
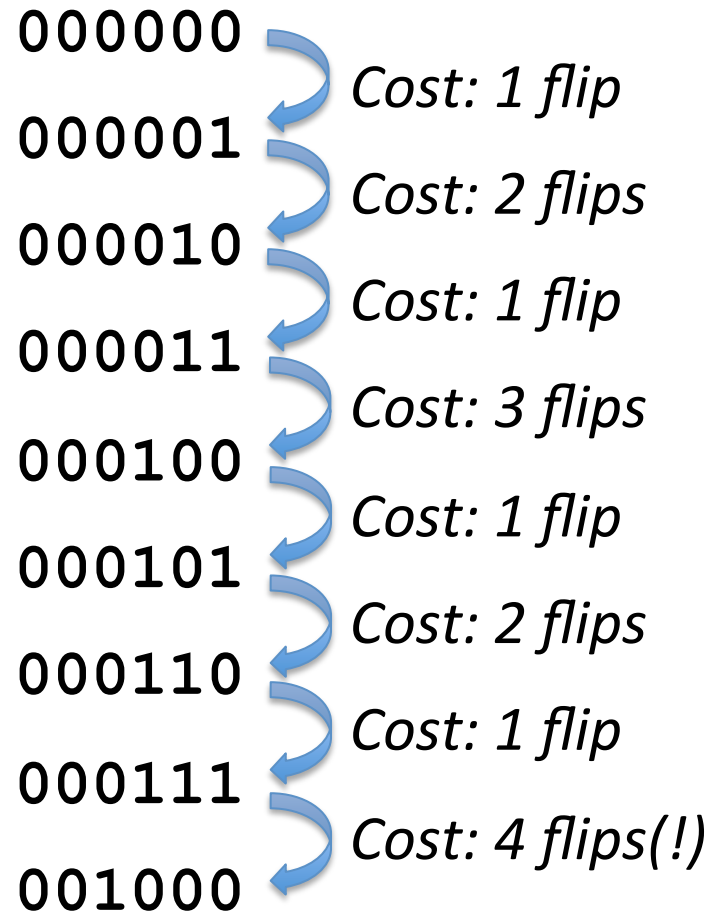


The k -bit counter

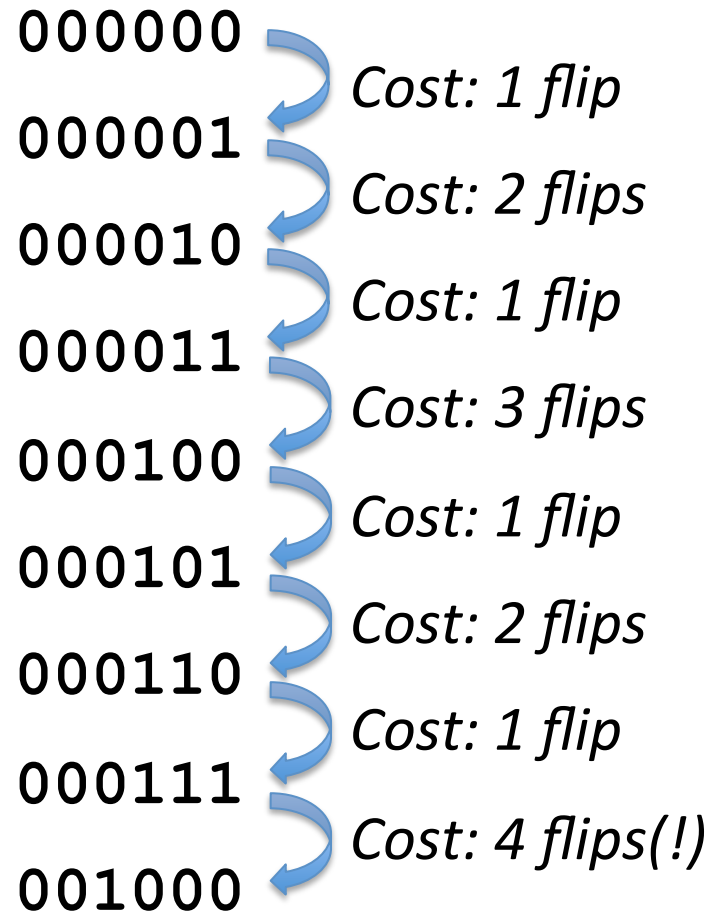
000000
000001
000010
000011
000100
000101
000110
000111
001000



