Theory of binaural interaction based on auditory-nerve data. IV. A model for subjective lateral position[a)]

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A model for the subjective lateral position of 500-Hz tones is presented and compared with experimental lateralization data. Previous papers in this series have explicitly described the auditory-nerve response to these stimuli and proposed a binaural displayer that interaurally compares the auditory-nerve firing times. The outputs of the displayer are postulated to represent the only information about detailed firing times that is available to the brain. In the present paper, lateral-position predictions are obtained by a central nonoptimal weighting of these outputs that depends on the interaural intensity difference of the tone. These predictions describe the results of lateralization-matching experiments more accurately and over a wider range of stimulus conditions than previous theories, except for those results which suggest that low-frequency binaural tones can generate multiple perceptual images. The predictions of our model are also consistent with the results of centering and laterality-comparison experiments. It is argued that the data discussed in this paper are generally incompatible with theories that propose a peripheral interaction of interaural timing and intensity information such as the latency hypothesis.

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INTRODUCTION

This is the fourth of a series of papers concerned with the development of a theory of binaural perception that is able to describe a wide variety of observed phenomena under a unified theoretical framework, and that is consistent with present knowledge of peripheral auditory physiology. In this paper we extend the previous work to describe the subjective lateral position of 500-Hz tones and we compare our predictions to the results of subjective lateralization-matching experiments. The work reported here is part of a project (Stern, 1976) that includes the application of this model to interaural discrimination and binaural detection experiments under the assumption that judgments in these experiments are based on subjective lateral position. We plan to discuss these aspects of the work in future publications.

In the previous papers of this series (Colburn, 1973, 1977a, 1977b; Colburn and Latimer, 1978) a model of binaural interaction is presented that describes essentially all binaural detection data. The model includes an explicit description of auditory-nerve activity and a display of interaural timing information. Discrimination and detection predictions were obtained in the previous papers by assuming an optimal linear combination of the displayer outputs. In Colburn and Latimer (1978) it is shown that the information available to the decision-making component of this model must be further restricted in order to describe the results of interaural time-discrimination experiments.

In this paper we postulate that available information is restricted to a single internal variable that describes subjective image position and that serves as the decision variable for interaural discrimination experiments. This postulate is motivated by our subjective experience that lateral position is the primary cue for interaural discrimination judgments and is supported by most of the results of Domnitz and Colburn (1977), whose experiments on lateral-position matching and interaural discrimination were designed to test this hypothesis. In the present paper, we attempt to relate their lateralization results to the description of auditory-nerve firing patterns and the binaural displayer used in the previous papers of this series. Specifically, we extend the model by proposing a (nonphysiologically based) mechanism that generates a position variable by combining the outputs of the binaural displayer with an intensity function that depends on the interaural amplitude ratio of the stimulus. The predictions of this extended model, which we call the position-variable model, are compared to data from lateralization-matching experiments in this paper. The position-variable model is of interest for the issues discussed in this paper and for its ability to predict results of discrimination and detection experiments (Stern, 1976). We plan to discuss these aspects of this model in later publications.

In Sec. I of this paper we state the assumptions specifying the position-variable model. In Sec. II we compare the predictions of the model to lateralization-matching, centering, and laterality-comparison data, and in Sec. III we compare our model to other models that describe the laterality of binaural pure tones.

I. THE MODEL

Our model is almost completely determined by a description of our fundamental internal variable $\hat{P}$ that is postulated to be monotonically related to subjective lateral position. In our model $\hat{P}$ is the centroid of the product of two functions called the timing and intensity functions, which are related to the corresponding interaural differences of the stimulus. In Fig. 1 we show the

