Modeling ethno-religious conflicts as Prisoner's Dilemma game in Graphs

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***Abstract*—In this paper, we present and analyze a multi-agent game theoretic model of conflicts in multi-cultural societies. Two salient factors responsible for violence in multi-cultural societies (that are identified in the social sciences literature) are (a) ethno-religious identity of the population and (b) spatial structure (distribution) of the population. It has also been experimentally shown by Lumsden that multi-cultural conflict can be viewed as a Prisoner's Dilemma (PD) game. Using the above observations, we model the multi-cultural conflict problem as a variant of the repeated PD game in graphs. The graph consists of labeled nodes corresponding to the different ethno-religious types and the topology of the graph encode the spatial distribution and interaction of the population. The agents play the game with neighbors of their opponent type and they update their strategies based on neighbors of their same type. This strategy update dynamics with different update neighborhood from game playing neighborhood distinguishes our model from conventional models of PD games in graphs. We present simulation results showing the effect of various parameters of our model to the propensity of conflict in a population consisting of two ethno-religious groups.**

I. Introduction

Ethno-religious conflict in multi-cultural societies has been one of the major causes of loss of life and property in recent history (e.g., violence in Yugoslavia [1], Sudan [2]). There has been substantial empirical research in sociology and conflict resolution literature on analyzing the causes of such violence (see [3] and references therein). Apart from the ethno-religious identity, other social and economic factors identified in the literature in violence prone areas are economic grievances, competition for scarce natural resources, historical precedents, territorial claims, influence of political/religious elites, and international influences. The spatial structure of the different population groups is also correlated to the occurrence of violence [3], [4].

More recently, there has been focus on computational study of civil conflict in societies (see [5], [6], [7] and references therein), some of which have also studied societies with multiple ethno-religious groups [4], [6]. Although the proposed computational models vary in detail, the common features of existing agent-based computational models are: (a) the agents are assumed to be distributed on a grid and the interaction between agents are restricted to a neighborhood around their position (usually their Moore-neighborhood, i.e., nearest 8 neighbors) and (b) the interaction between agents is non-adaptive (except in [6]), e.g., agents will die with a certain probability if they are in the neighborhood of opponent agents or they will migrate towards agents of only their type. Since it is well established that the structure of a social network of interacting population is not like a grid [8], we model the interaction topology among the agents as a graph. Moreover, we model the interaction between agents as a repeated prisoner's dilemma (PD) game where the agents update their strategies. Thus, in this paper, we present a simple multi-agent game-theoretic model for studying conflict situations in multi-cultural societies.

The primary cause of conflict in multi-cultural societies is the fear of the minority population about loss (or suppression) of their ethno-religious identity to the majority group. Lumsden demonstrated this through a series of experiments in the context of the Cyprus conflict between Greek Cypriots and Turkish Cypriots [9]. Both the Greek Cypriots and the Turkish Cypriots gave higher value to maintaining their position on *right to self-determination* (which is directly related to the importance the groups have for their own ethno-religious identity) than to compromise and modify their position even if the former meant war in the long run (that is detrimental to both) and the latter meant peace (that is beneficial to both). In other words, the payoff matrix of the conflict viewed as a 2×2 matrix game had the structure of the prisoner's dilemma (PD) game. In this paper, we use this insight of Lumsden combined with the ethno-religious identity and the spatial structure of the population to form a multi-agent game-theoretic model for studying conflict in multi-cultural societies.

We model the problem as a PD game on graphs where the nodes of the graph are the agents and the edges between the agents represent interaction between them. Each agent represents a group of people (e.g., a household or a family). The graph topology encodes the spatial distribution of the population and the interaction between the different groups of the population is abstracted as the PD game. The nodes in the graph have different labels denoting their ethno-religious identity. The agents play the game with members of their opponent groups and they update their strategies based on the members of their own group. We note that the different neighborhoods we use for an agent for game playing and strategy updating is a departure from the standard model of PD games on graphs (where the game playing and strategy updating neighborhoods are assumed to be the same). Assuming that there are two different ethno-religious types in the population, we present simulation studies (on synthetic data) showing the effect of our model parameter variations.

Contributions: The main contributions of our paper are: (1) we introduce a simple and novel game theoretic model for studying ethno-religious conflict, and (2) we present simulation studies showing the effect of various parameters of our model to a measure of the potential of conflict (that is defined by Equation 3). Since the model of the PD games in graphs that we introduced is new, we also present theoretical results that (partially) characterize the long term strategy update dynamics of our new PD game in graphs model. In particular, we prove that the strategy of an agent may not reach a fixed point by showing counter-examples of graphs where the strategies of the agents oscillate.

This paper is organized as follows: In Section II, we present a brief overview of the literature related to conflict modeling and PD games in graphs. In Section III, we present the formal definition of PD games in graphs and other definitions used in the paper. In Section IV, we present in detail our model for multi-cultural conflict modeling based on PD game in graphs. Thereafter, in Section VI, we present simulation results. In Section VII we present a discussion of some limitations of our model and indicate ways to overcome them. Finally, in Section VIII, we provide our conclusions and outline problems to be addressed in the future.

II. Related Work

There has been a variety of social and economic causes put forward for ethnic conflicts based on empirical fact-based research [3], [10], [11]. These causes can be divided into three generic categories [3], [12] (a) non-material causes like ethno-religious identity, culture, history of violence, mutual fear (b) material causes like uneven distribution of natural resources, uneven economic development and (c) use of ethno-religious identity based nationalism by political and religious elites. These factors are not independent of each other and can be thought of as factors that enhance the importance of ethno-religious identity in the population. Geographical factors like territorial indivisibility and the spatial distribution of the population has also been proposed as another reason for ethnic violence [3].

