18-452/18-750
Wireless Networks and Applications
Lecture 5: Physical Layer
Modulation and Diversity

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http://www.cs.cmu.edu/~prs/wireless24/

Outline

- RF introduction
- Modulation and multiplexing
- Channel capacity
- Antennas and signal propagation
- Modulation
- Coding and diversity
- OFDM and MIMO

(Limited) Goals

- Non-goal: turn you into electrical engineers
  - » Of course, some of you already are electrical engineers
- Basic understanding of how modulation can be done
- Understand the tradeoffs involved in increasing the bit rate

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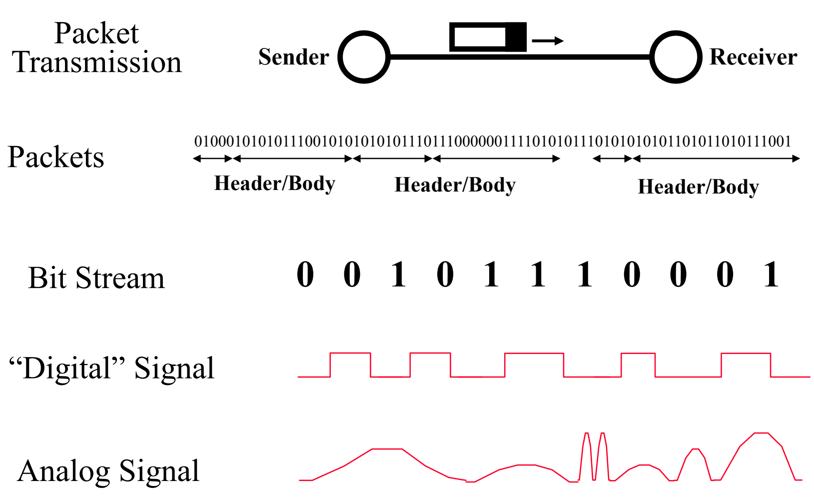
From Signals to Packets



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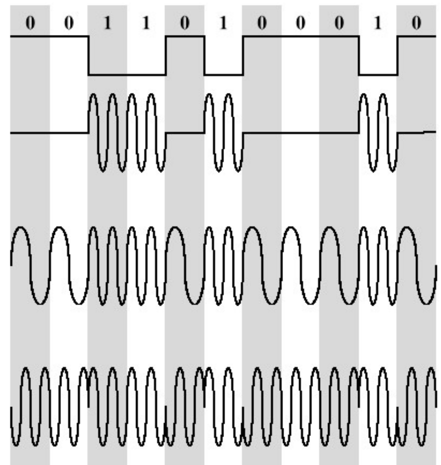
Basic Modulation Techniques

- Encode digital data in an analog signal
- Amplitude-shift keying (ASK)
  - » Amplitude difference of carrier frequency
- Frequency-shift keying (FSK)
  - » Frequency difference near carrier frequency
- Phase-shift keying (PSK)
  - » Phase of carrier signal shifted



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Amplitude-Shift Keying

- One binary digit represented by presence of carrier, at constant amplitude
- Other binary digit represented by absence of carrier

$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ 0 & \text{binary } 0 \end{cases}$$

  – where the carrier signal is $A\cos(2\pi f_c t)$
- Inefficient because of sudden gain changes
  - » Only used when bandwidth is not a concern, e.g. on voice lines (< 1200 bps) or on digital fiber
- A can be a multi-bit symbol

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How Can We Go Faster?

- Increase the rate at which we modulate the signal, or …
  - » I.e., a higher frequency base signal
  - » Signal time becomes short
- Modulate the signal with "symbols" that send multiple bits
  - » I.e., each symbol represents more information
  - » Longer signal time but more sensitive to distortion
- Which solution is the best depends on the many factors
  - » We will not worry about that in this course

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Binary Frequency-Shift Keying (BFSK)

- Two binary digits represented by two different frequencies near the carrier frequency

$$s(t) = \begin{cases} A\cos(2\pi f_1 t) & \text{binary } 1 \\ A\cos(2\pi f_2 t) & \text{binary } 0 \end{cases}$$

  – where f_1 and f_2 are offset from carrier frequency f_c by equal but opposite amounts
- Less susceptible to error than ASK
- Sometimes used for radio or on coax
- Demodulator looks for power around f_1 and f_2

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Multiple Frequency-Shift Keying (MFSK)

