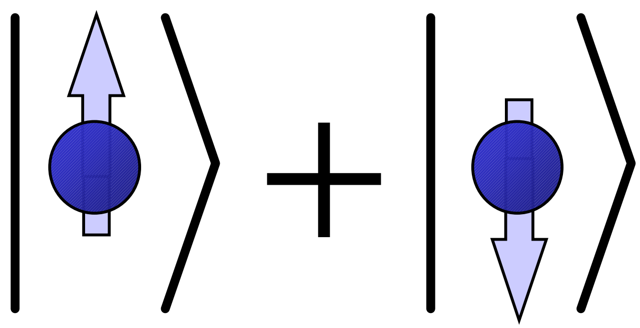
Memory Hierarchies for Quantum Computation



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Overview

- Introduction to Quantum Computing
- Error Correction
- Memory Hierarchy
- Future Work

Quantum Bits (qubit)

- 1 qubit probabilistically represents 2 states

$$|a\rangle = C_0|0\rangle + C_1|1\rangle$$

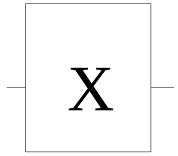
- Additional qubits double the number of states:

$$|ab\rangle = C_{00}|00\rangle + C_{01}|01\rangle + C_{10}|10\rangle + C_{11}|11\rangle$$

- *Quantum parallelism* on an exponential number of states
- Measurement collapses qubit waveform to a single classical value

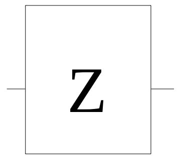
Quantum Gates

X Gate
Bit-flip, Not



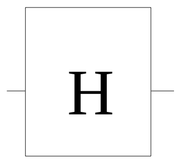
$$\equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \beta|0\rangle + \alpha|1\rangle$$

Z Gate
Phase-flip



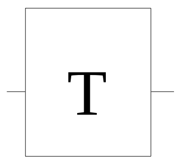
$$\equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle - \beta|1\rangle$$

H Gate
Hadamard



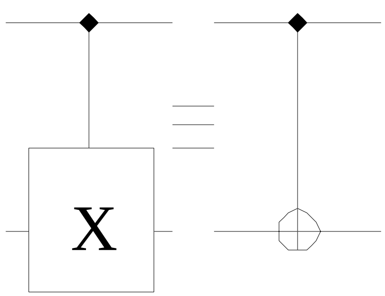
$$\equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{\alpha+\beta|0\rangle + \alpha-\beta|1\rangle}{\sqrt{2}}$$

T Gate
Rotate $\pi/8$



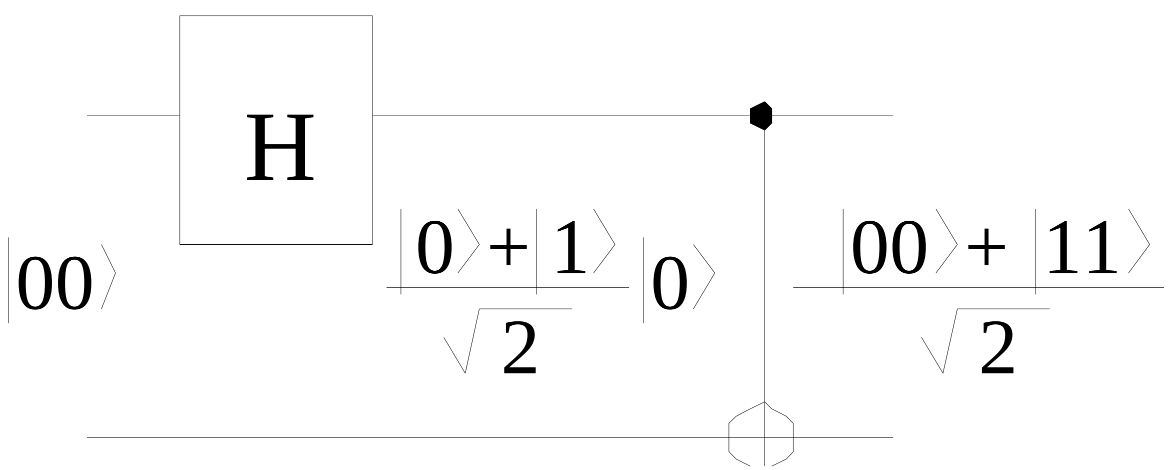
$$\equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle + e^{i\pi/4}\beta|1\rangle$$