Computational modeling for understanding/analyzing ethno-religious violence has received extensive attention recently [5], [4], [6], [7], [13]. Although [5], [6], [7], [13] view civil violence as a result of pent-up grievances, they vary significantly in their details. [5], [6], [7] look at the conflict as a function of mass mobilization whereas [13] considers it as a function of economic causes. However, all the models have the common feature that they consider the agents to be distributed on a grid and interact with their neighbors in the grid graph. In [6], the authors consider the interaction between the agents as an iterated game which they call as a PD game. However, their definition does not correspond to the standard definition of a PD game [14].

To demonstrate the importance of spatial structure of the population in ethnic conflicts, Lim et al. [4] proposed a mathematical model based on the dynamics of type separation. Type separation models were originally proposed to explain pattern formation in physical or chemical phase separation processes and has been used in the social segregation context in [15]. Lim at al. [4] assumed that the population consists of different types and occupy the nodes of a $2D$ grid with some empty nodes. The like agents move towards each other whereas unlike agents move away from each other. This leads to formation of patches of different types and a patch of one type surrounded by agents of another type is predicted as a site of violence. Thus, they demonstrate the importance of the spatial structure of the population for ethno-religious violence modeling. However, it is not apparent whether it is possible to extend this model to take into consideration other factors like effect of leaders or uneven distribution of natural resources.

Lumsden conducted experiments to elicit the structure of the Cyprus conflict in game-theoretic terms [9]. He concluded that the essential factors of the conflict can be captured as a two-party, two-choice game and showed that the payoff matrix of the game has the structure of the prisoner's dilemma game. The Prisoner's Dilemma game is a well known model for many social choice situations. In this two-player game, each player has two actions, cooperate (C) or defect (D). The payoff matrix for the game has the following characteristics: (a) the highest payoff is obtained by the defector against the cooperator, (b) the total payoff for mutual cooperation is the highest and (c) the defector's extra income (relative to its income for mutual cooperation) is less than the loss of the cooperator (please see Section III for a formal statement). In a single shot PD game, each player should choose to defect and this is the Nash equilibrium. However, the total payoff for both players in this case is lesser than the case when both play cooperate. In many social situations, cooperation emerges among self-interested people. The iterated PD game [16], [14] was proposed as a model to capture this and it was shown that cooperation indeed emerges in iterated PD games where the number of iterations are possibly infinite (for finite iterated PD games with the number of iterations known to the players, defect should be the best strategy; this can be shown by backward induction using the NE solution of the single-shot PD game at the last step).

The PD game in graphs have also received attention in the recent past. We will give a very brief discussion about the literature that is directly relevant to this work (for a more extensive review and discussion on evolutionary games on graphs in general, see [17]). In this literature also, emergence of cooperation has been demonstrated in social networks by simulation studies [18], [19], [20]. Santos et al. [19] presented simulation studies showing the emergence of cooperation in graphs of fixed topology for a range of values of the parameters in the payoff matrix. Their main goal was to study the effect of the variation of degree of the nodes on the evolution of cooperation. Zimmermann at al. [20] presented simulation results showing the evolution of cooperation in graphs with variable topology, where the dynamics of the network was much slower than the dynamics of the strategies. In this paper, the specific game model of PD games that we study is very similar to that in [19], [20]. One difference of our model from [20] is that we assume the network topology to be fixed. The main distinction of our model is that we consider two different types of agents form the nodes of the graph and thus the game-playing and strategy update neighborhood for the agents can be different.

III. PRELIMINARIES

In this section we introduce the notations and definitions that will be used in the remainder of the paper.

Undirected graph: An undirected graph G is an ordered pair, $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is a set of n nodes, and $E \subseteq V \times V$ is a set of edges. Two nodes v_i and v_j are called *neighbors* of each other if $(v_i, v_j) \in E$. The set $\mathcal{N}_i = \{v_j | (v_i, v_j) \in E\}$ is the set of v_i's neighbors, and $|\mathcal{N}_i|$ is defined as the *degree* of node v_i. Denote $\mathcal{N}_i^+ = \mathcal{N}_i \cup \{v_i\}$.

Scale-free network: A scale-free network is a graph where the degree distribution of nodes follow a power law [8], i.e., $N_d \propto d^{-\gamma}$, where N_d is the number of nodes of degree d and γ is a constant called the power law degree exponent (typically $\gamma \in [2, 3]$).

Prisoner's Dilemma Game: In its simplest form, the prisoner's dilemma game is a single-shot two-player game where the players have two available strategies – cooperate (C) and defect (D). The payoff's of the players is given by the following table

	C	D
C	σ_1, σ_2	a_1, b_2
D	b_1, a_2	δ_1, δ_2

where the index 1 corresponds to the row player and the index 2 to the column player. The entries in the payoff matrix of each player should satisfy $b_i > \sigma_i > \delta_i > a_i, i = 1, 2$. The payoff's of both players are usually assumed to be identical. In this paper, we assume $\sigma_1 = \sigma_2 = \sigma$, $a_1 = a_2 = a$, $\delta_1 = \delta_2 = \delta$. b represents the extent of an agent's playing defection, i.e., an agent with higher b has more incentive to defect than an agent with lower b. For repeated PD games an additional constraint is $2\sigma > a + b_i, i = 1, 2$. We further follow the convention in Nowak [14] and set $a = 0$, to reduce the number of parameters.

