15-122: Principles of Imperative Computation

Recitation 26 Solutions   Josh Zimmerman

Exercises

1. Verify that the relation for union-find (u and v are related if and only if there's a path from u to v) is an equivalence relation by showing that it is reflexive, symmetric and transitive. (You can assume we're working with undirected graphs.)

Solution: Reflexive: There is a path from any vertex to itself: the zero-length path.

Symmetric: Since we're working with undirected graphs, if there is a path from u to v, there is a path from v to u — simply go in the opposite direction.

Transitive: If there's a path from u to v and one from v to s, there's one from u to s — just go from u to v and then from there to s.

2. What's the worst case cost of adding an edge, assuming we never update canonical representations of the vertices in the larger equivalence class and that we have n vertices total?

Solution: If we have $\frac{n}{2}$ vertices in each equivalence class, we'll have to update $\frac{n}{2}$ vertices no matter what. So, at worst we must do $O(n)$ updates to the array of canonical representations.