Controlled Not
Controlled X
CNot



$$\equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle$$

CAT State Creation



Quantum Algorithms

- Unordered Search: $O(n^{1/2})$ vs. $O(n)$ [Grover96]
- Large Number Factorization [Shor94]
  - $O(n^3)$ vs. $O(2^{n/2})$ for known classical alg's
  - Quantum Fourier Transform
  - Periodicity of Modular Exponentiation
- Quantum Encryption
- Quantum Teleportation

Science Fiction?

- 5 and 7-bit machines have been built

  [Vandersypen00, Laflamme99]

  – NMR, ion-trap and other technologies
  – Shor’s and Grover’s algorithms demonstrated
- Larger machines are proposed
- Solid-state technologies are coming

  [Kane98,Vrijen99,Nakamura99,Mooij99]

General Purpose Machines

will require:

- thousands or millions of qubits
- better technology
- practical error rates are 10^{-6} to 10^{-9}
- billions or trillions of operations

  (e.g., factoring a 1024-bit number: $5x10^{11}$ ops)
- hence, error correction

Quantum Error Correction

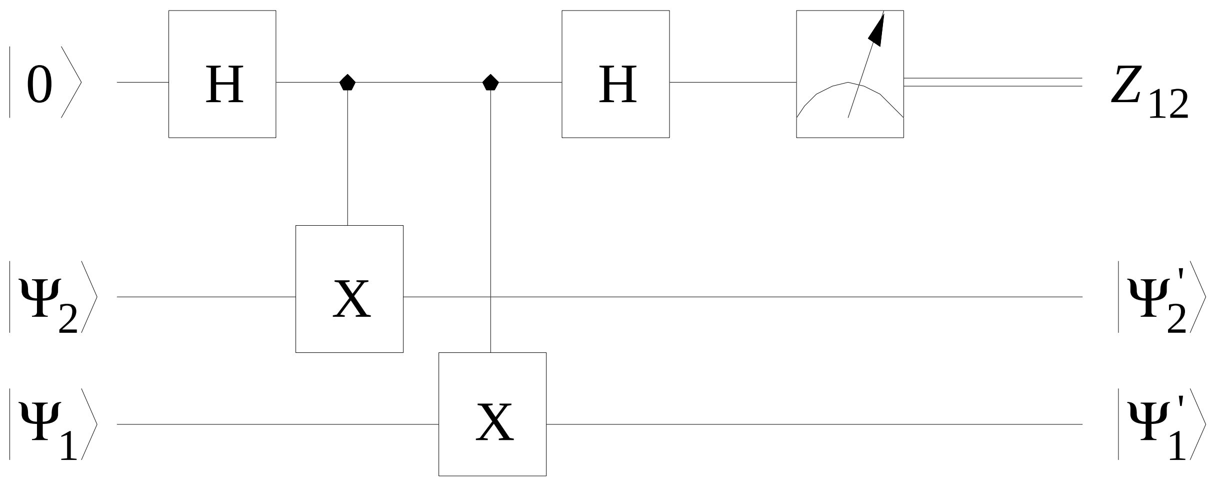
- Based on classical “linear” codes
- Some codes have efficient operations
  - Steane’s [[7,1]] code
- Operate on encoded data, error correct after each operation
- Arbitrary level of encoding for reliability
- Need to be reliable enough to run program to completion