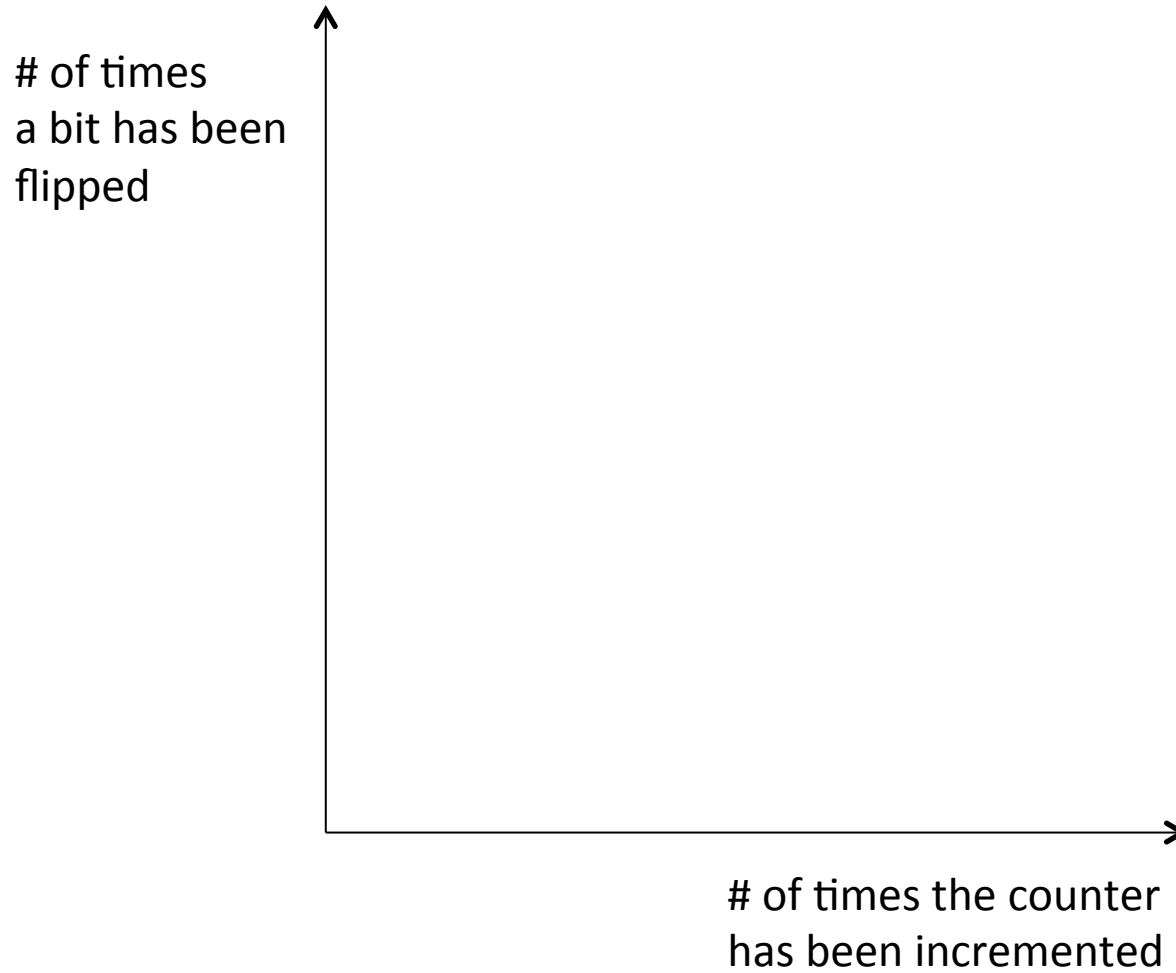
The k -bit counter



The k -bit counter



The k -bit counter



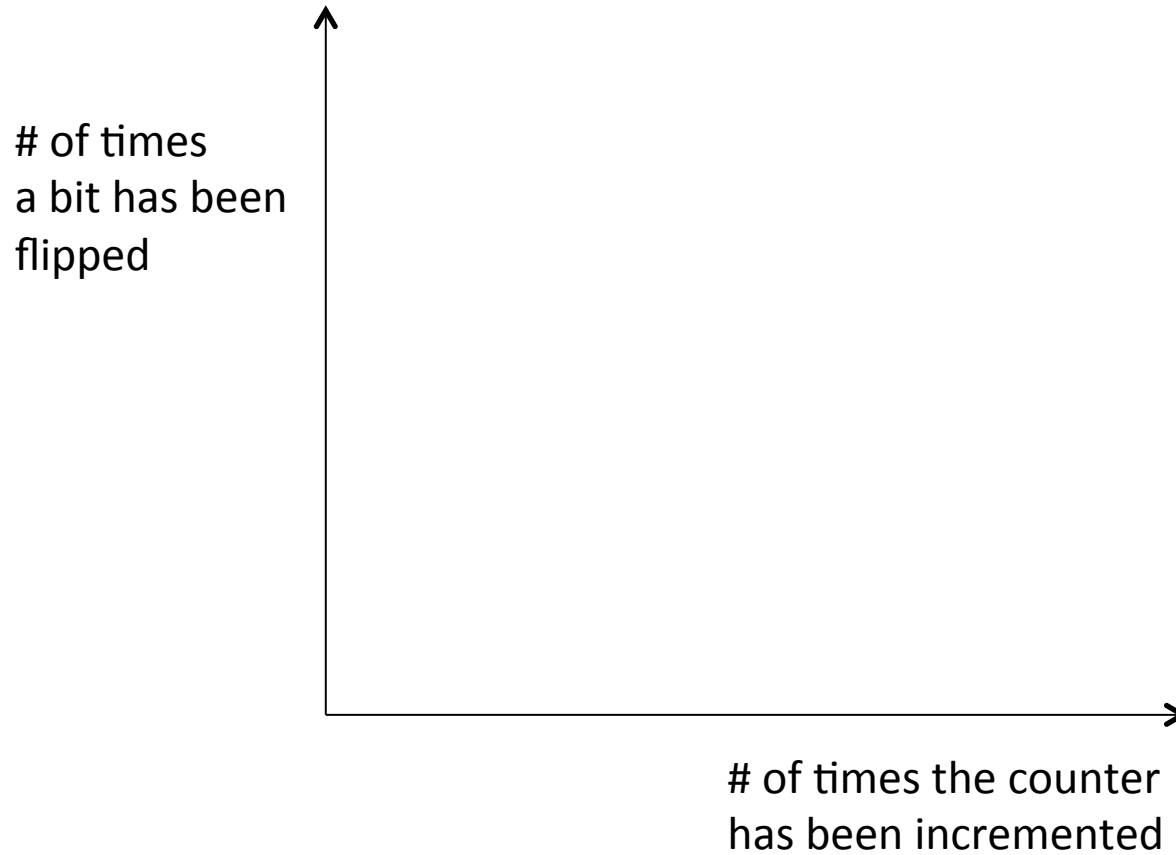
The k -bit counter

Worst case cost of a single INCREMENT operation:

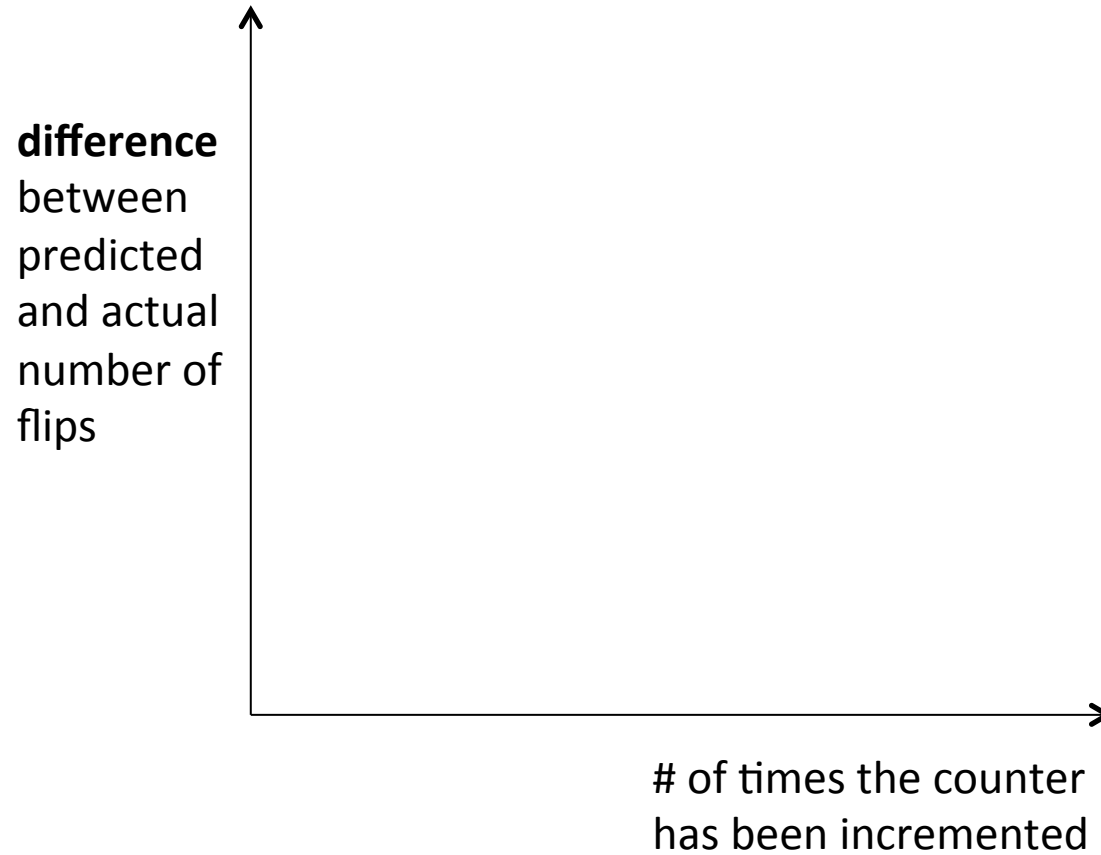
Total cost of n INCREMENT operations (starting from all 0s):

So a single INCREMENT operation...










Amortized analysis



Amortized analysis



Amortized analysis

| | | | | |
|----|----|--------|---|-------------------------|
| 0 | 0 | 000000 |  | <i>Cost: 1 flip</i> |
| 2 | 1 | 000001 |  | <i>Cost: 2 flips</i> |
| 4 | 3 | 000010 |  | <i>Cost: 1 flip</i> |
| 6 | 4 | 000011 |  | <i>Cost: 3 flips</i> |
| 8 | 7 | 000100 |  | <i>Cost: 1 flip</i> |
| 10 | 8 | 000101 |  | <i>Cost: 2 flips</i> |
| 12 | 10 | 000110 |  | <i>Cost: 1 flip</i> |
| 14 | 11 | 000111 |  | <i>Cost: 4 flips(!)</i> |
| 16 | 15 | 001000 |  | |

Predicted Actual

Amortized analysis

Invariant: we have ONE TOKEN for each “1” digit

Amortized analysis

Invariant: we have ONE TOKEN for each “1” digit

110110

100000

011001

Amortized analysis

Invariant: we have ONE TOKEN for each “1” digit

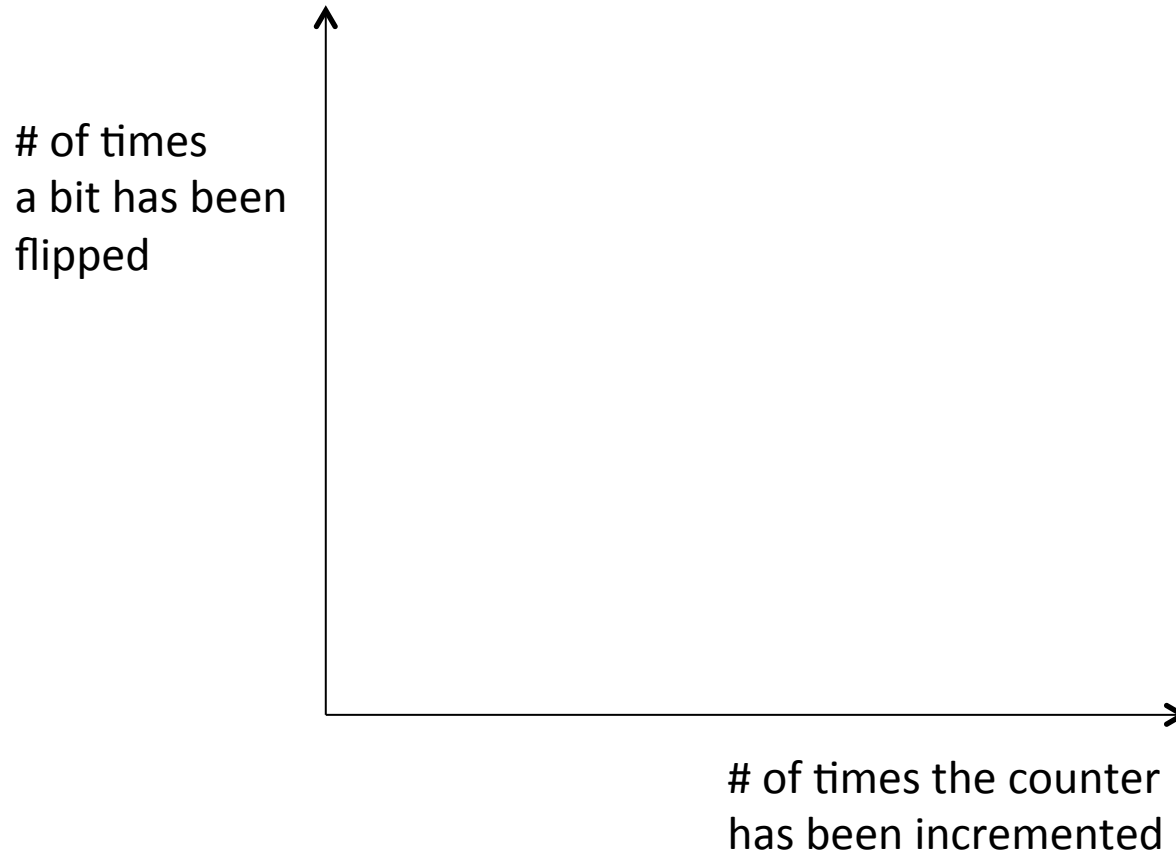
True initially?