- More than two frequencies are used
- Each symbol represents L bits

$$s_i(t) = A\cos 2\pi f_i t \qquad 1 \le i \le M$$

  - $f_i = f_c + (2i - 1 - M)f_d$
  - L = number of bits per signal element
  - M = number of different signal elements = 2^L
  - f_c = the carrier frequency
  - f_d = the difference frequency
- More bandwidth efficient but more susceptible to error
    - » Symbol length is $T_s = LT$ seconds, where T is bit period

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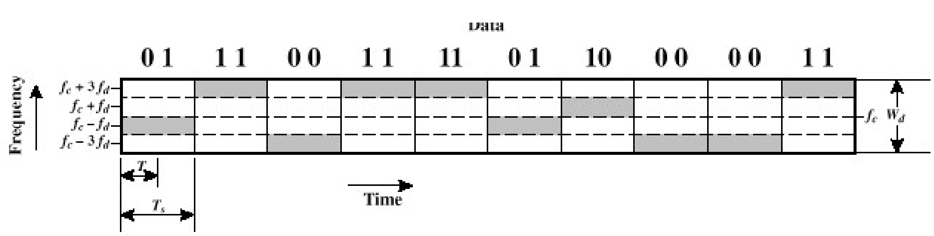
Multiple Frequency-Shift Keying (MFSK)



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Phase-Shift Keying (PSK)

- **Two-level PSK (BPSK)**
  - » **Uses two phases to represent binary digits**

$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ A\cos(2\pi f_c t + \pi) & \text{binary } 0 \end{cases}$$

$$= \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ -A\cos(2\pi f_c t) & \text{binary } 0 \end{cases}$$

- **Differential PSK (DPSK)**
  - » **Phase shift with reference to previous bit**
    - **Binary 0 – signal of same phase as previous signal burst**
    - **Binary 1 – signal of opposite phase to previous signal burst**

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Phase-Shift Keying Four Level PSK

- **Each element represents 2 (or more) bits**

$$s(t) = \begin{cases} A\cos\left(2\pi f_c t + \frac{\pi}{4}\right) & 11 \\ A\cos\left(2\pi f_c t + \frac{3\pi}{4}\right) & 01 \\ A\cos\left(2\pi f_c t - \frac{3\pi}{4}\right) & 00 \\ A\cos\left(2\pi f_c t - \frac{\pi}{4}\right) & 10 \end{cases}$$

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Outline

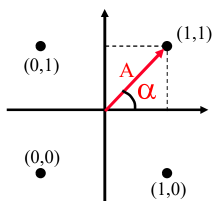
- RF introduction
- Modulation and multiplexing
- Channel capacity
- Antennas and signal propagation
- **Modulation**
  - » Basic techniques
  - » QAM
- Coding and diversity
- OFDM and MIMO

Quadrature Amplitude Modulation - QAM

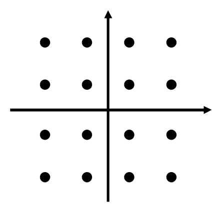
- Modulation is based on both phase and amplitude
- QAM has many benefits
  - » Since two signal properties are used, QAM can scale up to very large symbol sizes
  - » It has a simple mathematical representation (next slide)
  - » Bonus: it has a very intuitive representation (figures!)
- QAM is the dominating modulation technique for modern, high performance wireless technologies
  - » 4G, 5G, all recent WiFi standards

Quadrature-Amplitude Modulation

- Modulation is based on changing the phase and amplitude of the signal
  - » Phase – in “degrees”
  - » Amplitude – signal strength
- The values of the signals can be represented as points in a 2D plane



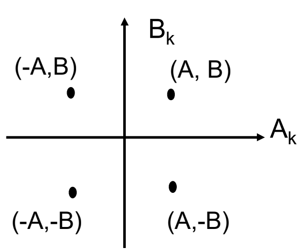
2 bit symbol (phase only)



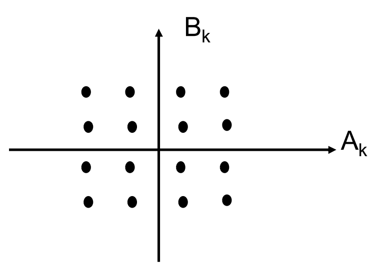
4 bit symbol

Signal Constellations

- Each pair (A_k, B_k) defines a point in the plane
- ***Signal constellation*** **is the set of signal points**