Quantum Error Correction

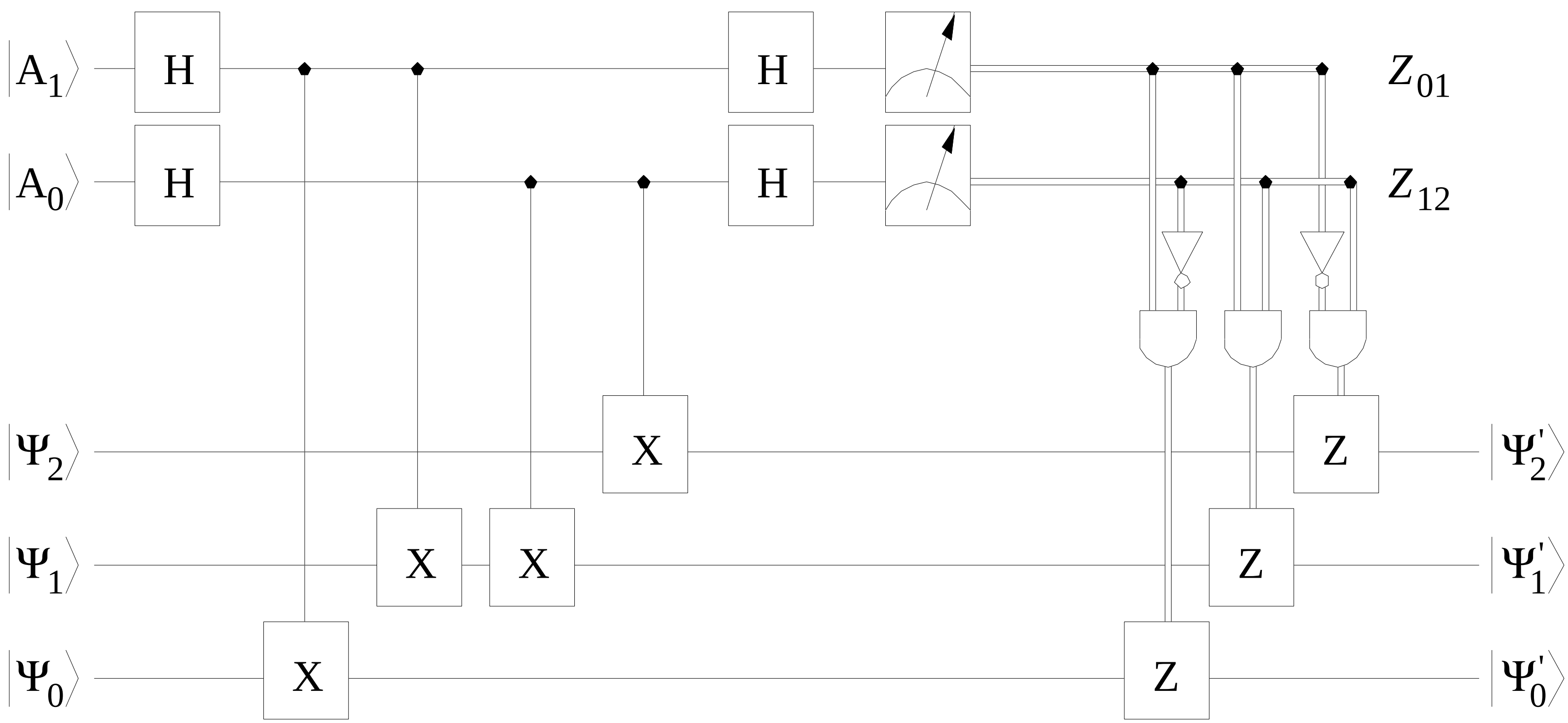
Three Qubit Code

$\mathbf{Z_{01}}$	$\mathbf{Z_{12}}$	**Error Type**	**Action**
+1	+1	no error	no action
-1	+1	bit 0 flipped	flip bit 0
-1	-1	bit 1 flipped	flip bit 1
+1	-1	bit 2 flipped	flip bit 2