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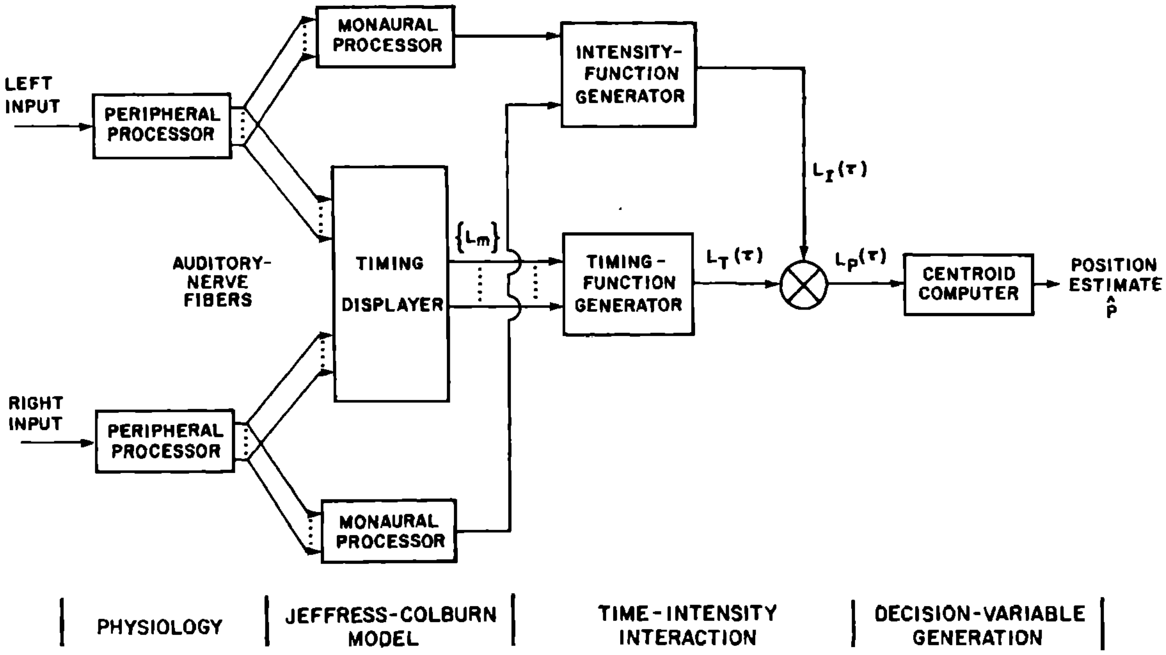

FIG. 1. Block diagram of the position-variable model. The peripheral processors and timing displayer with outputs L_m have been described by Colburn (1973, 1977a). The timing function $L_T(\tau)$ is the density of interaural coincidences in auditory-nerve event times recorded by the timing displayer with respect to the internal delays τ of the fiber pairs. The intensity function $L_I(\tau)$ is Gaussian-shaped and its location along the τ-axis depends on the interaural intensity difference of the stimulus. The position estimate $\hat{P}$ is obtained by computing the centroid of the position function $L_P(\tau)$, which is the product of the timing and intensity functions.

generation of $\hat{P}$ from the acoustical signals. The signal entering each ear is transformed into temporal patterns of firings on auditory-nerve fibers by probabilistic peripheral processors (Colburn, 1973, 1977b). The firing times are interaurally compared by the timing displayer (Colburn, 1977a), which generates the set of outputs L_m from which the timing function $L_T(\tau)$ is generated. The timing function $L_T(\tau)$ is related to the interaural crosscorrelation of the binaural stimulus, as described below, and the variable τ can be thought of as the argument of a crosscorrelation function. The intensity function $L_I(\tau)$ is postulated to be a deterministic, pulse-shaped function which has a fixed width (and shape) and a location along the τ axis that depends on the interaural intensity difference of the stimulus.[1] As stated above, the position estimate, $\hat{P}$ is equal to the centroid of the product of the time and intensity functions. Note that the random nature of the auditory-nerve description implies that the $\{L_m\}$, $L_T(\tau)$, and $\hat{P}$ are all probabilistic quantities [in contrast to $L_I(\tau)$ which is not] and, further, that no additional source of variability is present in the model.

A. Description of the model

Our specific assumptions for the description of auditory-nerve activity are given quantitatively in Colburn (1973). These assumptions are selected to describe the results of physiological studies by Kiang and his associates (Kiang *et al.*, 1965; Kiang, 1968) rather than any specific psychoacoustical phenomena, and are an extension of the theoretical work of Siebert (1968, 1970). The firing times of the auditory-nerve fibers are described as sample functions of statistically independent nonhomogeneous Poisson processes (defined in Parzen, 1962), with instantaneous expected rates of firing that depend on the acoustical stimulus. Expressions for these rates, which are given in Appendix A, describe the frequency-selectivity of the cochlea, the synchrony of the firing patterns to the detailed time structure of a low-frequency stimulus, and the saturation of the time-averaged firing rate to intense stimuli.