PD Game in Graphs: A PD game in a graph is a repeated game where the n-players form the nodes of the graph and the game proceeds in two phases: (i) game playing phase (ii) strategy update phase. The parameters that define different versions of PD games in graphs are: (a) topology of the graph (fixed or variable) (b) game playing and strategy update neighborhood (c) strategy update rule (d) assumptions on synchronous or asynchronous strategy update. The version of the PD game on graphs that is most relevant to this paper is defined below.

PD game in fixed graphs with synchronized strategy update is a repeated game where each iteration of the game proceeds in the following two phases: (a) In the game playing phase the players play the PD game with all their neighbors with a fixed strategy and compute their total payoff. (b) In the strategy update phase, each player compares the payoff's of all its neighbors (including itself) and chooses the strategy of its neighbor with the highest payoff for the next iteration. In other words, our strategy update rule is: *imitate your best/wealthiest neighbor*.

In this version of the game, the agents are not distinguished by group labels, and the neighborhoods for game playing and strategy update are assumed to be same. As we shall discuss in the next section, in our model, we assigned agents to different groups (so each agent will have a new property of group label) and distinguish the game playing and strategy update neighborhood of each agent based on its group label.

IV. PROBLEM MODEL

In this section, we present our agent-based model for conflict among different groups in multi-cultural societies. We model the whole multi-cultural population in a geographical region as a collection of agents. An agent represents a collection of individuals. Since an individual interacts with few other individuals in a society, we model the collection of interacting population as a graph where the nodes are the agents and the edges denote interaction between the agents. We assume that the population consists of two different ethno-religious groups (i.e., there are two different types of nodes in the graph). The interaction between agents in two different groups is modeled as the prisoner's dilemma game. In this context, the strategy cooperate (C) implies the willingness of the agent to compromise with the other group whereas the strategy defect (D) implies unwillingness to compromise with the other group. Thus, the fraction of links between the two groups where both agents play D can be used as a measure of tension between the two groups.

Network Construction: As stated in Section III, for defining the PD game on graphs we need to first define the graph on which the game should be played. We use an undirected graph $G = (V, E)$ to represent the agents of two groups and their connections. $V = \{v_i | i = 1, \ldots, n\}$. Let n_1 and n_2 be the number of agents in the two groups, respectively with $n = n_1 + n_2$. We construct G in two steps:

1) Construct the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ for each group separately, where $|V_1| = n_1$ and $|V_2| = n_2$.
2) Construct the set of edges $E_3 \subseteq V_1 \times V_2$ such that each agent in one group is connected to at least one agent in the other group. The edges are added by picking two nodes from the two different groups uniformly at random. Let the average number of edges connecting an agent in one group to agents in the other group be k (a parameter that captures the degree of connectivity between the two groups).

Thus, after the two steps of construction, we get the graph $G = (V, E)$, with $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E_3$. Figure 1 illustrates the network structure of G. By construction, in the graph G, each agent i has two types of neighborhood: (a) neighborhood of agents of same type $\mathcal{NS}_i = \{v_j | (v_i, v_j) \in E_1 \cup E_2\}$ and (b) neighborhood of agents of different type $\mathcal{ND}_i = \{v_j | (v_i, v_j) \in E_3\}$.

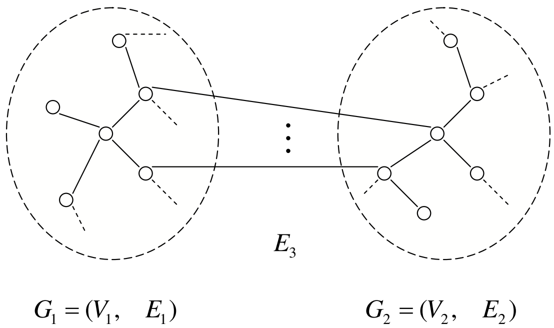

Fig. 1. The network structure G of the model. $G = (V_1 \cup V_2, E_1 \cup E_2 \cup E_3)$. On the left (or right) oval is a sub-graph of network G_1 (or G_2). Edges connecting nodes in the two ovals represent E_3, the set of edges between two groups.

Phases in each round of the game: As stated before, the PD game on graphs proceeds in rounds where each round (iteration) consists of the game playing phase and the strategy update phase. We assume that the network structure, G, is fixed and the strategy update is synchronous. Each agent plays the PD game with all agents of the other type in its neighborhood, i.e., $\mathcal{ND}_i$ is the game playing neighborhood of each agent. Let $s_i(t)$ denote the strategy of agent i at round t, where $s_i(t) = 0$ implies cooperation (C) and $s_i(t) = 1$ implies defection (D). We assume that at each iteration, each agent plays the same strategy with all agents in its game playing neighborhood. The aggregate payoff, $p_i(t)$, of agent i in iteration t can be computed by summing up the individual payoffs obtained from playing with agents in $\mathcal{ND}_i$.

$$p_i(t) = \sum_{j: v_j \in \mathcal{ND}_i} \sigma(1 - s_i(t))(1 - s_j(t)) + b s_i(t)(1 - s_j(t)) + \delta s_i(t) s_j(t) \quad (1)$$