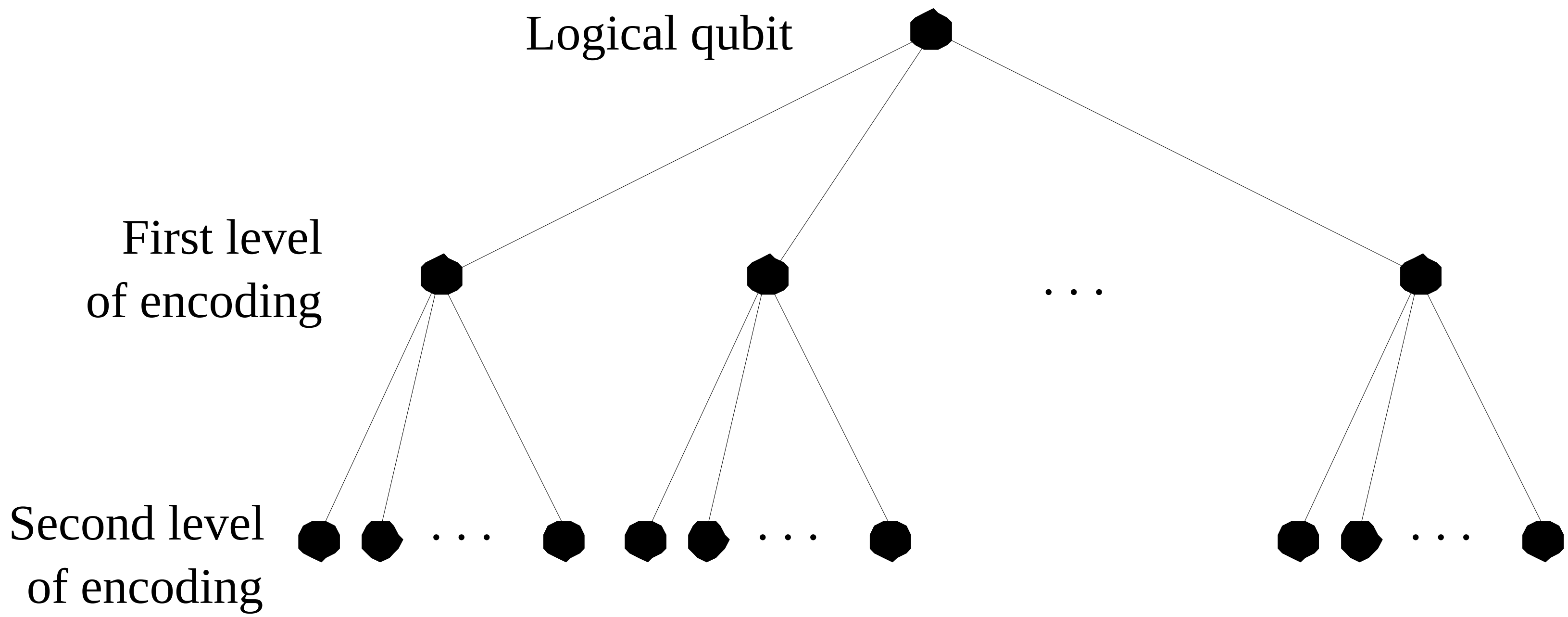
Error-Syndrome Measurement



3-bit Error Correction



Concatenated Codes