The timing displayer has been described in detail by Colburn (1977a) and is similar to a model proposed by Jeffress (1948). In brief, it consists of a network of binaural units, each taking input from two auditory-nerve fibers with the same frequency characteristics, one from each ear, with a small fixed delay inserted on one side. Each unit counts coincidences in firing times of the two input fibers (after the delay) within a short time interval (10 μs). Figure 2(a) shows the expected relative number of firing-time coincidences recorded by these units as a function of their internal delay τ, in response to a 500-Hz binaural tone presented with the signal to the left ear delayed by 500 μs.[2] The delays are fixed for each pair and distributed over the pairs independently of all other model parameters according to a density function $p(\tau)$ shown in Fig. 2(b). The specific shape of the function $p(\tau)$ was chosen to describe the dependence of antiphasic binaural detection thresholds on frequency (Colburn, 1969).

The timing function $L_T(\tau)$ is defined to be the number of coincidences occurring during the stimulus presentation from all fibers in a fixed range of characteristic or "best" frequencies as a function of internal delay. This range of characteristic frequencies is chosen to optimize

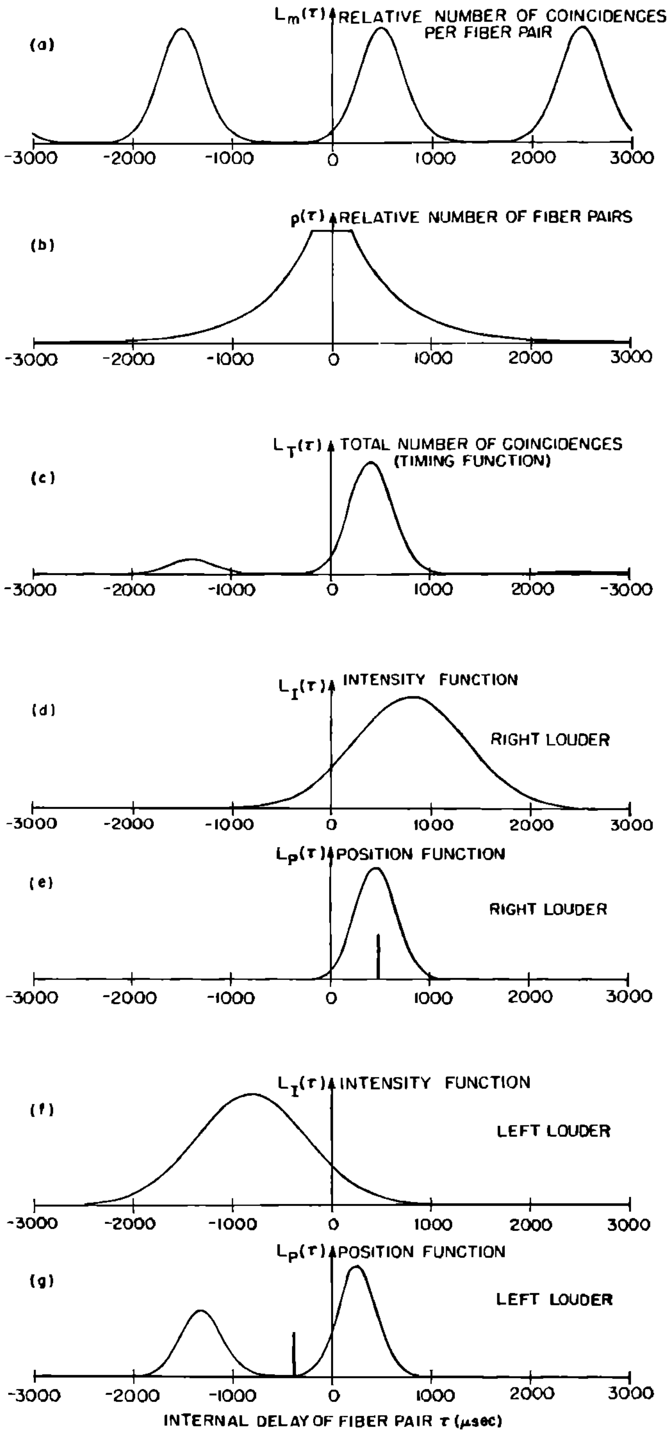

FIG. 2. Generation of the timing function $L_T(\tau)$ and position functions $L_P(\tau)$ for 500-Hz tone with +500-μs interaural time delay. Shown as a function of the internal delay τ of the fiber pairs are (a) $L_m(\tau)$, the relative number of coincidences observed by a single fiber pair, (b) $p(\tau)$, the relative number of fibers pairs, and (c) the timing function $L_T(\tau)$, which is the product of $L_m(\tau)$ and $p(\tau)$. These functions are plotted assuming that all fibers are synchronized to the stimulus tone. In (d) and (e) we show the generation of the position function for a reinforcing (right leading and louder) combination of time and intensity differences. The position function $L_P(\tau)$ shown in (e) is the product of the timing function $L_T(\tau)$ in (c) and the intensity function $L_I(\tau)$ in (d). Similarly, in (f) and (g) we show the generation of the position function for a canceling (right leading; left louder) combination of interaural intensity differences. For these stimulus conditions $L_P(\tau)$ in (g) is the product of $L_T(\tau)$ in (c) and $L_I(\tau)$ in (f). In (e) and (g) $\bar{P}$, the centroid of $L_P(\tau)$, is indicated by the vertical line. Note that in (a), (c), (e), and (g) we plot the expected value of random functions, while the functions in (b), (d), and (f) are deterministic. All functions are plotted using an arbitrary vertical scale, and for reasons of clarity we use intensity functions $L_I(\tau)$ that are considerably more narrow than those used in the actual computations. [To make this figure more intuitive (right-lateralized image on the right side of the graph), we consider the internal delays to be in the channels from the right ear (rather than the left as in our previous papers), so that the variable τ in this figure corresponds to $-\tau_m$ in previous papers of this series.]