Amortized analysis

Invariant: we have ONE TOKEN for each “1” digit

**If we get 2 new tokens for every INCREMENT,
Is the invariant preserved by every operation?**

Amortized analysis



Amortized analysis

Invariant:

True initially

**If we get 2 new tokens for every INCREMENT,
Is the invariant preserved by every operation?**

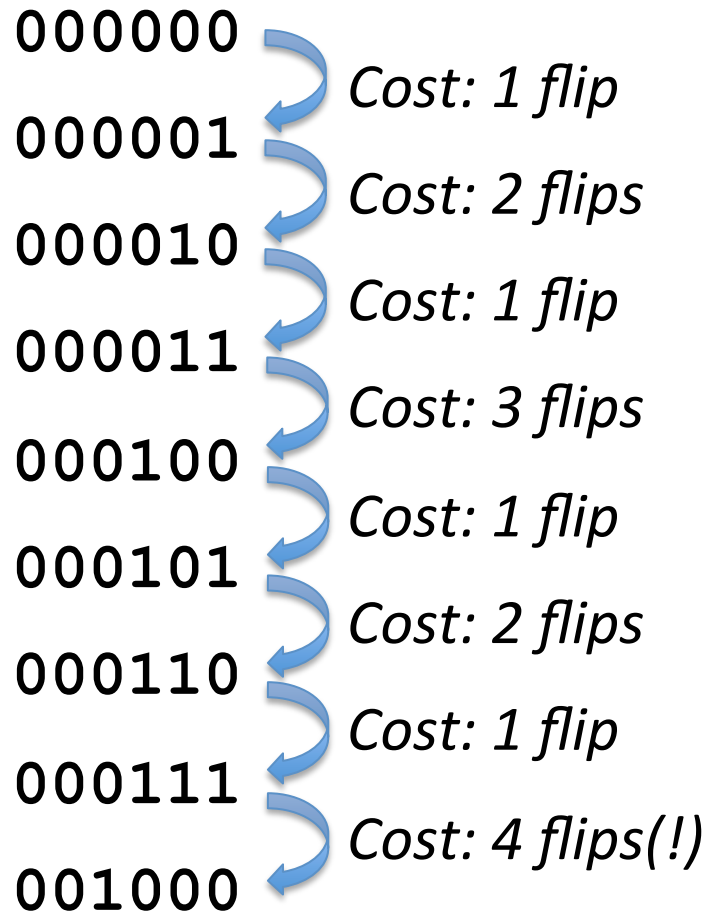
Amortized analysis

110110

100000

011001

The k -bit counter



Definitely $O(k)$
worst case...

..is cost $O(kn)$
for a sequence of
 n flips?