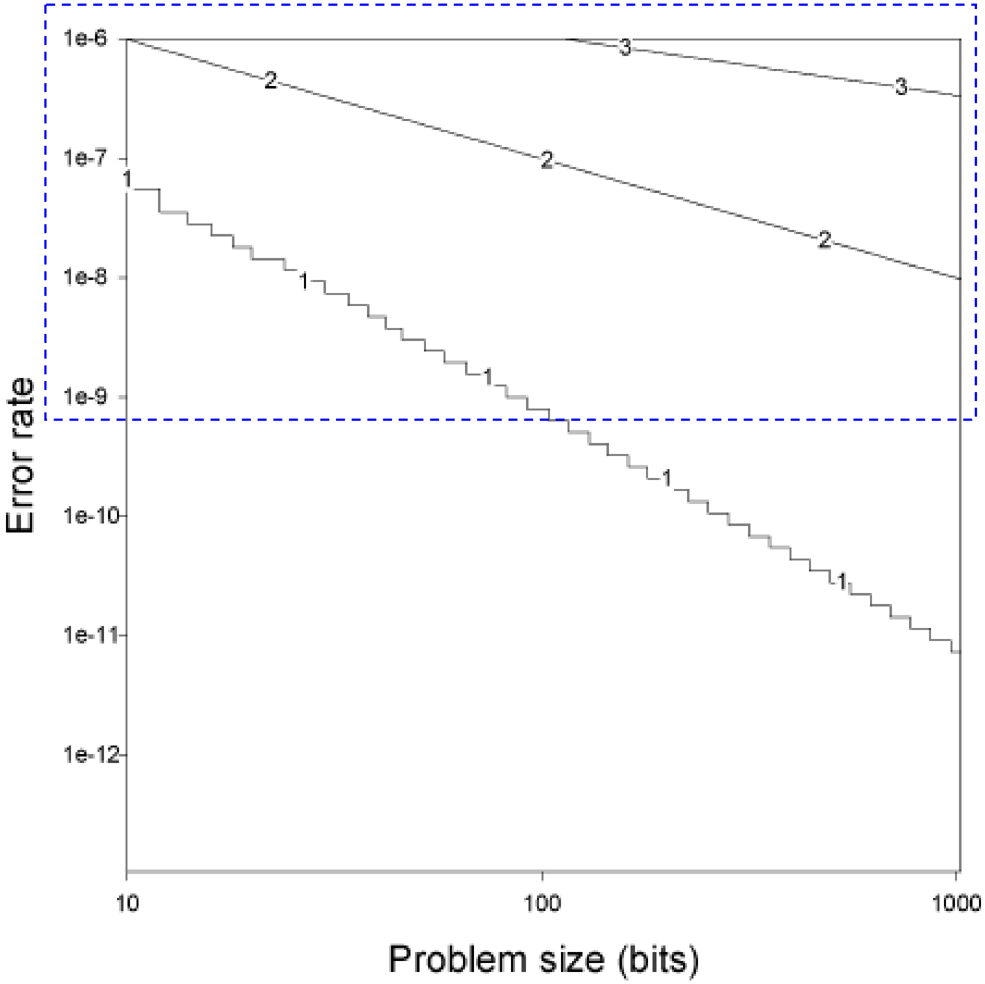
Error Correction Overhead

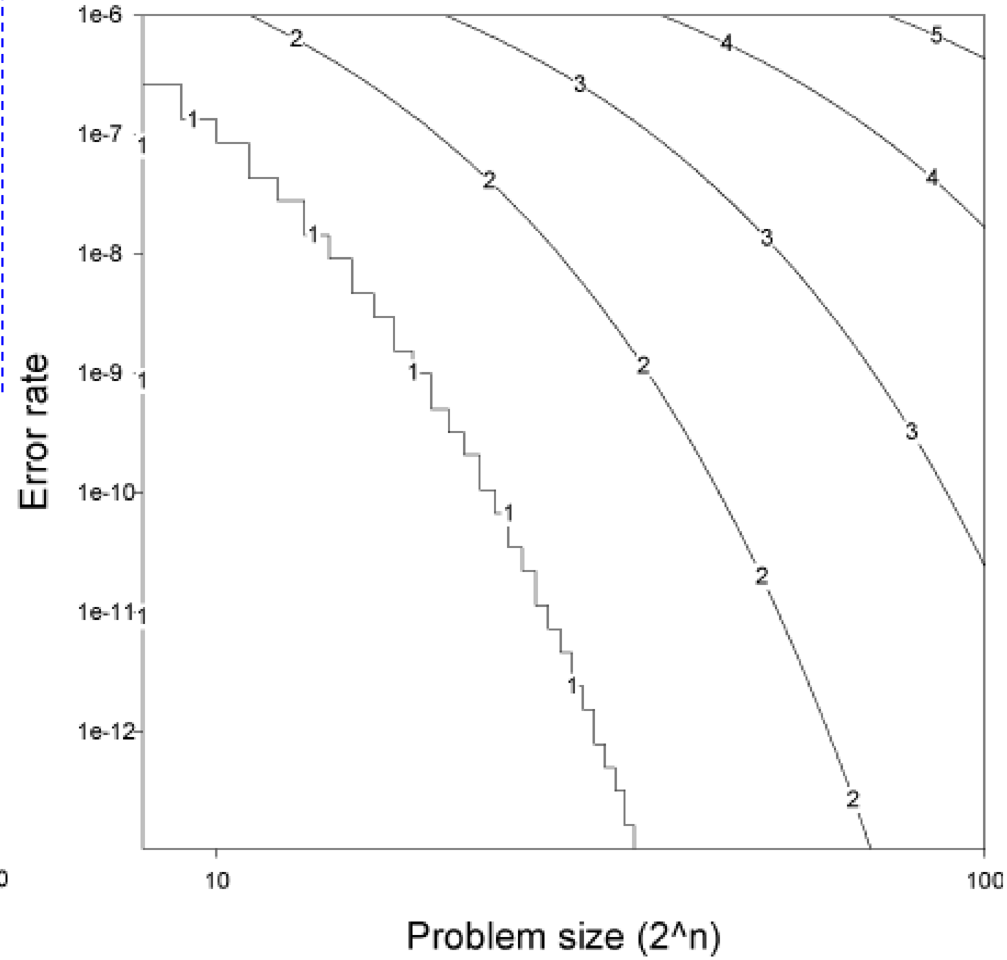
7-qubit code [Steane96], applied recursively