interaural discrimination performance predicted by the model at moderate levels (55 dB SPL), and is 190–3350 Hz for 500-Hz puretone stimuli.[3] Expressions for the timing function are derived in Appendix A. Because the set of fibers used to compute $L_T(\tau)$ is fixed, some of the coincidences included in the function $L_T(\tau)$ will be from fiber pairs that are not synchronized to the stimulus when the stimuli are presented with low overall intensities and/or large interaural amplitude ratios. The expected value of the timing function for a 500-Hz tone presented with a 500-μs time delay is shown in Fig. 2(c), assuming that all fibers pairs in the fixed range of frequencies are synchronized to the stimulus tone. In this case $L_T(\tau)$ is proportional to the product of the functions $L_m(\tau)$ and $p(\tau)$ shown in Figs. 2(a) and 2(b).

The intensity function $L_I(\tau)$ is assumed to be a deterministic Gaussian function of τ, with a fixed width W_I. The location of this pulse-shaped function along the τ axis depends on α_s, the interaural intensity difference of the stimulus, according to the function $M_I(\alpha_s)$.[4] The parameter W_I and the function $M_I(\alpha_s)$ are chosen to fit data in lateral-position-matching experiments, as specified below.

The position function $L_P(\tau)$ is the product of the timing function $L_T(\tau)$ and the intensity function $L_I(\tau)$; that is, for each τ,

$$L_P(\tau) = L_I(\tau)\, L_T(\tau). \tag{1}$$

The position variable $\hat{P}$ is the centroid along the τ axis of the position function L_P:

$$\hat{P} = \int_{-\infty}^{\infty} \tau L_P(\tau) d\tau \Big/ \int_{-\infty}^{\infty} L_P(\tau) d\tau. \tag{2}$$

Figures 2(c)–2(g) show how the position function $L_P(\tau)$ and the position variable $\hat{P}$ are calculated for a typical set of reinforcing (louder side leading in time) or cancelling (louder side lagging in time) set of interaural parameters for a 500-Hz tone.[5] In Figs. 2(c)–2(e) we plot the timing function $L_T(\tau)$, the intensity function $L_I(\tau)$, and their product, the position function $L_P(\tau)$, for the reinforcing stimulus conditions; these functions for canceling stimulus parameters are shown in Figs. 2(c), 2(f), and 2(g). The heavy vertical bar in Figs. 2(e) and 2(g) shows the location of the centroid of $L_P(\tau)$, which is the expected value of the position variable $\hat{P}$. A comparison of Figs. 2(d)–2(g) illustrates that the location of the intensity weighting function $L_I(\tau)$ along the τ axis varies with the interaural intensity difference of the stimulus, causing $L_I(\tau)$ to emphasize either the primary or secondary peak of the timing function, $L_T(\tau)$. For the canceling-cue case (right leading; left louder) in Fig. 2(g) the centroid of the position function lies between its two peaks, and to the left of the origin. Clearly there exists a smaller interaural intensity difference with the left side louder that will give rise to a centered position-function centroid, corresponding to a "trading" of the interaural time and intensity differences.