000000
000001
000010
000011
000100
000101
000110
000111
001000

Pay 1

Pay 2

Pay 1

Pay 3

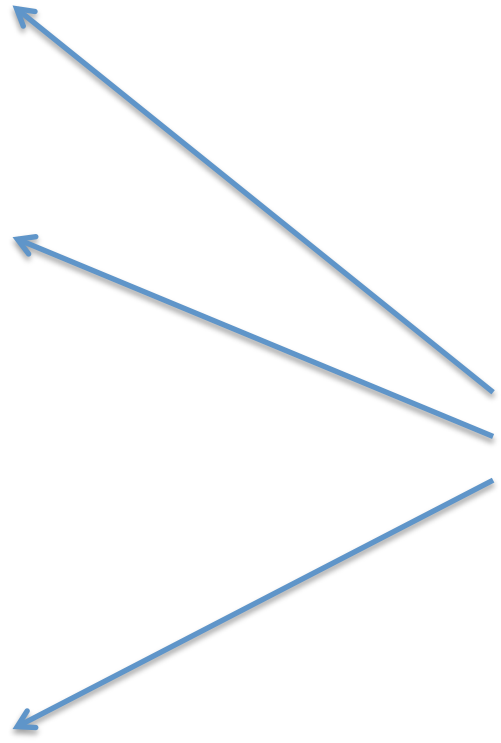
Pay 1

Pay 2

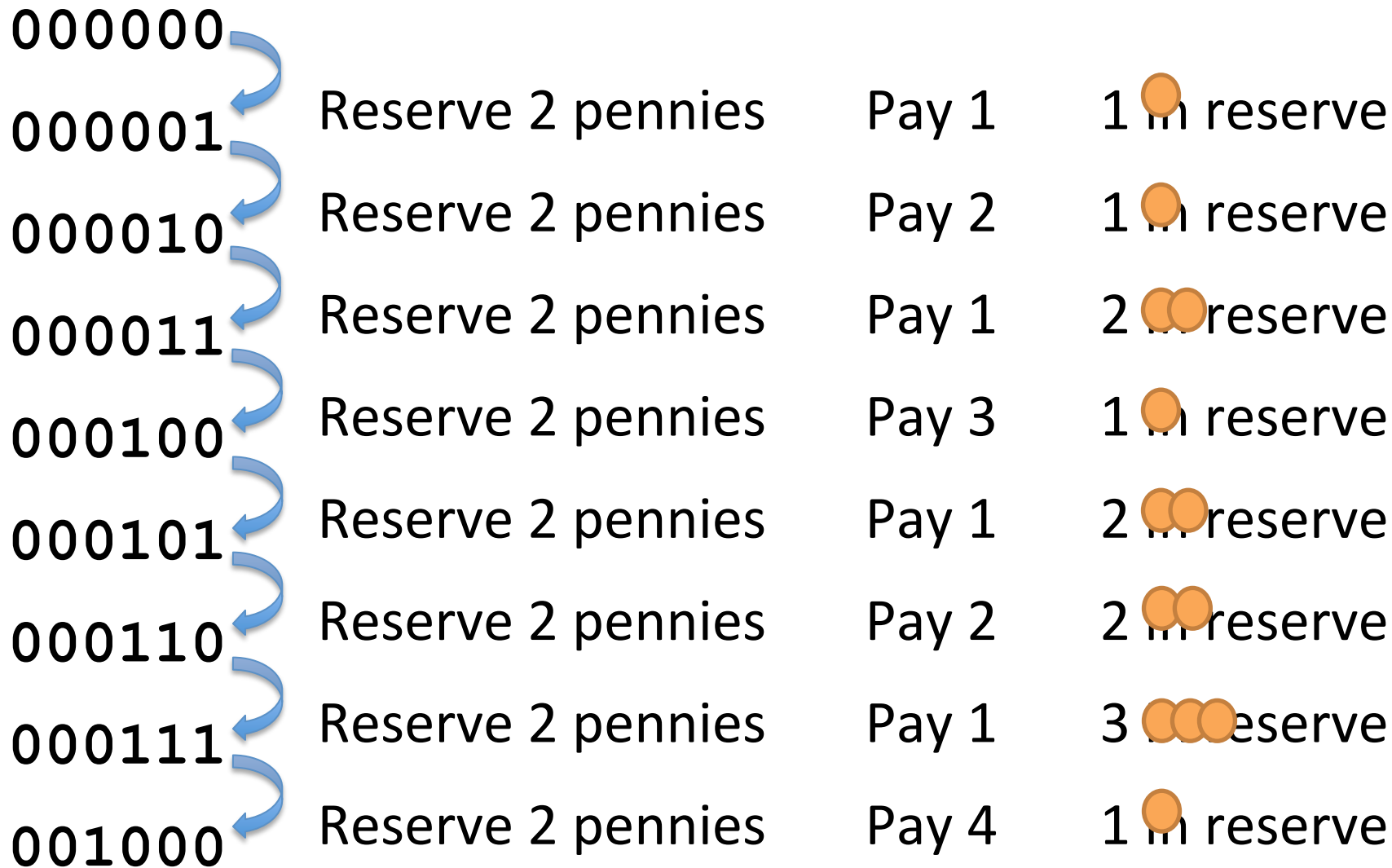
Pay 1

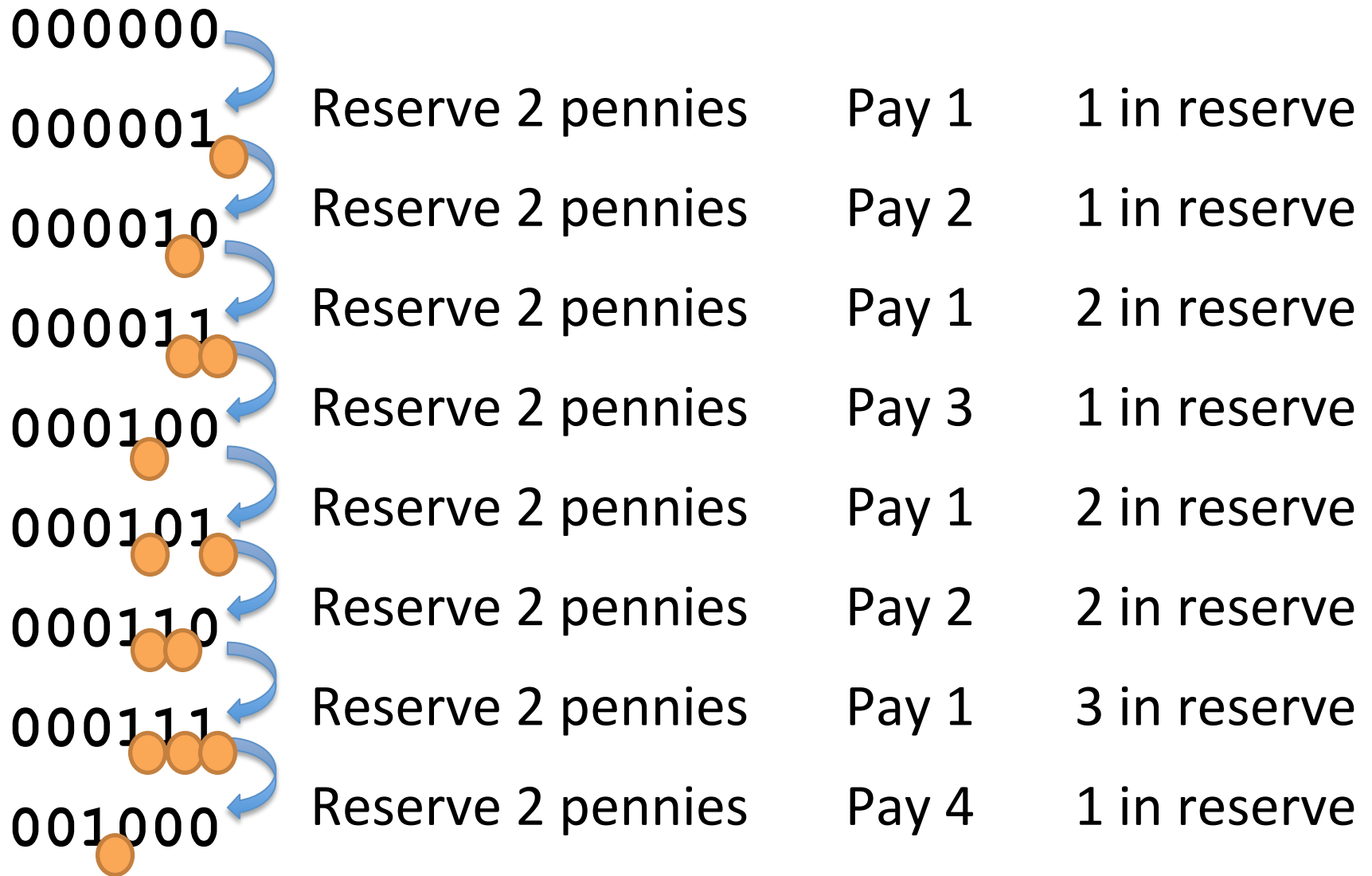
Pay 4

Definitely $O(k)$
worst case

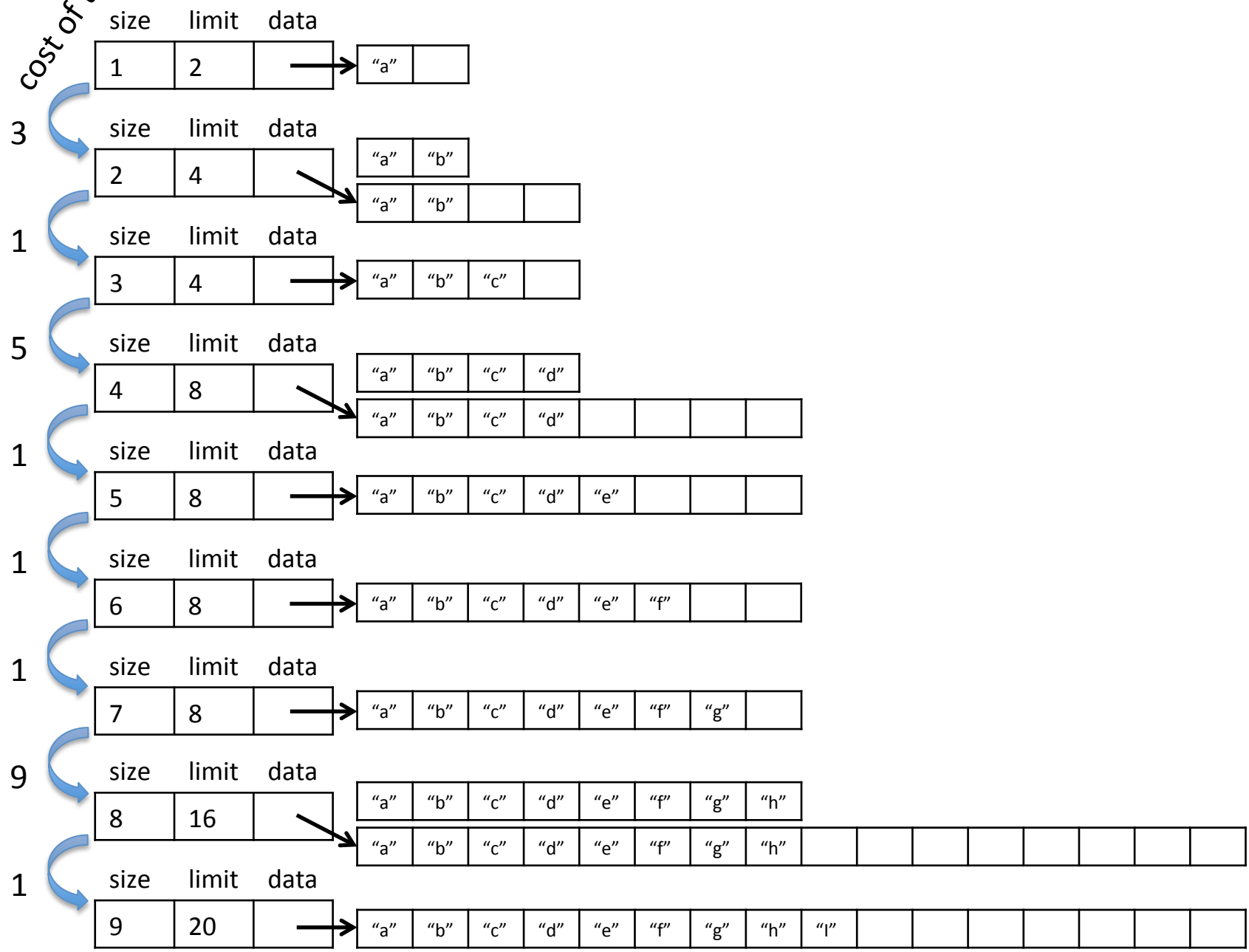


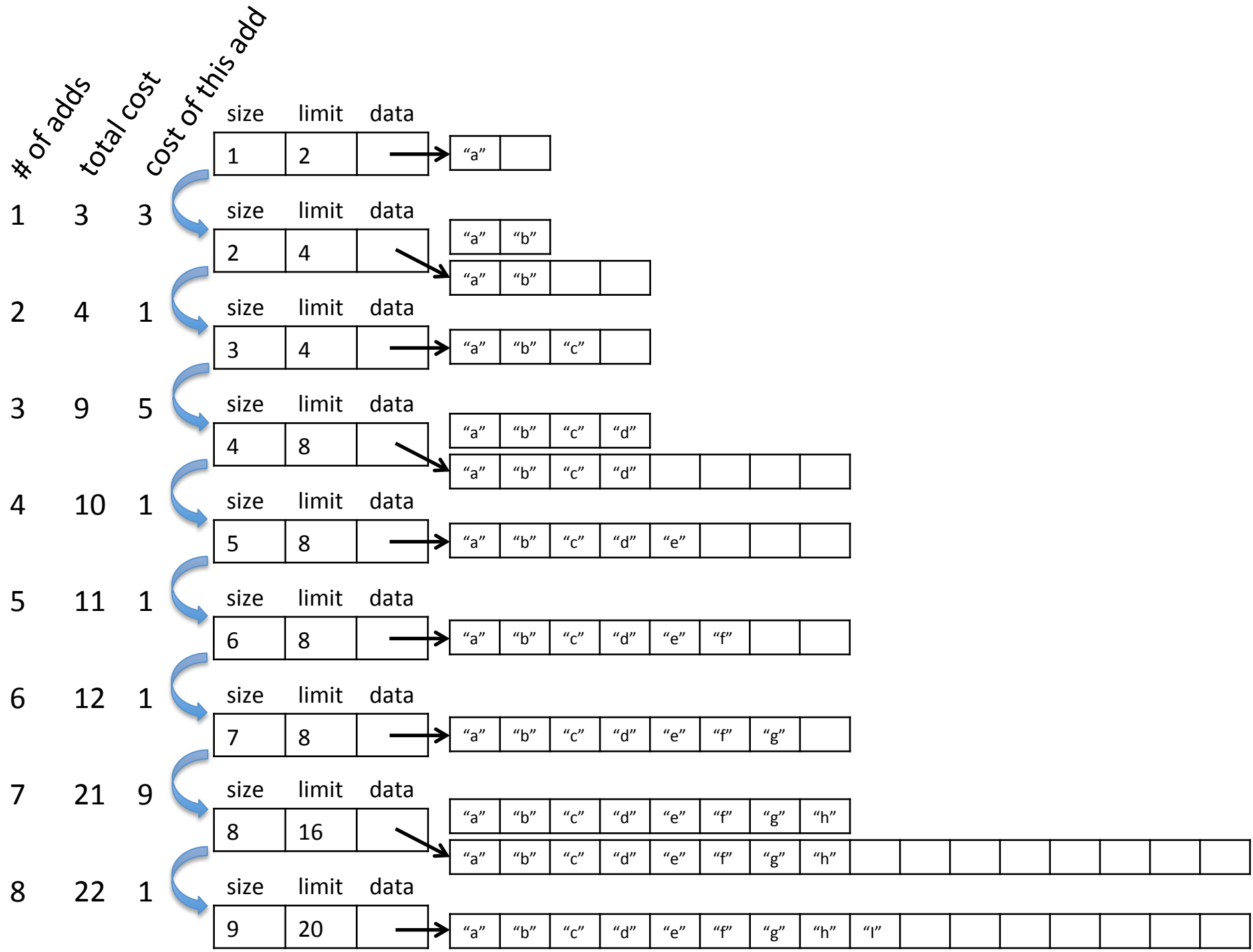
| | | | |
|--------|-------------------|-------|--------------|
| 000000 | | | |
| 000001 | Reserve 2 pennies | Pay 1 | 1 in reserve |
| 000010 | Reserve 2 pennies | Pay 2 | 1 in reserve |
| 000011 | Reserve 2 pennies | Pay 1 | 2 in reserve |
| 000100 | Reserve 2 pennies | Pay 3 | 1 in reserve |
| 000101 | Reserve 2 pennies | Pay 1 | 2 in reserve |