Recursion (k)	**Storage (7^k)**	**Operations (153^k)**	**Min. time (5^k)**
0	1	1	1
1	7	153	5
2	49	23,409	25
3	343	3,581,577	125
4	2,401	547,981,281	625
5	16,807	83,841,135,993	3125

Recursion Requirements

from: [Oskin, et al, 2002]

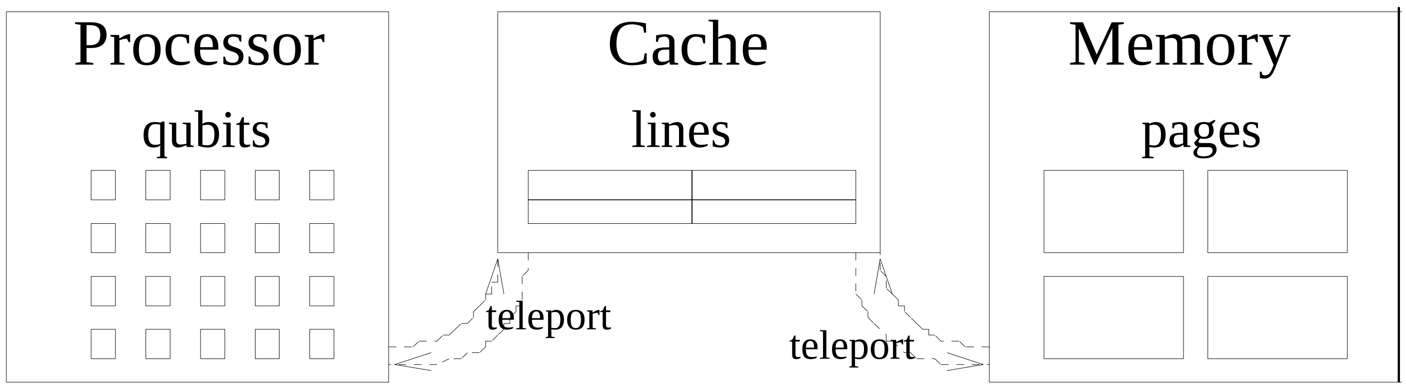