In the strategy update phase, each agent i imitates the strategy of the agent with highest payoff at previous iteration from a set $\mathcal{C}_i^+ = \mathcal{C}_i \cup \{v_i\}$ where $\mathcal{C}_i$ is the strategy update neighborhood. In this paper we choose the strategy update neighborhood of an agent to be the neighborhood containing agents of the same ethno-religious type, i.e., $\mathcal{C}_i = \mathcal{NS}_i$ (we will discuss another choice for this below). If there is more than one agent with the highest payoff, an agent randomly selects one of the agents and imitate its strategy. Thus the strategy update for agent i can be written as:

$$s_i(t) = s_j(t-1)$$
$$\text{where } j = \arg\max_{k \in \mathcal{C}_i^+} (p_k(t-1)) \quad (2)$$

Let $S(t) = [s_1(t), \ldots, s_n(t)]$ and $P(t) = [p_1(t), \ldots, p_n(t)]$ the strategy vector and the aggregate payoff vector for all n agents at iteration t, respectively. The initial strategy vector $S(1)$ is randomly chosen so that the probability that one agent's initial strategy is cooperation is f_c (which represents the initial fraction of cooperators). Figure 2 illustrates the evolution of strategy vector $S(t)$ and payoff vector $P(t)$.

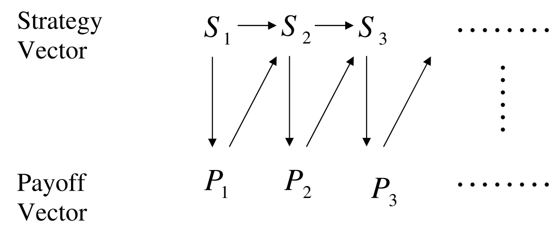

Fig. 2. The evolution of $S(t)$ and $P(t)$. The arrows represent the dependence relationship of the variables. The value of the arrows' destination is given by the values of the arrows' origins.

Game playing and strategy update neighborhood: In our model, we chose our game playing neighborhood for an agent as $\mathcal{ND}_i$ (i.e., agents in its neighborhood that are of different type). Intuitively, this encodes the fact that we are interested in inter-cultural disputes and not in disputes within the culture. For the strategy update neighborhood we have chosen $\mathcal{NS}_i$. This encodes the assumption that an agent gives more importance to the opinions of neighbors of its own type regarding the opposite type than the opinion of the other type for its own type. Mathematically, there is another possibility that the strategy update neighborhood for an agent can be all agents within its neighborhood, i.e., $\mathcal{NS}_i \cup \mathcal{ND}_i$. However, we have done some simulations showing that the results obtained with the two choices are qualitatively very similar (also see [21] for details). Therefore we do not consider the second case any further in the paper.

In our model of the PD game in graphs we have a separate neighborhood for game playing and strategy update. This is different from the conventional case where the game playing and strategy update neighborhood is the same. We make the distinction in the game playing and strategy update neighborhood to encode the fact that there are two different types of agents in the graph. We emphasize this difference because the literature on PD games in graphs mostly talks about the emergence of cooperation, whereas we will show that considering two different types of agents lead to the emergence of defection between the two groups.

Model parameters: The PD game in graphs that we defined has a number of parameters: (a) The parameters defining the payoff matrix σ, δ, a, b_1 and b_2. We have already mentioned that we choose $a = 0$. We can also choose $\sigma = 1$ without any loss of generality. The main parameters of interest in the payoff matrix are b_1 and b_2. They represent the payoff to an agent of group 1 (group 2) when an agent in group 2 (group 1) compromises on its stand. According to the results in [9], agents in smaller groups have more incentive to defect than those in larger group. That is, b for agents in smaller group is larger than for agents in larger group. So we distinguish b values for the two groups: b_1, b_2 and if $n_1 > n_2$ we have $b_1 < b_2$. We choose the value of the parameter δ, the payoff for $D-D$ pair, to be 0.1. δ does not change the results significantly as long as it is much smaller than σ (increasing the value of δ will increase the fraction of $D-D$ links). (b) The ratio of population in the two groups n_1/n_2 is another parameter. (c) The initial fraction of cooperators in the PD game that roughly encodes the prevalent level of animosity between the two groups (e.g., due to historical reasons or due to occurrence of an external event). (d) The topology of the graphs and the average number of edges from one group to another. This last parameter is an assumption that we make since we cannot know the exact interaction patterns between individuals in the population. In Section VI, we will present simulation results obtained by systematically varying these parameters.

Measure of Conflict Potential: The measure of conflict potential between two different groups should consider the interactions between agents in different groups, which can be expressed as the strategy pairs for two neighboring agents belonging to different groups (in the steady state). So we use the fraction of $D-D$ links (two neighboring agents both play defection) between two groups as a measure of the potential of conflict between the two groups, which can be computed as follows:

$$f_{dd} = \frac{\sum_{(v_i,v_j)\in E_3} s_i \cdot s_j}{|E_3|} \tag{3}$$

V. Analysis

As discussed in Section IV, the strategy vector $S(t)$ of all the agents will evolve according to equation 2. We define the state of each agent as its strategy s_i and therefore the vector $S(t)$ is the state vector of the whole graph. Thus equation 2 gives the state evolution equations of the whole dynamical system. We will call the set of all possible states as the state space of the system. Since each component of the state vector can take on only two possible values (i.e., $s_i(t) = 0$ or 1), the state space is discrete with cardinality 2^n. In this section, we will give partial characterization of the long term behavior of the state vector $S(t)$ for the model as the iteration progresses.