| 000110 | Reserve 2 pennies | Pay 2 | 2 in reserve |
| 000111 | Reserve 2 pennies | Pay 1 | 3 in reserve |
| 001000 | Reserve 2 pennies | Pay 4 | 1 in reserve |

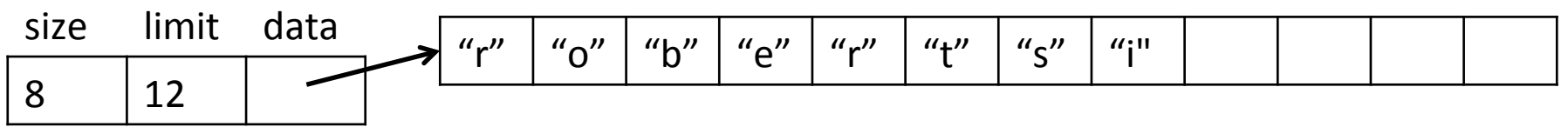
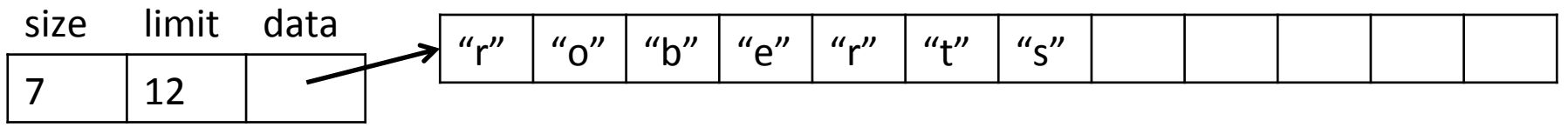
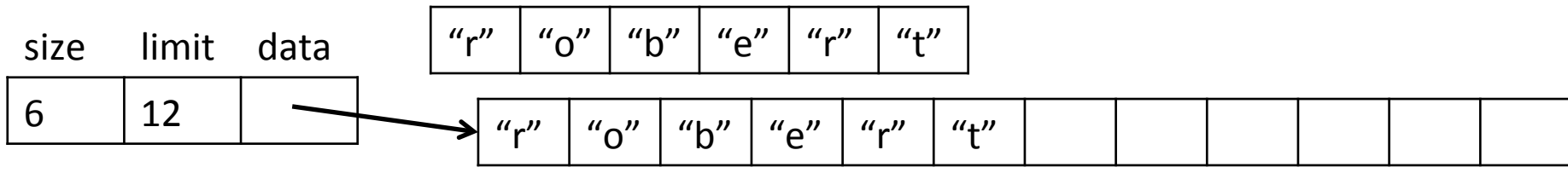
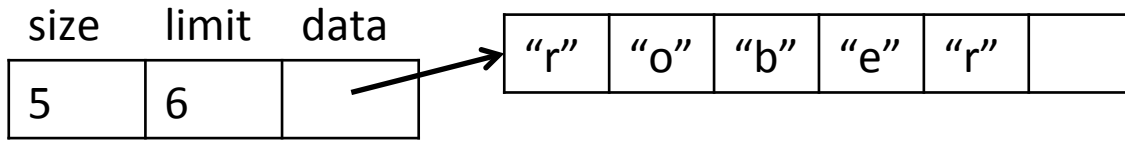
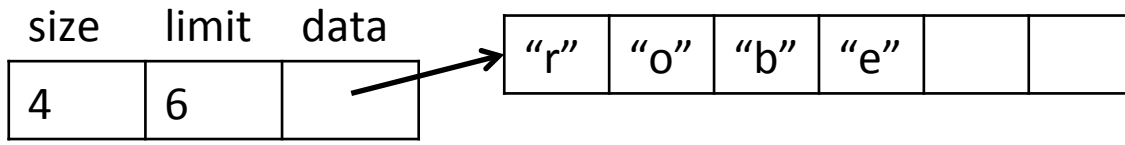
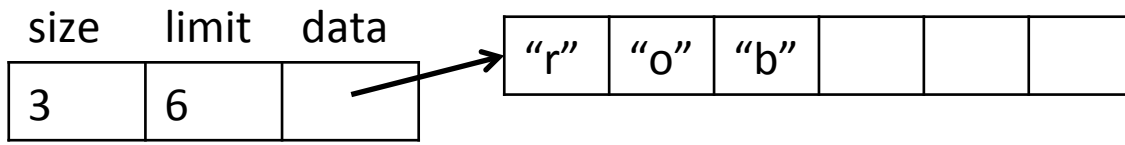


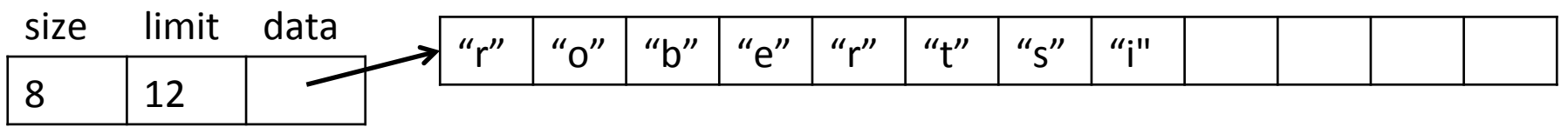
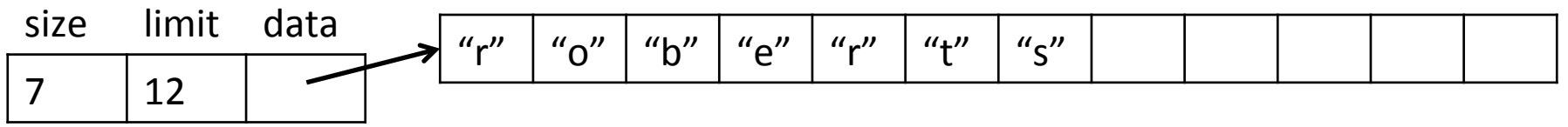
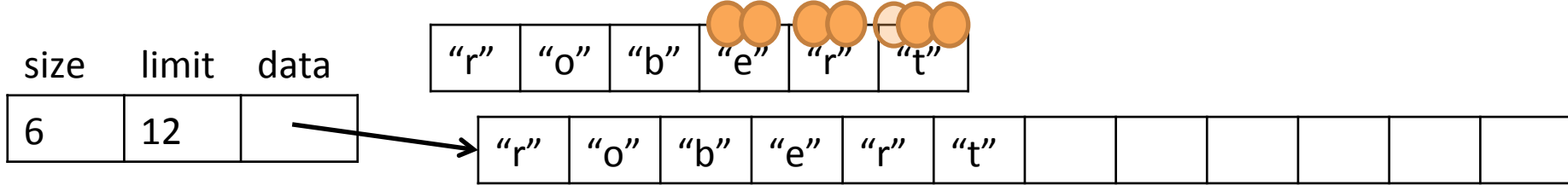
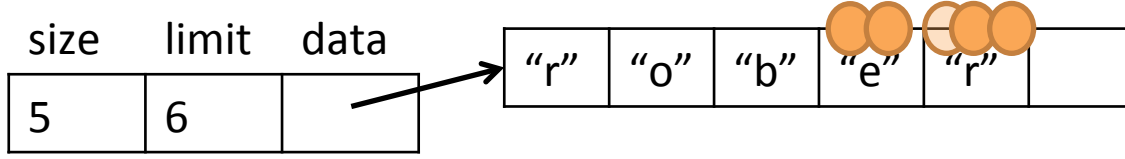
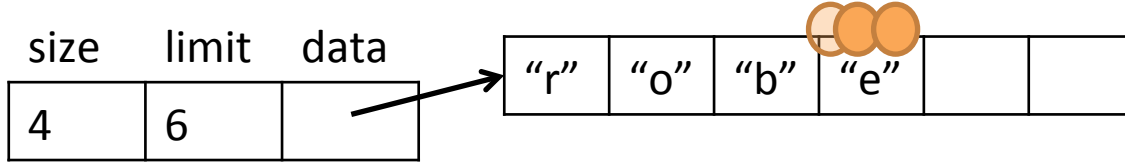
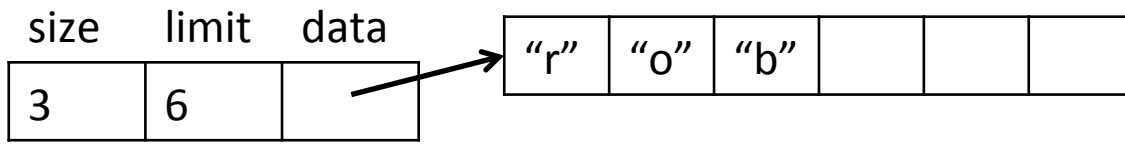