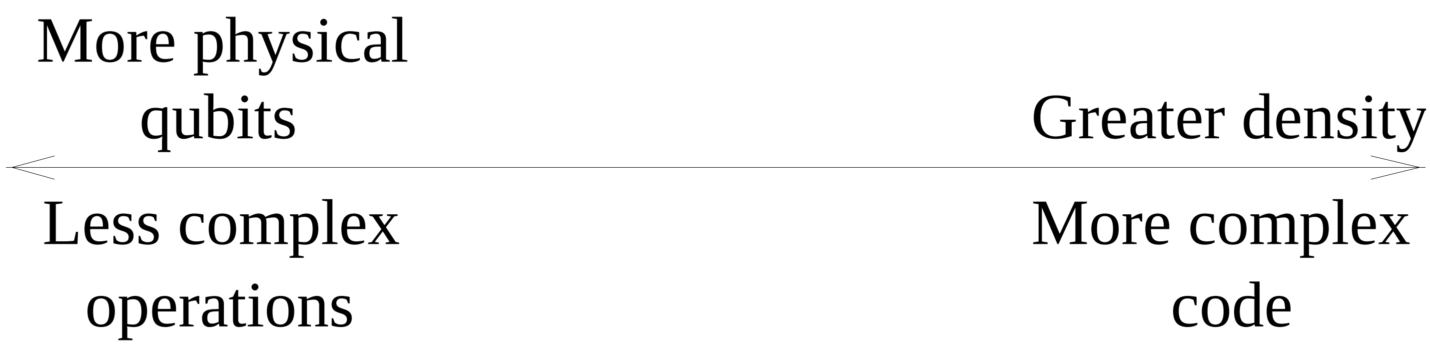


Shor's

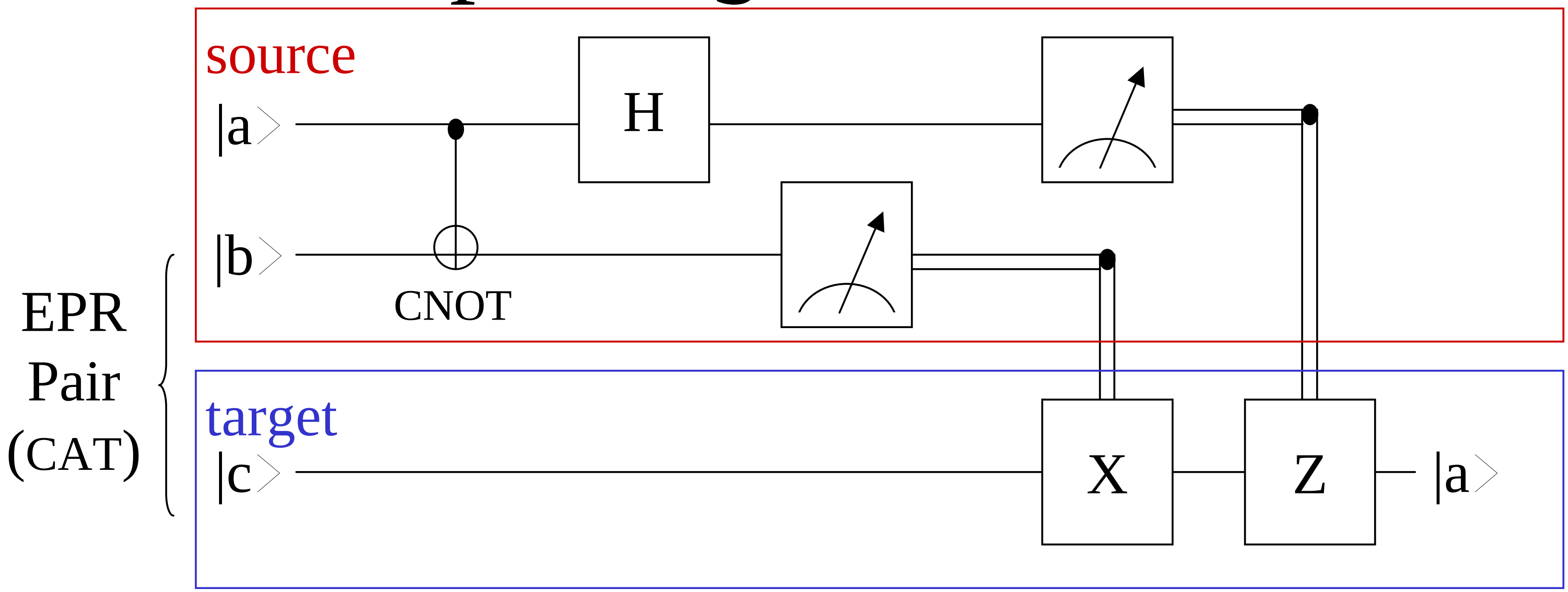
Grover's

Memory Hierarchy





Teleporting Between Codes



- Source generates |bc⟩ EPR pair
- Pre-communicate |c⟩ to target with retry
- Classical communication to set value
- Can be used to convert between codes