The first question one might ask for this dynamical system is whether the state vector will converge to a fixed state (or fixed point in the state space). Our main result here is that the state vector $S(t)$ whose components evolve according to equation 2 may not converge to a fixed state, which means the system's conflict measure in this model may fluctuate depending on the network structure.

Lemma 1: For the model of PD game in graphs defined in Section IV, as $t \to \infty$, the state vector $S(t)$, may not converge to a fixed point.

Proof: We will prove this lemma by constructing an example of oscillation for the model in Section IV. At iteration t, the fact that the system has reached the oscillatory state means that the sequence of strategy vectors $\{S(t), \dots, S(t+k-1)\}, k > 1$ will repeat forever (where k is the period of the oscillation).

Figure 3 shows an example of oscillation with two agents in the middle switching their strategies forever (oscillation with period 4). Agents in the first row belong to one group and agents in the second row belong to the other group. The two agents in the middle column will alternate between C and D and their strategies will traverse all alternatives: (C,D), (D,D), (D, C) and (C, C). In Figure 3, the notation $\underline{C}$ or $\underline{D}$ means that the agent will switch strategy at next iteration; the italic dot-underlined C or D means that the agent just switched strategy at previous iteration; the bold $\mathbf{C}$ or $\mathbf{D}$ means that the agent's payoff has changed due to the fact that its neighboring agent from the other group has switched strategy. The four states will repeat forever. ■

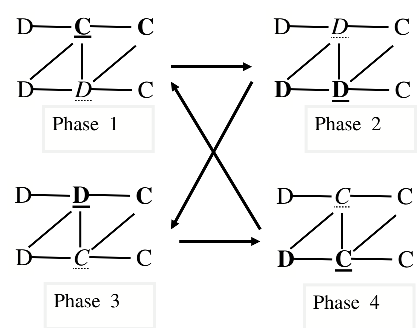

Fig. 3. Oscillation in our model with period 4.

Although we have shown above that there are cases where the strategy vector $S(t)$ do not reach a fixed point, there are also cases when the strategy vector will reach a fixed point. In fact, in our simulations we have observed that the strategy vector reaches a fixed point in most cases. In general, whether we will reach a fixed state depends on the initial strategy distribution and the structure of the graph. We can construct trivial examples of fixed states for the model by setting identical the strategies of agents in the group where the strategy update neighborhood is defined. For example, one group of agents play C while the other group of agents play D or all agents play C (or D). It is easy to conclude that on a complete graph, the steady state can only be trivial examples of fixed states above. However, there can be many other non-trivial fixed states on a general graph. Below we try to capture some common features of them.

Given a node i, define $l(i)$ as i's neighboring node whose strategy i imitates. $l(i)$ can be computed according to Equation 2, and is time-varied until the system reaches a fixed state. At a fixed state, if we keep each node i and each edge $(i, l(i))$ (*imitation edges*), and remove other edges in G, we will get a new graph

$$G' = (V, E') \text{ where } E' = \{(i, l(i)) | i \in V\}$$

that represents imitation relation of agents in fixed states. Since we removed non-imitation edges in G to form G', G' may not be a connected graph as G. According to the strategy update rule, the imitation edges can not form any cycles. So G' can be represented as a set of trees:

$$G' = \cup_{i:i=l(i)} T_i$$

where T_i is a tree (*imitation trees*) composed of root node i imitating itself, all other nodes connecting to i through imitation edges in E', and the imitation edges between them. In each imitation tree, each node j will have one parent node $l(j)$. All agents in each imitation tree will play the same strategy and the payoff of any ancestor node must be greater than or equal to that of offspring nodes. So the imitation trees can be classified as cooperative imitation tree or defective imitation tree.

Starting from an agent i in the imitation tree T_j, we can trace the parent node until we reach the root node j to form a chain of agents: $L(i) = \{i = l^0(i), l(i), l^2(i), \ldots, l^{p-1}(i) = j\}$. Then we can conclude that there will not be edges crossing each other between two chains derived from different imitation trees.

Proposition 1: Considering two chains from different imitation trees: $L(i)$ with length p_1 and $L(j)$ with length p_2, $\forall 0 \le a_1 < a_2 \le p_1$, $0 \le b_1 < b_2 \le p_2$, if $\{l^{a_1}(i), l^{b_2}(j)\} \in E$, $\{l^{a_2}(i), l^{b_1}(j)\} \notin E$.
Proof: Proof by contradiction. Suppose $\{l^{a_1}(i), l^{b_2}(j)\} \in E$, $\{l^{a_2}(i), l^{b_1}(j)\} \in E$.