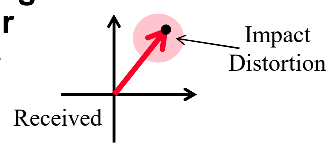
4 possible points per *T* sec.
2 bits QAM (phase only)

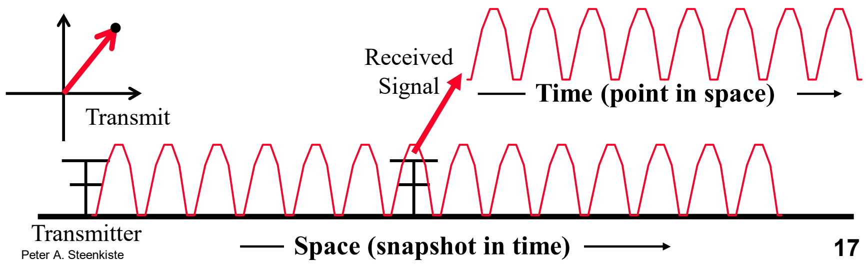


16 possible points per *T* sec.
4 bits / symbol

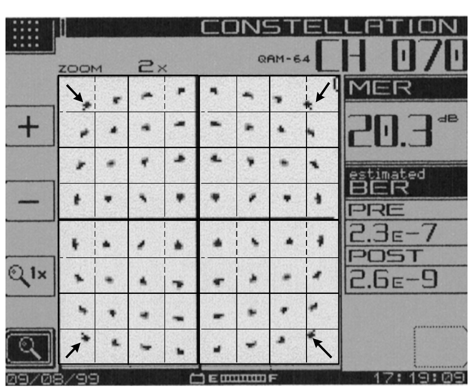
Impact of the Channel on the Signal

- The channel can change both the amplitude and phase of the signal
- This that the "location" of the signal in the 2D plane is different at the receiver compared with the sender side
- As a result, the receiver can misinterpret the signal





How Does Distortion Impact a Constellation Diagram?

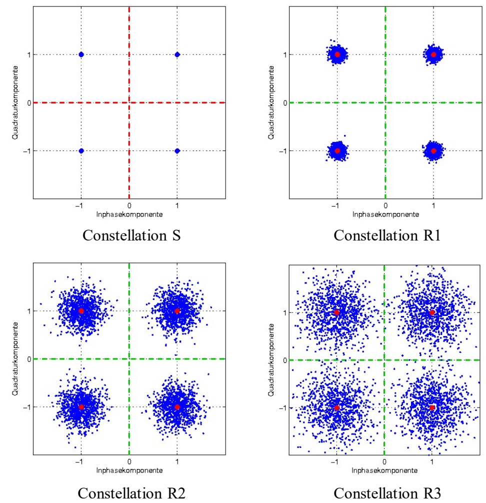


- Changes in amplitude, phase or frequency move the points in the constellation
- Large shifts can create uncertainty on what symbol was transmitted
- Larger symbols are more susceptible
- Can adapt symbol size to channel conditions to optimize throughput

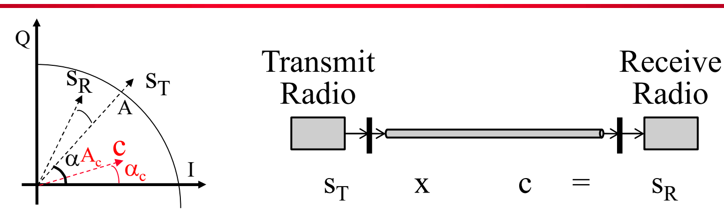
www.cascaderange.org/presentations/Distortion_in_the_Digital_World-F2.pdf

Impact of Channel on Received Signal

Sender: https://commons.wikimedia.org/wiki/File:4qam_constellation_new.png
Receiver 1: https://commons.wikimedia.org/wiki/File:4qam_constellation_noisy_sigma001.png
Receiver 2: https://commons.wikimedia.org/wiki/File:4qam_constellation_noisy_sigma01.png
Receiver 3: https://commons.wikimedia.org/wiki/File:4qam_constellation_noisy_sigma025.png



Modeling the Channel



- The channel can also be modeled as a vector S_c
  - » α_c the phase change inflicted on the signal (additive)
  - » A_c the change in amplitude (multiplicative)
- This is called the Channel State Information (CSI)
  - » These two parameters determine how hard or easy it will be to correctly demodulate the signal at the receiver
  - » It can also be represented as a point in the space
- How can we use this model to optimize performance?