Denser Error Correction Codes

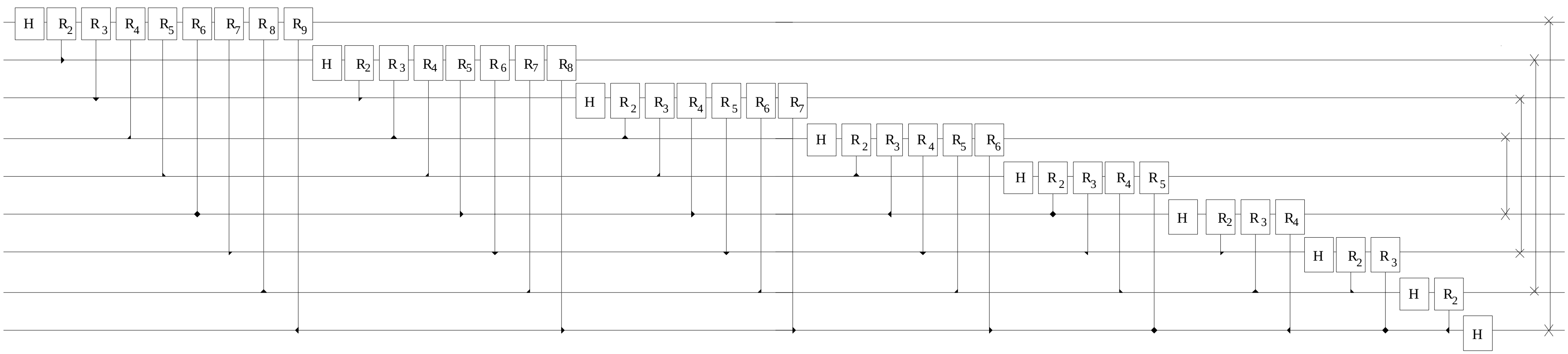
- [[7,1]] code [Steane96] – Efficient operations
- [[5,1]] code [Laflamme, et al, 96]
- [[8,3]] code [Steane96, Gottesman 96]

Overhead per qubit

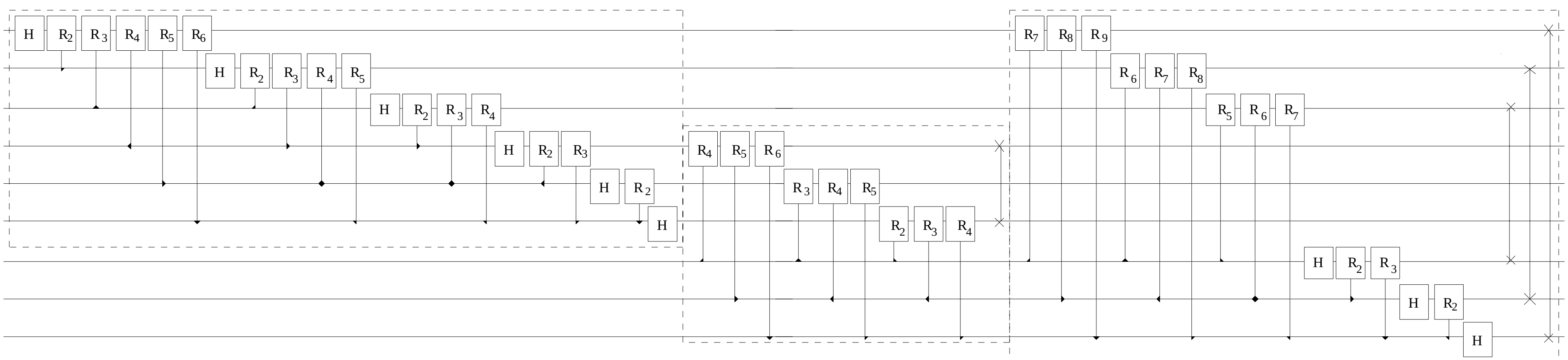
Code	**Storage**	**Operations**
[[7x7x7,1]]	343	3.5×10^6
[[5x7x7,1]]	245	3.1×10^6
[[8x7x7,3]]	130.67	2.2×10^6
[[8x8x8,3x3x3]]	< 19	0.8×10^6

Quantum Fourier Transform

Generic 9-qubit QFT



Blocked for 3-qubit accesses



Conclusion

- Classical memory hierarchies reduce cost by accessing frequently used data fast
- Quantum memory hierarchies reduce cost by decreasing the overhead of error correction
- Our proposal, in a nutshell
  - Calculate on sparse encoding
  - Use denser codes for cache and RAM
  - Teleport between codes
- Allows for solution of larger problems

Future Work

- Investigate different physical media
  - Different technologies, different primitives
  - Decoherence-free subspaces
  - High-genus surfaces
- Investigate larger “linear” codes
- Investigate non-linear codes
  - Toric codes
  - Iteratively decoded codes