Since $\{l^{a_1}(i), l^{b_2}(j)\} \in E$, we have $p_{l^{a_1+1}(i)} \ge p_{l^{b_2}(j)}$ Also according to the imitation rule, the payoff of any ancestor nodes must be no less than that of offspring nodes, so

$$p_{l^{a_2}(i)} \ge p_{l^{a_1+1}(i)}, p_{l^{b_2}(j)} \ge p_{l^{b_1+1}(j)}$$

From the equations above, we get $p_{l^{a_2}(i)} \ge p_{l^{b_1+1}(j)}$. Since $\{l^{a_2}(i)), l^{b_1}(j)\} \in E$, $l^{b_1}(j)$ will imitate $l^{a_2}(i)$ instead of $l^{b_1+1}(j)$, which contradicts the fact. (Even when $p_{l^{a_2}(i)} = p_{l^{b_1+1}(j)}$, according to the imitation rule, $l^{a_2}(i)$ has higher priority to be imitated than $l^{b_1+1}(j)$.)■

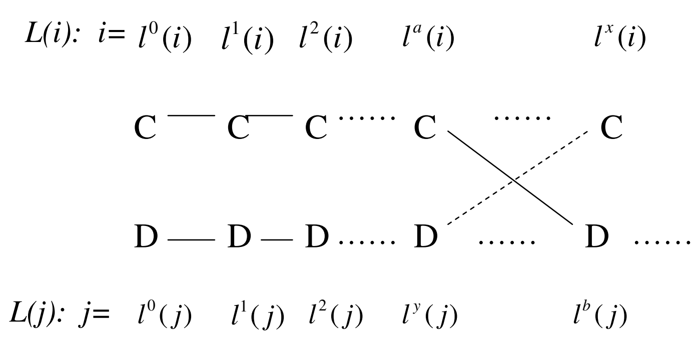

Fig. 4. When there is an edge in E from $l^a(i)$ to $l^b(j)$ between two chains there cannot be an edge between $l^x(i)$ and $l^y(j)$.

The characteristic of the steady state captured in Proposition 1 means that as iteration progresses, the network will evolve to form some kind of tree structure of "organization" based on the imitation relationship and the connectivity among different trees should remain some kind of social hierarchy (tree levels) according to Proposition 1.

VI. Simulation Results

In this section, we present simulation results showing the effect of changing the various parameters that define our model. As stated before, our measure of conflict is the fraction of $D-D$ links between the two groups. The parameters we consider include: (i) the initial fraction of cooperators, f_c, (ii) the number of nodes in the two groups, n_1, n_2, (iii) the average number of edges from an agent in one group to the other group, k, and (iv) the payoffs (b_1 or b_2) to the agent playing D when its opponent plays C.

The default values for the parameters in our simulation are: $f_c = 0.5$, $n_1 = n_2 = 300$, $k = 5$, $b_1 = b_2 = b = 1.5$. When we change the value of one parameter, other parameters have their default values unless otherwise stated. The graphs G_1 and G_2 used for the simulations are scale-free networks generated by using the Barabasi-Albert algorithm [8]. The set of edges E_3 between nodes in V_1 and V_2 are generated randomly. However, we ensure that each node in V_1 is connected to at least one other node in V_2 and the average number of edges between the two groups is k. Each data point in the figures for fraction of final $D-D$ links were generated using an average of 500 iterations on randomly generated graphs as described in Section IV. In the simulation, we observed that after a transient time of 30 iterations, the strategy vector either converges to a fixed state or occasionally comes to an oscillation state with small magnitude and period. So we compute the fraction of final $D-D$ links by averaging over 5 iterations after a transient time of 30 iterations.

Figure 5 shows the effect of the fraction of initial cooperators (f_c) on the final fraction of $D-D$ links for various values of b with the other parameters remaining constant. For a given b, as f_c is increased, the final fraction of $D-D$ links between the two groups decrease (as expected). However, as the value of b increases, even when the initial fraction of cooperators is as high as 0.9, our model still predicts a high fraction of final $D-D$ links (e.g., more than 0.6, for $b = 1.5$), implying that the model captures the potential of conflict between two different groups. To verify that this is due to the fact that we are considering two different groups in our model, we ran simulation on the same graphs where we do not distinguish between the two types of agents (i.e., we simulate a conventional PD game). In those cases, we found (although we do not show it here) that the fraction of final $D-D$ links is quite small when f_c is high (as expected and reported in the literature), which means cooperation will emerge there.

To investigate the influence of the number of nodes in the network on the evolution of conflict, we ran two sets of simulations by increasing n (the number of nodes in each group) from 100 to 1000, and increasing k (the number of average edges from an agent in one group to agents in the other group) from 1 to 20, respectively. Figure 6 shows that different n do not influence the fraction of final $D-D$ links for our model (it stays within 0.78 to 0.79 as n varies from 300 to 1000). However, in Figure 7, we find that with k increasing, the fraction of final $D-D$ links does increase when k is

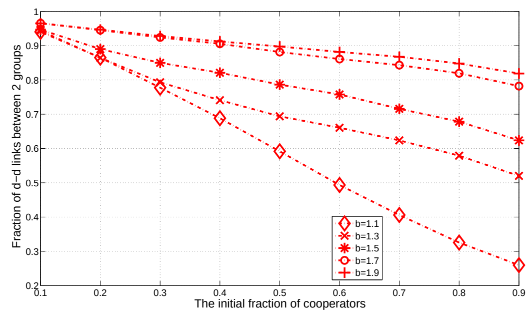

Fig. 5. The plot of fraction of $D-D$ links between two groups as function of initial fraction of cooperators of all agents for different b from 1.1 to 1.9. $n = 300, k = 5$.

sufficiently large ($k \geq 5$). Since k represents the connectivity of the two groups, we can see that in the conflict model, the more interactions between two groups, the more conflicts they will have.