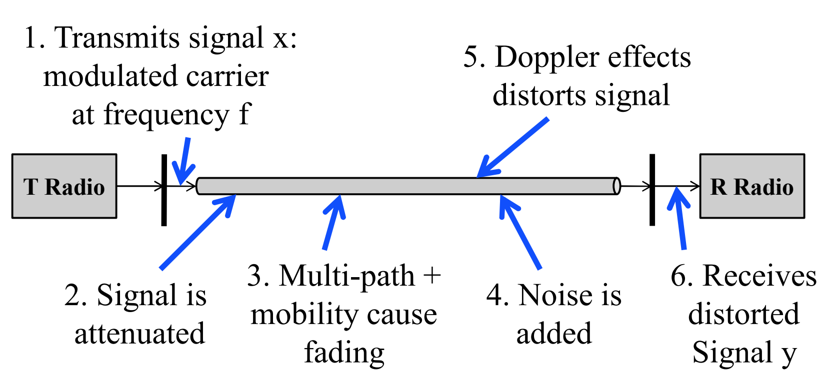
QAM Signals and Channel State

- **The symbol values and channel state can also be modeled as complex number**
  - » **Project the vector on the I and Q axis**
- **This is a big deal: we can represent wireless communications using a simple equation**

$$S_R = S_T \times C$$

  - » **If the sender knows C, it can estimate S_R**
  - » **How about if the receiver knows C?**
- **C changes over time due to mobility: C(t)**
  - » **This change is continuous but it can be captured as a sequence of samples c_i**
- **C depends on carrier frequency: c(f)**
  - » **Frequency selective fading or attenuation**
  - » **The dependency on f is a concern for wide-band signals**

Channel Model



$$S_T \quad x \quad C \quad + \quad N \quad = \quad S_R$$

Tradeoff: Bit Rate versus Error Rate - Informal

- Amplitude and phase modulation places transmitted symbols into 2D space
  - » Represented by a complex number
- Channel distortion "moves" the symbol
  - » Large shift can map it onto another symbol
- Large symbols means denser packing of symbols in the plane
  - » Results in high bit rate but distortions are more likely to result in errors
- Smaller symbols are more conservative
  - » Lower bit rate but more resistant to errors

Good channels

Bad channels

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Adapting to Channel Conditions

- Channel conditions can be very diverse
  - » Affected by the physical environment of the channel
  - » Changes over time as a result of slow and fast fading
- Fixed coding/modulation scheme will often be inefficient
  - » Too conservative for good channels, i.e. lost opportunity
  - » Too aggressive for bad channels, i.e. lots of packet loss
- Adjust coding/modulation based on channel conditions – "rate" adaptation
  - » Controlled by the MAC protocol
  - » E.g. 802.11a: BPSK – QPSK – 16-QAM – 64 QAM

Bad ⟷ Good Channel

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Example: 802.11ax

Modulation and coding schemes

MCS index[i]	Modulation type	Coding rate	Data rate (Mbit/s)[i]							
			20 MHz channels		40 MHz channels		80 MHz channels		160 MHz channels	
			1600 ns GI[ii]	800 ns GI	1600 ns GI	800 ns GI	1600 ns GI	800 ns GI	1600 ns GI	800 ns GI
0	BPSK	1/2	8	8.6	16	17.2	34	36.0	68	72
1	QPSK	1/2	16	17.2	33	34.4	68	72.1	136	144
2	QPSK	3/4	24	25.8	49	51.6	102	108.1	204	216
3	16-QAM	1/2	33	34.4	65	68.8	136	144.1	272	282
4	16-QAM	3/4	49	51.6	98	103.2	204	216.2	408	432
5	64-QAM	2/3	65	68.8	130	137.6	272	288.2	544	576
6	64-QAM	3/4	73	77.4	146	154.9	306	324.4	613	649
7	64-QAM	5/6	81	86.0	163	172.1	340	360.3	681	721
8	256-QAM	3/4	98	103.2	195	206.5	408	432.4	817	865
9	256-QAM	5/6	108	114.7	217	229.4	453	480.4	907	961
10	1024-QAM	3/4	122	129.0	244	258.1	510	540.4	1021	1081
11	1024-QAM	5/6	135	143.4	271	286.8	567	600.5	1134	1201