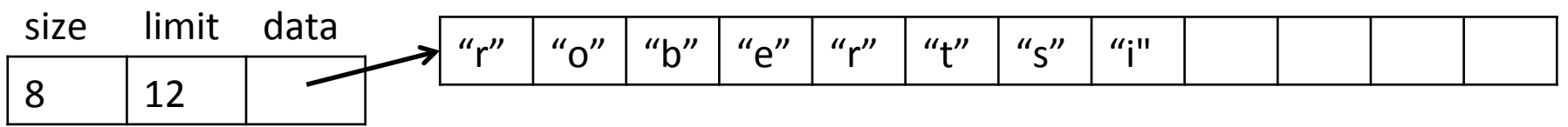
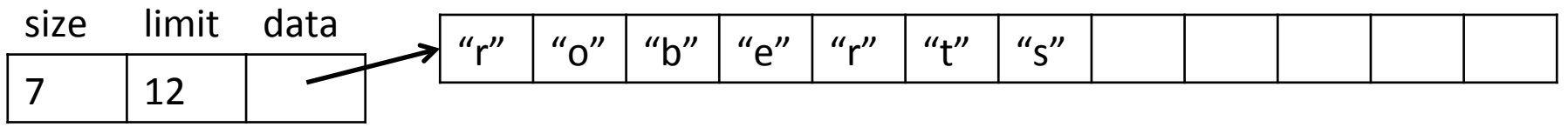
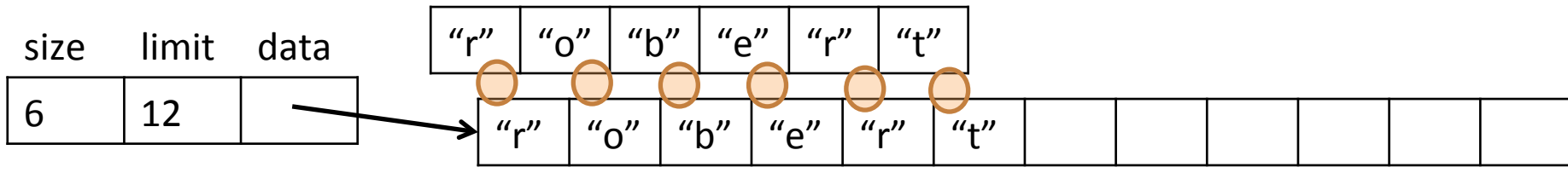
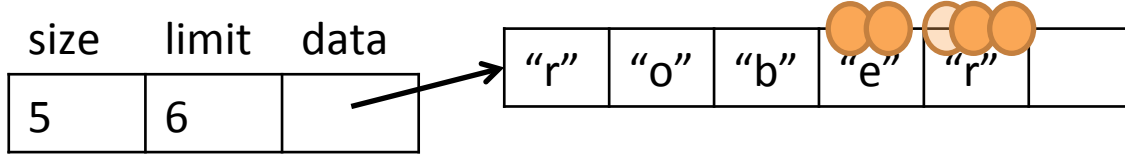
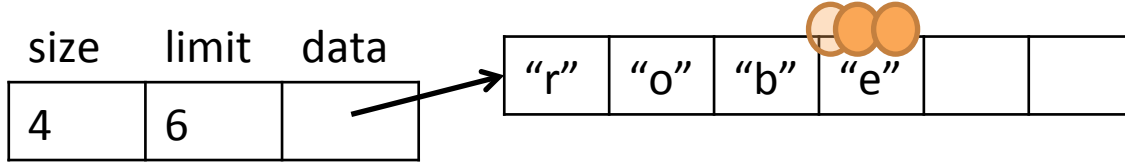
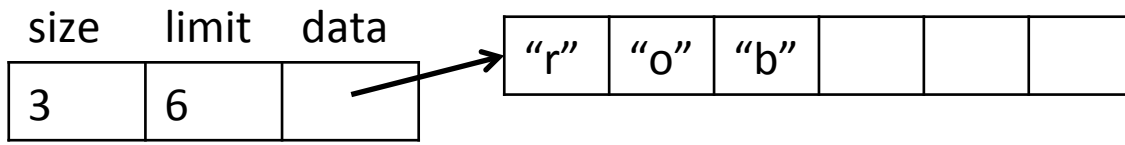
cost of this add

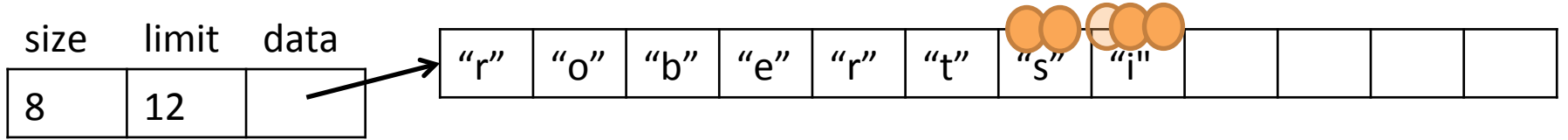
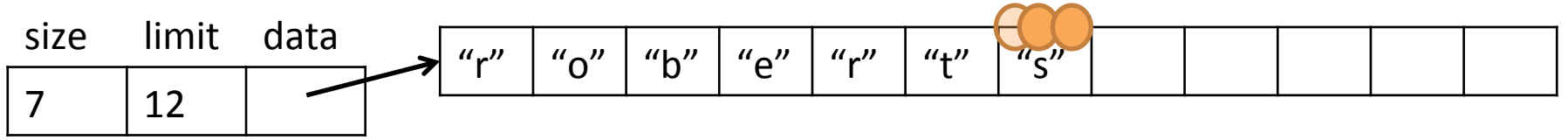
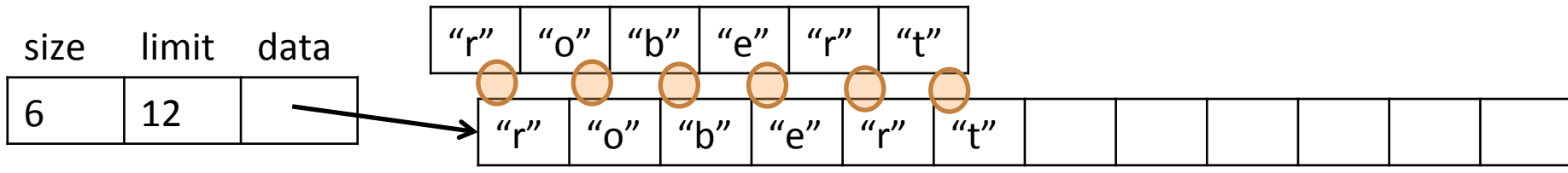
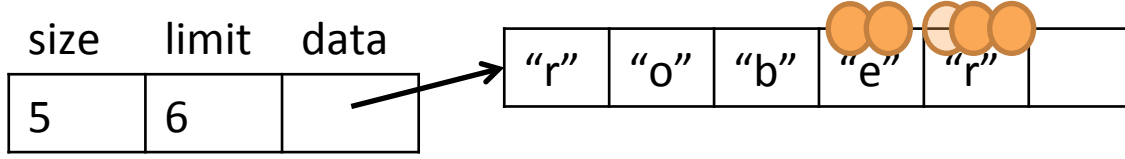
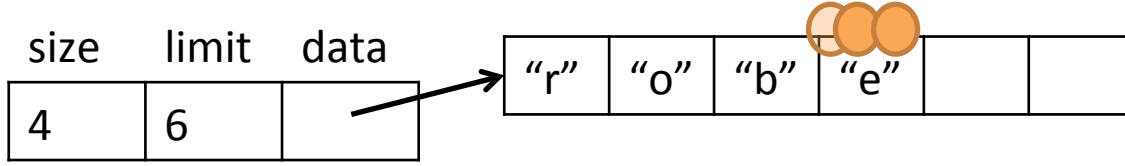
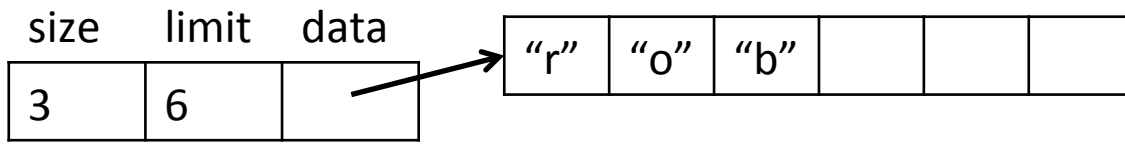