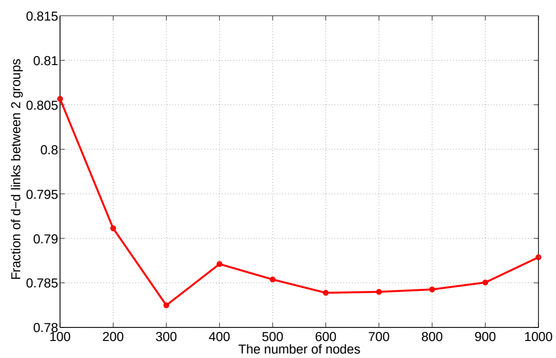

Fig. 6. The plot of fraction of d-d links between two groups as function of the number of nodes in each group. $f_c = 0.5, k = 5, b = 1.5$.

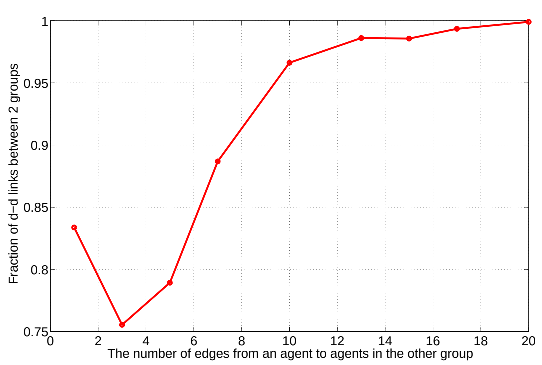

Fig. 7. The plot of fraction of $D-D$ links between two groups as function of k. $n = 300, f_c = 0.5, b = 1.5$.

Figure 8 shows that when b increases from 1.1 to 1.9, the fraction of final $D-D$ links will increase. That is because b represents the incentive of agents to play defection. With higher b, the higher payoff values of defection versus cooperation encouraged agents to play defection instead of cooperation.

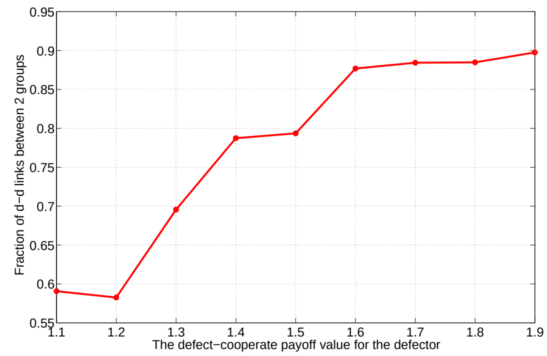

Fig. 8. The plot of fraction of $D-D$ links between two groups as function of b. $n = 300, f_c = 0.5, k = 5$.

In the simulation above, we assume that both groups have the same size $n_1 = n_2$. Next we remove the assumption to consider the effect of having two group with different sizes $n_1 \neq n_2$. We fixed the size of the larger group $n_1 = 500$, and increase $\frac{n_2}{n_1}$, the relative size of the smaller group to the larger one, from 0.1 to 0.9. Meanwhile according to the results in [9], smaller groups are more defective than larger group, so we distinguish b values for the two groups: $b_1 < b_2$. Figure 9 shows the result when we change the relative size of the two groups and b values for them. From the figure, we can find that the conflict will increase when the relative size of the two groups decrease, e.g., the conflict is more drastic (the fraction of $D-D$ links is almost 1) between a larger group and a smaller one with $\frac{1}{10}$ size of the former.

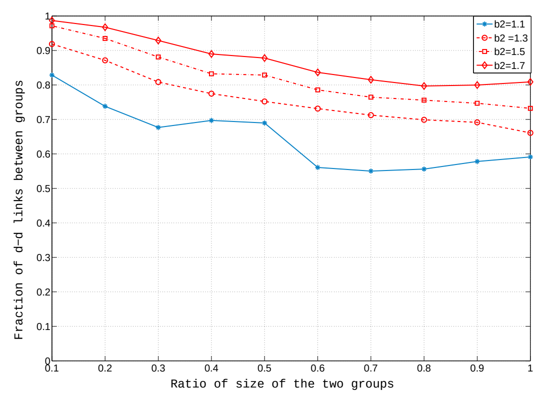

Fig. 9. The plot of fraction of $D-D$ links between two groups as function of $\frac{n_2}{n_1}$ corresponding to $b_1 = 1.1$ and b_2 increased from 1.1 to 1.7. $f_c = 0.5, k = 5$.

VII. Discussion

In the very simple model that we developed for studying conflict in multi-cultural societies, we considered only the effects of ethno-religious identity and spatial distribution of the population. There are other important factors like (a) effect of leaders, (b) migration of population, and (c) uneven distribution of natural resources, that should also be considered for making the model more realistic. We state below the possible extensions (or variations) of our model to take these factors into account.

Since we model the population of each type as a graph, we can use the degree of the nodes of the graph as a measure of its social influence. Consequently, high degree nodes can be thought of as the agents with high influence on the population, i.e., the leader nodes. Thus by multiplying the payoff's of the agents at each stage by a factor proportional to the degree of their nodes, we can ensure that the strategies of the leader nodes have high payoffs. Consequently, during the strategy update phase, the leaders will have more significant effect on the strategies of the whole population compared to low degree nodes. In other words, the willingness of the leaders to compromise or not will play a significant role in the potential of conflict between the two groups. The migration of population can be taken into consideration by using a variable topology graph instead of a fixed topology graph. The migration of a group of population would correspond to the breaking of some existing edges and adding new edges corresponding to the new spatial distribution. The effect of migration on an existing state of potential conflict in the society can thus be studied. The uneven distribution of resources can be taken into account by using a value of b, the incentive to defect as a function of the space. For example, minority agents at places with uneven distribution of natural resources may have a higher value of b than at places where such inequity do not exist. We are currently working on adding these features to our model.