Source https://en.wikipedia.org/wiki/Wi-Fi_6

Summary

- **Key properties of channels:**
  - » **Channel state that concisely captures many of the factors degrading the channel**
  - » **The power budget expresses the power at the receiver**
  - » **Channel reciprocity**
- **Modulation changes the signal based on the data to be transmitted**
  - » **Can change amplitude, phase or frequency**
  - » **The transmission rate can be increased by using symbols that represent multiple bits**
    - – **Can use hybrid modulation, e.g., phase and amplitude**
  - » **The symbol size can be adapted based on the channel conditions – results in a variable bit rate transmission**
  - » **Details do not matter!**

Outline

- RF introduction
- Modulation and multiplexing
- Channel capacity
- Antennas and signal propagation
- Modulation
- Diversity and coding
  - » Space, time and frequency diversity
- OFDM

Diversity Techniques

- Key idea: increase the odds of the receiver receiving the data by using diverse channels
  - » "Don't put all your eggs in one basket"
- Space diversity: use multiple nearby antennas and combine signals
  - » Both at the sender and the receiver
- Time diversity: spread data out over time
  - » Useful for burst errors, i.e., errors are clustered in time
- Frequency diversity: spread signal over multiple frequencies
  - » For example, spread spectrum
- Distribute data over multiple "channels"
  - » "Channels" experience different frequency selective fading, so only part of the data is affected

Space Diversity

- Use multiple antennas that transmit and/or receive the signal in slightly different locations
- If antennas are sufficiently separated, instantaneous channel conditions are independent
  - Antennas should be separated by ½ wavelength or more
- If one antenna experiences deep fading, the other antenna may have a strong signal
- This represents a wide class of techniques
  - Use on transmit or receive side, or both
  - Level of sophistication of the algorithms used
  - Can use more than two antennas!

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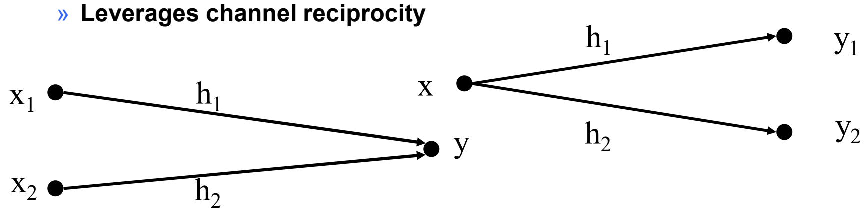
Selection Diversity

- Receiver diversity: receiver picks the antenna with the best SNR
  - Very easy
- Transmit diversity: sender picks the antenna that offers the best channel to the receiver
  - Transmitter can learn the channel conditions based on signals sent by the receiver
  - Leverages channel reciprocity



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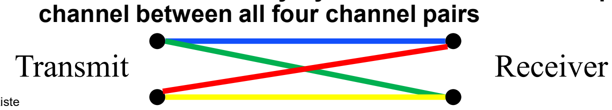
Simple Algorithm used in old 802.11 versions

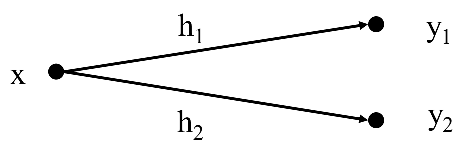
- Combine transmit + receive selection diversity
  - » Assume packets are acknowledged – why?
- How to explore all channels to find the best one … or at least the best transmit antenna
- Receiver:
  - » Uses the antenna with the strongest signal
  - » Always use the same antenna to send the acknowledgement – gives feedback to the sender
- Sender:
  - » Picks an antenna to transmit and learns about the channel quality based on the ACK
  - » Needs to occasionally try the other antenna to explore the channel between all four channel pairs



Receiver Diversity Can we Do Better?

- But why not use both signals?
  - » 2 Signals contain more information than 1
  - » What can go wrong?
- Simply adding the two signals has drawbacks:
  - » The carrier signals may be out of phase
    - – Similar to multi-path - can reduce the signal strength!
  - » We want to make sure we do not amplify the noise
- Maximal ratio combining: combine the signals with a weight that is based on their SNR
  - » Using a weight will favor the strongest signal (highest SNR)

Receiver Diversity Optimization



$$\vec{y} = \vec{h} * x + \vec{n}$$

$$\vec{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

- **Multiply $\vec{y}$ with the complex conjugate $\vec{h}^*$ of the channel vector $\vec{h}$**
  - » **This aligns the phases of the two signals so they amplify each other**
  - » **It also scales the signals according to their SNR so the effect of noise is not amplified**
- **Receiver can learn $\vec{h}$ based on training data**

