKES.ssn}(STUDENT \times TAKES)$

| Ssn | Name  | Address    | ssn | cid    | grade |
|-----|-------|------------|-----|--------|-------|
| 123 | smith | main str   | 123 | 15-413 | A     |
| 234 | jones | forbes ave | 123 | 15-413 | A     |
| 123 | smith | main str   | 234 | 15-413 | B     |
| 234 | jones | forbes ave | 234 | 15-413 | B     |

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### Cartesian product

$\sigma_{cid=15-415}(\sigma_{STUDENT.ssn=TAKES.ssn}(STUDENT \times TAKES))$

| Ssn | Name  | Address    | ssn | cid    | grade |
|-----|-------|------------|-----|--------|-------|
| 123 | smith | main str   | 123 | 15-413 | A     |
| 234 | jones | forbes ave | 123 | 15-413 | A     |
| 123 | smith | main str   | 234 | 15-413 | B     |
| 234 | jones | forbes ave | 234 | 15-413 | B     |

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### Cartesian product

$\pi_{name}(\sigma_{cid=15-415}(\sigma_{STUDENT.ssn=TAKES.ssn}(STUDENT \times TAKES)))$

| Ssn | Name  | Address    | ssn | cid    | grade |
|-----|-------|------------|-----|--------|-------|
| 123 | smith | main str   | 123 | 15-413 | A     |
| 234 | jones | forbes ave | 123 | 15-413 | A     |
| 123 | smith | main str   | 234 | 15-413 | B     |
| 234 | jones | forbes ave | 234 | 15-413 | B     |

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### FUNDAMENTAL Relational operators

|                     |                         |
|---------------------|-------------------------|
| • selection         | $\sigma_{condition}(R)$ |
| • projection        | $\pi_{att-list}(R)$     |
| • cartesian product | MALE $\times$ FEMALE    |
| • set union         | $R \cup S$              |
| • set difference    | $R - S$                 |

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### Relational ops

- Surprisingly, they are enough, to help us answer almost any query we want!!
- derived operators, for convenience
  - set intersection
  - join (theta join, equi-join, natural join)  $\bowtie$
  - 'rename' operator  $\rho_{R'}(R)$
  - division  $R \div S$

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### Joins

- Equijoin:  $R \bowtie_{R.a=S.b} S = \sigma_{R.a=S.b}(R \times S)$

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### Cartesian product

- A:  $\dots\dots\dots \sigma_{STUDENT.ssn=TAKES.ssn}(STUDENT \times TAKES)$

| Ssn | Name  | Address    | ssn | cid    | grade |
|-----|-------|------------|-----|--------|-------|
| 123 | smith | main str   | 123 | 15-413 | A     |
| 234 | jones | forbes ave | 123 | 15-413 | A     |
| 123 | smith | main str   | 234 | 15-413 | B     |
| 234 | jones | forbes ave | 234 | 15-413 | B     |

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### Joins

- Equijoin:  $R \bowtie_{R.a=S.b} S = \sigma_{R.a=S.b}(R \times S)$
- theta-joins:  $R \bowtie_{\theta} S$   
generalization of equi-join - any condition  $\theta$

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### Joins

- very popular: natural join:  $R \bowtie S$
- like equi-join, but it drops duplicate columns:  
STUDENT(ssn, name, address)  
TAKES(ssn, cid, grade)

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### Joins

- nat. join has 5 attributes  $STUDENT \bowtie TAKES$

| Ssn | Name  | Address    | ssn | cid    | grade |
|-----|-------|------------|-----|--------|-------|
| 123 | smith | main str   | 123 | 15-413 | A     |
| 234 | jones | forbes ave | 123 | 15-413 | A     |
| 234 | jones | forbes ave | 234 | 15-413 | B     |
| 234 | jones | forbes ave | 234 | 15-413 | B     |

- equi-join: 6  $STUDENT \bowtie_{STUDENT.ssn=TAKES.ssn} TAKES$

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### Natural Joins - nit-picking

- if no attributes in common between R, S:
- nat. join -> cartesian product:

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### Overview - rel. algebra

- fundamental operators
- derived operators
  - joins etc
  - rename
  - division
- examples

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### rename op.

- Q: why?  $\rho_{AFTER}(BEFORE)$
- A: shorthand; self-joins; ...
- for example, find the grand-parents of 'Tom', given PC(parent-id, child-id)

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### rename op.

- PC(parent-id, child-id)  $PC \bowtie PC$

| PC    |      | PC    |      |
|-------|------|-------|------|
| p-id  | c-id | p-id  | c-id |
| Mary  | Tom  | Mary  | Tom  |
| Peter | Mary | Peter | Mary |
| John  | Tom  | John  | Tom  |

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### rename op.