VIII. Conclusion

In this paper we developed and analyzed a model for studying conflict in multi-cultural societies that is based on the prisoner's dilemma games in graphs. Our model captures two essential causes of multi-cultural conflict (a) ethno-religious identity of the different groups and (b) spatial distribution of the population. The prisoner's dilemma game in graphs usually encourages the evolution of cooperation but we show that dividing the population into two groups lead to an increase in the fraction of $D - D$ links in the two groups (which is our measure of the tendency of conflict).

We evaluated our model by running several sets of simulations showing the effect of different parameters defining our model. We showed that the fraction of D-D pairs is relatively insensitive to the number of nodes in the two groups, i.e., the number of agents. Thus the exact number of agents present in the society is not very relevant to the tendency of conflict. We found that, as expected, with higher initial fraction of cooperators, there is a lower tendency of conflict. However, the tendency of conflict increases with the number of edges between two groups (which represents the interaction between the groups of agents). We also showed that the tendency of conflict is much more between two groups of different size than between two groups of similar size. This is mainly because the smaller groups have more incentive to defect than the larger groups [9].

There are various directions for extending this work. Currently we are working on exhaustive testing of our model in simulation. A natural extension is to extend the model and analysis to more than two groups of different cultures. We also plan to test our model on real world data (from conflicts in Yugoslavia and Sudan) to see how well the fraction of $D - D$ links correlates with the actual occurrence of violent incidents.

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References

[1] P. Ball, W. Betts, F. Scheuren, J. Dudukovich, and J. Asher, "Killings and refugee flow in Kosovo, march-june 1999," *Report to the Intl. Criminal Tribunal for the former Yugoslavia*, January 2002.

[2] G. Prunier, *Darfur: The ambiguous genocide*. Cornell University Press, Ithaca, NY, 2005.

[3] M. D. Toft, *The Geography of Ethnic Violence: Identity, Interests, and the Indivisibility of Terrirtory*. Princeton University Press, Princeton, NJ, 2006.

[4] M. Lim, R. Metzler, and Y. Bar-Yam, "Global Pattern Formation and Ethnic/Cultural Violence," *Science*, vol. 317, no. 5844, pp. 1540–1544, 2007.

[5] J. Epstein, "Modeling civil violence: An agent-based computational approach," *Proceedings of the National Academy of Sciences*, vol. 99, no. Suppl. 3, pp. 7243–7250, May 2002.

[6] C. K. Goh, H. Y. Quek, K. C. Tan, and H. A. Abbass, "Modeling civil violence: An evolutionary multi-agent, game theoretic approach," in *IEEE Congress on Evolutionary Computation, 2006.*, pp. 1624–1631.

[7] A. Srbljinovic, D. Penzar, P. Rodik, and K. Kardov, "An agent based model of ethnic mobilisation," *Journal of Artificial Societies and Social Simulation*, vol. 6, no. 1, 2003.

[8] L.-A. Barabasi and R. Albert, "Emergence of Scaling in Random Networks," *Science*, vol. 286, no. 5439, pp. 509–512, 1999.

[9] M. Lumsden, "The Cyprus conflict as a prisoner's dilemma game," *Journal of Conflict Resolution*, vol. 17, no. 1, pp. 7–32, 1973.

[10] R. T. Gurr, *Peoples Versus States: Minorities at Risk in the New Century*. United States Institute of Peace Press, Washington, D.C., 2000.

[11] D. L. Horowitz, *Ethnic Groups in Conflict*. University of California Press, Berkeley, CA, 2005.

[12] J. Ackelson, "Mapping ethnic violence," *The International Studies Review*, vol. 8, no. 3, pp. 492–494, 2006.

[13] R. Bhavnani, D. Miodownik, and J. Nart, "REsCape: an agent-based framework for modeling resources, ethnicity, and conflict," *Journal of Artificial Societies and Social Simulation*, vol. 11, no. 2, 2008.

[14] M. Nowak and K. Sigmund, "Game-dynamical aspects of the prisoner's dilemma," *Appl. Math. Comput.*, vol. 30, no. 3, pp. 191–213, 1989.

[15] T. C. Schelling, "Dynamic models of segregation," *Journal of Mathematical Sociology*, vol. 1, pp. 143–186, 1971.

[16] R. Axelrod, *The evolution of cooperation*. Basic Books, New York, 1984.

[17] G. Szabo and G. Fath, "Evolutionary games on graphs," *Physics Reports*, vol. 446, no. 4-6, pp. 97 – 216, 2007.

[18] H. Ebel and S. Bornholdt, "Coevolutionary games on networks," *Phys. Rev. E*, vol. 66, no. 5, Nov 2002.

[19] F. C. Santos, J. M. Pacheco, and T. Lenaerts, "Evolutionary dynamics of social dilemmas in structured heterogeneous populations," *Proceedings of the National Academy of Sciences*, vol. 103, no. 9, pp. 3490 – 3494, Feb 2006.

[20] M. G. Zimmermann and V. M. Eguíluz, "Cooperation, social networks, and the emergence of leadership in a prisoner's dilemma with adaptive local interactions," *Phys. Rev. E*, vol. 72, no. 5, Nov 2005.

[21] L. Luo, N. Chakraborty, and K. Sycara, "Prisoner's dilemma on graphs with heterogeneous agents," in *Late breaking papers at GECCO '09*, July 2009.