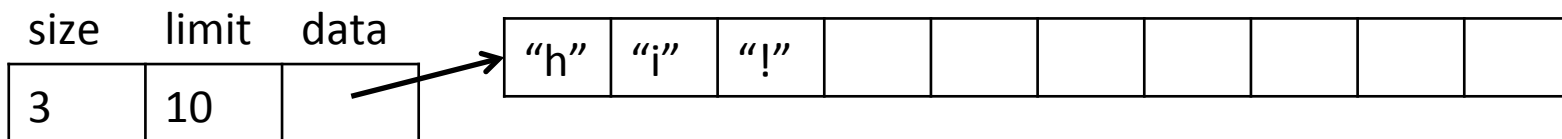
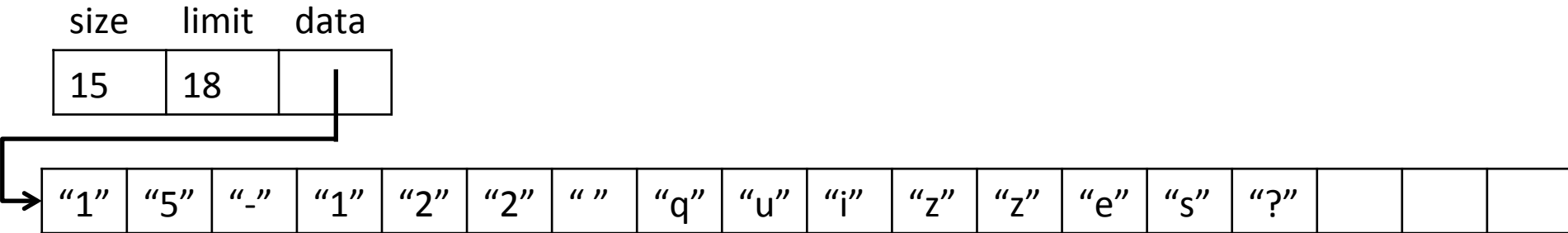
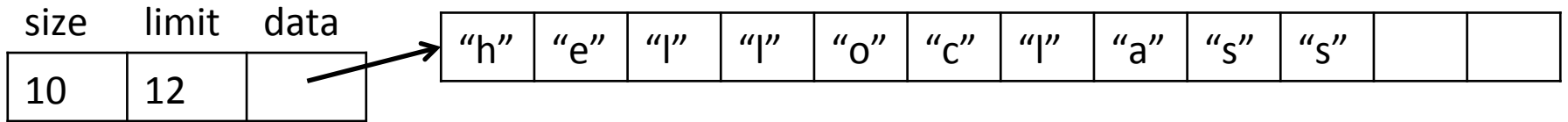
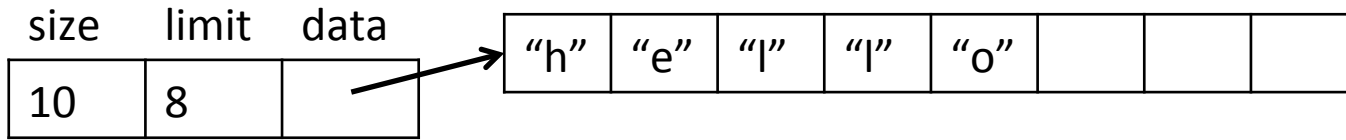


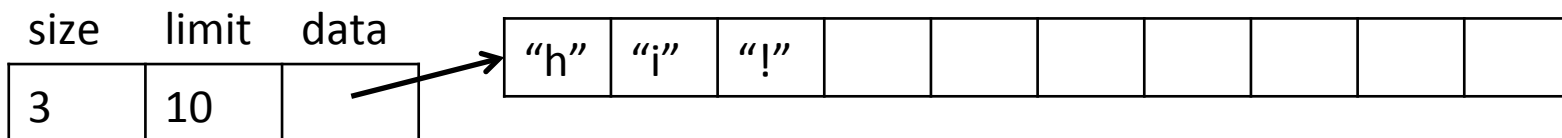
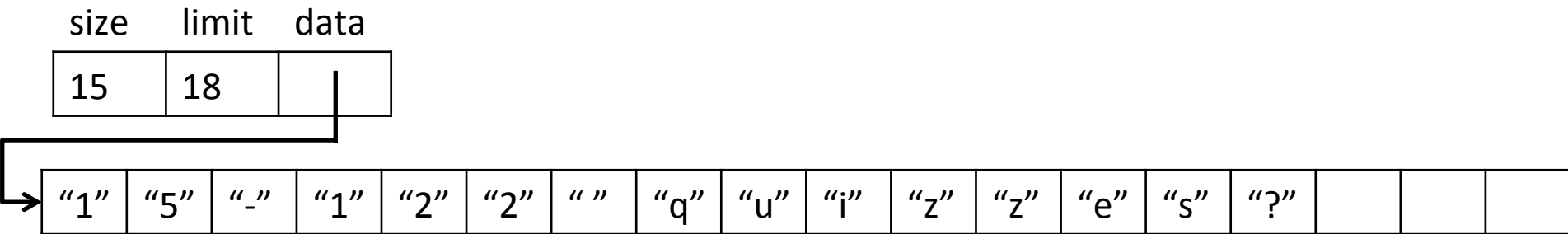
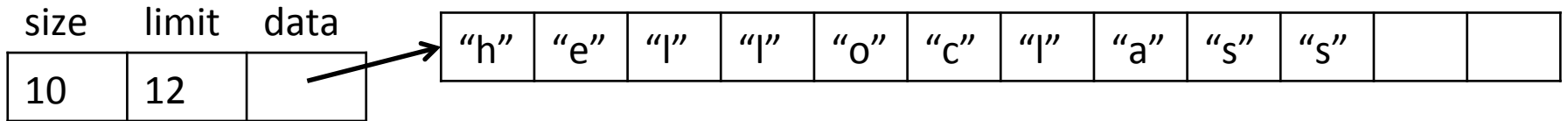
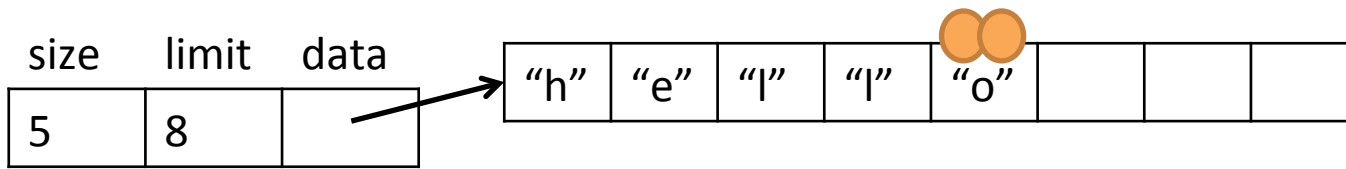