- first, WRONG attempt:  ~~$PC \bowtie PC$~~
- (why? how many columns?)
- Second WRONG attempt:  ~~$PC \bowtie_{PC.c-id=PC.p-id} PC$~~

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### rename op.

- we clearly need two different names for the same table - hence, the 'rename' op.

$$\rho_{PC1}(PC) \bowtie_{PC1.c=id=PC.p-id} PC$$

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### Overview - rel. algebra

- fundamental operators
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### Division

- Rarely used, but powerful.
- Example: find suspicious suppliers, ie., suppliers that supplied all the parts in A\_BOMB

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### Division

| SHIPMENT |    |
|----------|----|
| s#       | p# |
| s1       | p1 |
| s2       | p1 |
| s1       | p2 |
| s3       | p1 |
| s5       | p3 |

$\div$

| ABOMB |  |
|-------|--|
| p#    |  |
| p1    |  |
| p2    |  |

$=$

| BAD_S |  |
|-------|--|
| s#    |  |
| s1    |  |

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### Division

- Observations: ~reverse of cartesian product
- It can be derived from the 5 fundamental operators (!!)
- How?

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### Division

- Answer:

$$r \div s = \pi_{(R-S)}(r) - \pi_{(R-S)}[(\pi_{(R-S)}(r) \times s) - r]$$

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### Overview - rel. algebra

- fundamental operators
- derived operators
  - joins etc
  - rename
  - division
- examples

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### Sample schema

find names of students that take 15-415

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

| CLASS  |        |       |
|--------|--------|-------|
| c-id   | c-name | units |
| 15-413 | s.e.   | 2     |
| 15-412 | o.s.   | 2     |

  

| TAKES |        |       |
|-------|--------|-------|
| SSN   | c-id   | grade |
| 123   | 15-413 | A     |
| 234   | 15-413 | B     |

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### Examples

- find names of students that take 15-415

$$\pi_{name} [\sigma_{c-id=15-415} (STUDENT \bowtie TAKES)]$$

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### Sample schema

find course names of 'smith'

| STUDENT |       |            |
|---------|-------|------------|
| Ssn     | Name  | Address    |
| 123     | smith | main str   |
| 234     | jones | forbes ave |

| CLASS  |        |       |
|--------|--------|-------|
| c-id   | c-name | units |
| 15-413 | s.e.   | 2     |
| 15-412 | o.s.   | 2     |

  

| TAKES |        |       |
|-------|--------|-------|
| SSN   | c-id   | grade |
| 123   | 15-413 | A     |
| 234   | 15-413 | B     |

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### Examples

- find course names of 'smith'

$$\pi_{c-name} [\sigma_{name='smith'} (STUDENT \bowtie TAKES \bowtie CLASS)]$$

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### Examples

- find ssn of 'overworked' students, i.e., that take 412, 413, 415

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### Examples

- find ssn of 'overworked' students, ie., that take 412, 413, 415: almost correct answer:

~~$$\sigma_{c-name=412}(TAKES) \sqcup \sigma_{c-name=413}(TAKES) \sqcup \sigma_{c-name=415}(TAKES)$$~~

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### Examples

- find ssn of 'overworked' students, ie., that take 412, 413, 415 - Correct answer:

$$\pi_{SSN} [\sigma_{c-name=412}(TAKES)] \sqcup \pi_{SSN} [\sigma_{c-name=413}(TAKES)] \sqcup \pi_{SSN} [\sigma_{c-name=415}(TAKES)]$$

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### Examples

- find ssn of students that work at least as hard as ssn=123 (ie., they take all the courses of ssn=123, and maybe more)

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### Sample schema

| STUDENT |       |            | CLASS  |        |       |
|---------|-------|------------|--------|--------|-------|
| Ssn     | Name  | Address    | c-id   | c-name | units |
| 123     | smith | main str   | 15-413 | s.e.   | 2     |
| 234     | jones | forbes ave | 15-412 | o.s.   | 2     |

| TAKES |        |       |
|-------|--------|-------|
| SSN   | c-id   | grade |
| 123   | 15-413 | A     |
| 234   | 15-413 | B     |

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### Examples

- find ssn of students that work at least as hard as ssn=123 (ie., they take all the courses of ssn=123, and maybe more)

$$[\pi_{SSN, c-id}(TAKES)] \div \pi_{c-id}[\sigma_{SSN=123}(TAKES)]$$

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### Conclusions

- Relational model: only tables ('relations')
- relational algebra: powerful, minimal: 5 operators can handle almost any query!
- most non-trivial op.: join

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