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The Details

- **Complex conjugates: same real part but imaginary parts of opposite signs**

$$\vec{h}^* * \vec{y} = \vec{h}^* * (\vec{h} * x + \vec{n})$$

$$\text{Where } h^* = [h_1^* \;\; h_2^*] = [\, a_1{+}b_1 i \;\; a_2{-}b_2 i]$$

- **Result:**

  signal x is scaled by $a_1^2 + b_1^2 + a_2^2 + b_2^2$

  noise becomes: $h_1^* * n_1 + h_2^* * n_2$

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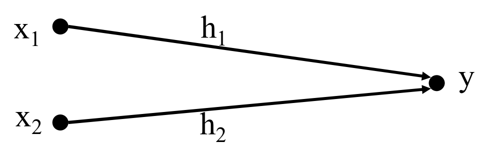
Transmit Diversity

- Same as receive diversity but the transmitter has multiple antennas
- Maximum ratio combining: sender "precodes" the signal
  - » Pre-align the phases at receiver and distribute power over the transmit antennas (the total power is fixed)
- How does transmitter learn channel state?
  - » Channel reciprocity: learn from packets received Y



$$y = \vec{h} * \vec{x} + n$$

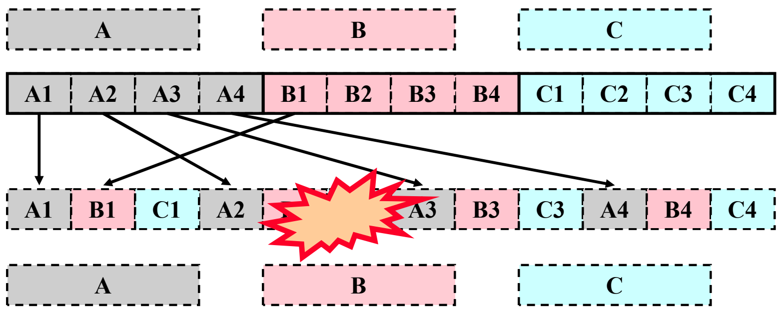
$$\vec{h}^{*} * x$$

Adding Redundancy

- Protects digital data by introducing redundancy in the transmitted data.
  - » Error detection codes: can identify certain types of errors
  - » Error correction codes: can fix certain types of errors
- Block codes provide Forward Error Correction (FEC) for blocks of data.
  - » (n, k) code: n bits are transmitted for k information bits
  - » Simplest example: parity codes
  - » Many different codes exist: Hamming, cyclic, Reed-Solomon, …
- Convolutional codes provide protection for a continuous stream of bits.
  - » Coding gain is n/k
  - » Turbo codes: convolutional code with channel estimation

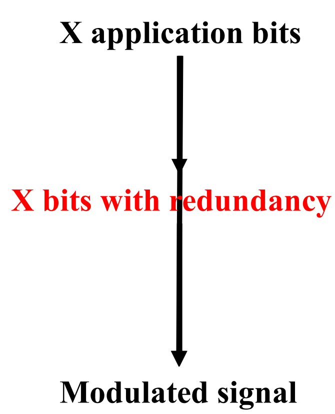
Combine Redundancy with Time Diversity

- Fading can cause burst errors: a relatively long sequence of bits is corrupted
- Spread blocks of bytes out over time so redundancy can help recover from the burst
  - » Example: only need 3 out of 4 to recover the data



Bits, Symbols, and Chips

- Redundancy and time diversity can be added easily at the application layer
  - » Wireless protocol can then directly modulate the application bits
- Better idea: add a coding layer inside the wireless protocol
  - » WiFi and Cellular
- This makes it possible to adapt much more quickly to the channel conditions
  - » Also provides access to much more accurate channel state information



Discussion

- Error coding increases robustness at the expense of having to send more bits
  - » Technically this means that you need more spectrum
- But: since you can tolerate some errors, you may be able to increase the bit rate through more aggressive modulation
- Coding and modulation combined offer a lot of flexibility to optimize transmission
- Next steps:
  - » Apply a similar idea to frequency diversity - spread spectrum
  - » Combine coding with frequency and time diversity - OFDM

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Summary

- Space diversity really helps in overcoming fading
  - » Very widely deployed
  - » Will build on this when we discuss MIMO
- Coding is also an effective way to improve throughput
  - » Widely used in all modern standards
  - » Coding, combined with modulation, can be adapt quickly to channel conditions

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