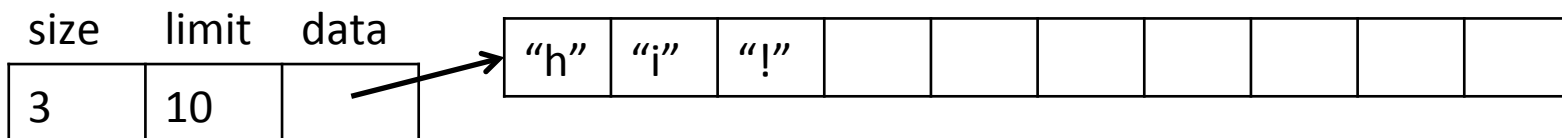
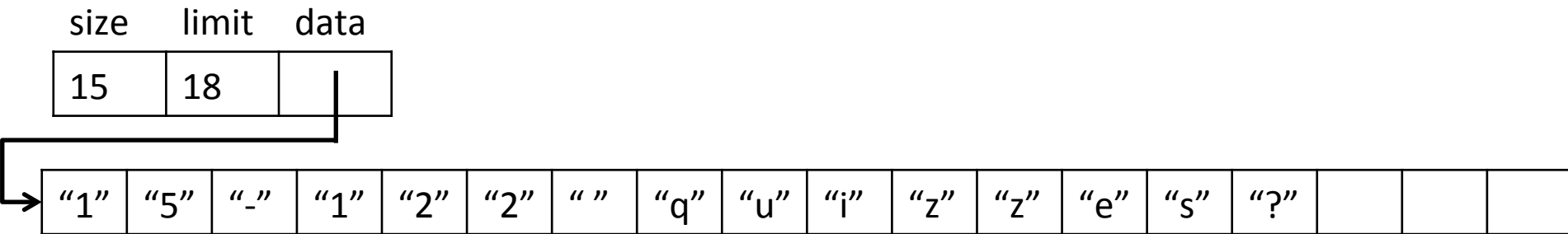
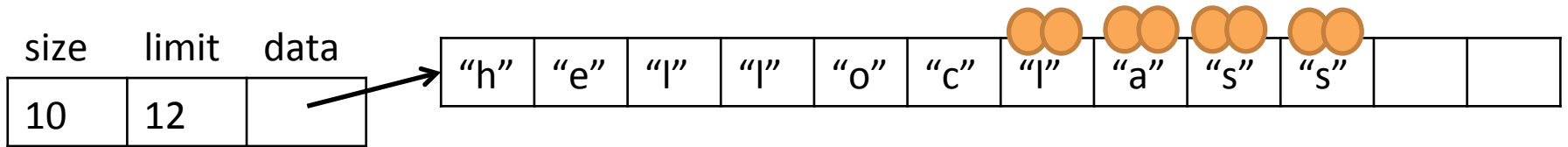
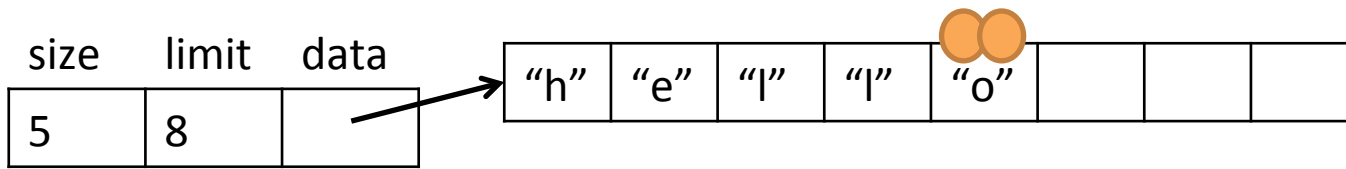


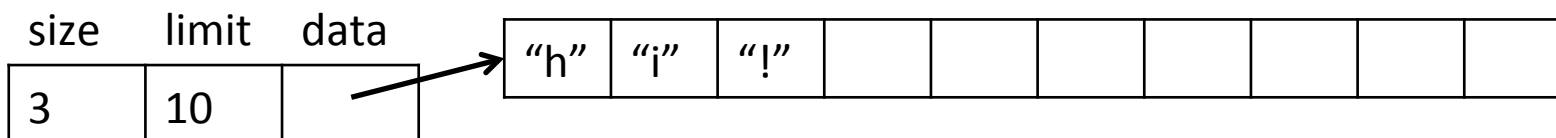
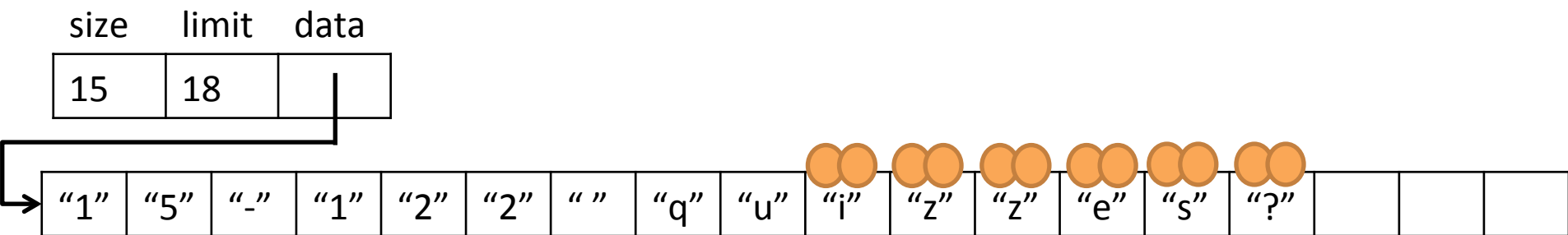
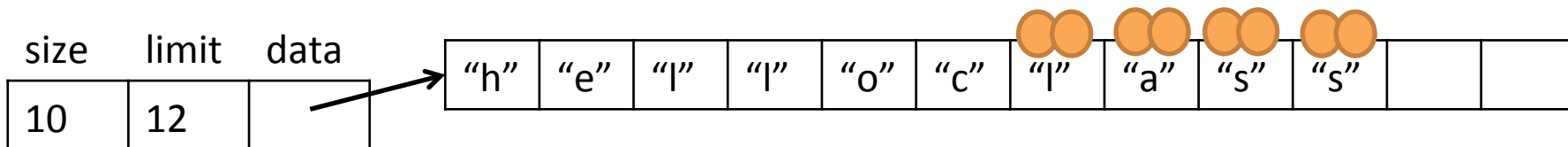
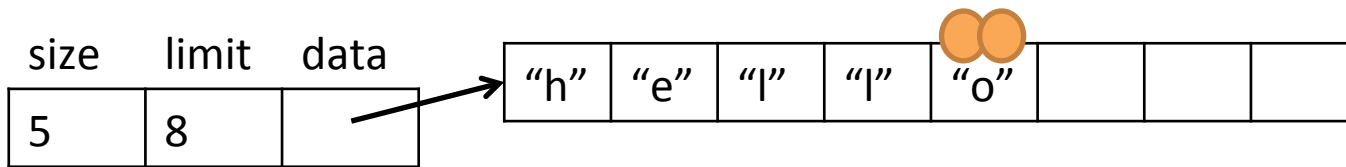


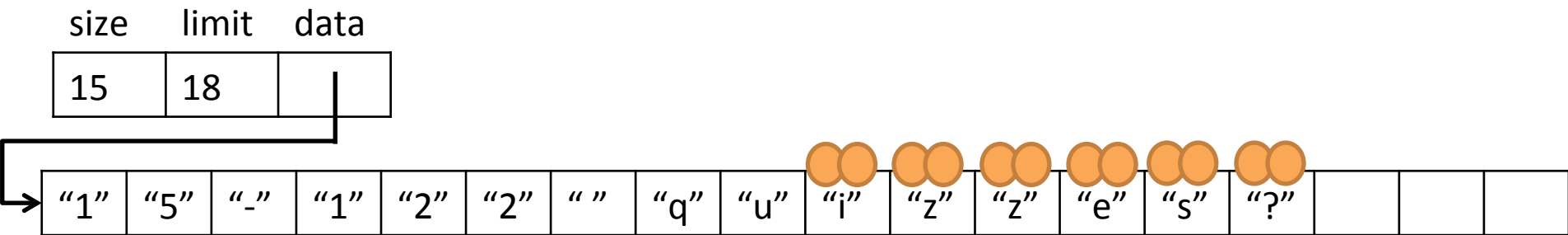
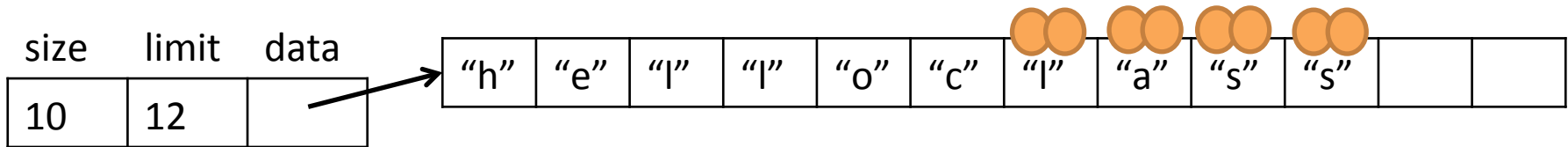
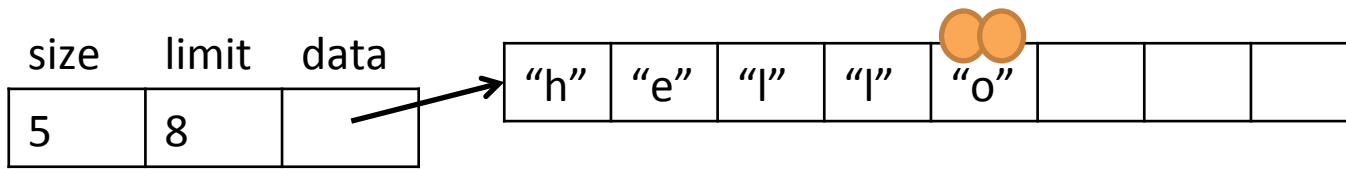












● ???