B. Comments

An implicit assumption of our model is that the spatial attributes of a binaural tone are adequately described by the single variable $\hat{P}$ related to the lateral position of a single image, and that the shape of that image does not change significantly when the stimulus parameters are varied. This assumption is not directly tested by the experimental results of Domnitz and Colburn (1977) because their subjects were instructed to match the "center" of the pointer tone to the "center" of the test tone. On the other hand, the results of some previous lateralization studies (Whitworth and Jeffress, 1961; Sayers, 1964; and Hafter and Jeffress, 1968) imply that a binaural pure tone may be perceived with more than one spatial image. Other lateralization results (e.g., Shaxby and Gage, 1932; Moushegian and Jeffress, 1959) exhibit a very low trading ratio, implying that the dependence of subjective lateral position on the interaural intensity difference of the stimulus is small. Our single-image intensity-dependent predictions necessarily differ from these results. We assume the existence of only a single unimodal binaural image in the theoretical predictions described in this paper primarily for reasons of simplicity. A second subjective image similar to the ones described by Whitworth and Jeffress (1961) and Hafter and Jeffress (1968) could easily be obtained from the present model by computing the centroid of the unweighted timing function $L_T(\tau)$. We plan to discuss the significance of our single-position assumption further in a future publication concerned with interaural discrimination experiments.

We close this section with several comments about the interaction of time and intensity information in our model. First, the multiplicative weighting of the coincidence counts by the deterministic intensity-dependent function was chosen because it offers a relatively simple combination of interaural timing and intensity information with a relatively small number of fitted parameters. Given the structural assumptions defining the model (including the assumed range of fiber frequencies which are included in computations), the only free parameters to be fitted to the data are the width parameter W_I of the intensity function $L_I(\tau)$, and the function $M_I(\alpha)$ describing the dependence of its location along the τ-axis on interaural intensity difference. The position predictions of the model do not appear to be very sensitive to the exact shape of the weighting function $L_I(\tau)$, although its width is important. The multiplicative weighting by the intensity function and the centroid computation constitute, in effect, a nonoptimum weighting of the timing displayer outputs $\{L_m\}$ by coefficients proportional to $\tau_m L_I(\tau_m)$. [These are different from the nonoptimal weighting coefficients discussed in Colburn and Latimer (1978).]

We wish also to note that interaural intensity differences affect the predictions of the model in two ways. First, the interaural intensity difference determines the location of the intensity function $L_I(\tau)$ and thereby affects the position variable (as illustrated in Fig. 2). Second, as the signal to one ear is attenuated, the function $L_m(\tau)$ becomes flatter, because an increasingly large percentage of fiber pairs with one or both fibers discharging spontaneously is included. Since spontaneous discharges are not phase locked to the stimulus tone, coincidences with spontaneous discharges are equally likely to be recorded at any value of τ. The contribution of these fiber pairs to the position variable, then, would be independent of the time delay of the stimulus, and the overall effect of the interaural time delay of the stimulus on the position variable $\hat{P}$ will decrease. In the limiting case of a monaural presentation the expected number of coincidences per fiber pair [Fig. 2(a)] would be independent of the internal delay of the fiber pair for all pairs, and the timing function would depend only on the relative number of fiber pairs $p(\tau)$.

Finally, our description of the auditory-nerve patterns dictates that the locations of the modes of the timing function are unaffected by changes in the interaural intensity difference of the stimulus. If we had assumed a peripheral conversion of intensity to an equivalent time delay, as in the latency hypothesis (cf. Deatherage and Hirsh, 1959), the locations of the modes of the timing function as well as the position function would depend on the interaural intensity difference of the stimulus.

II. COMPARISONS OF PREDICTIONS TO DATA

We now compare the predictions of the position-variable model to experimental results based on lateraliza-

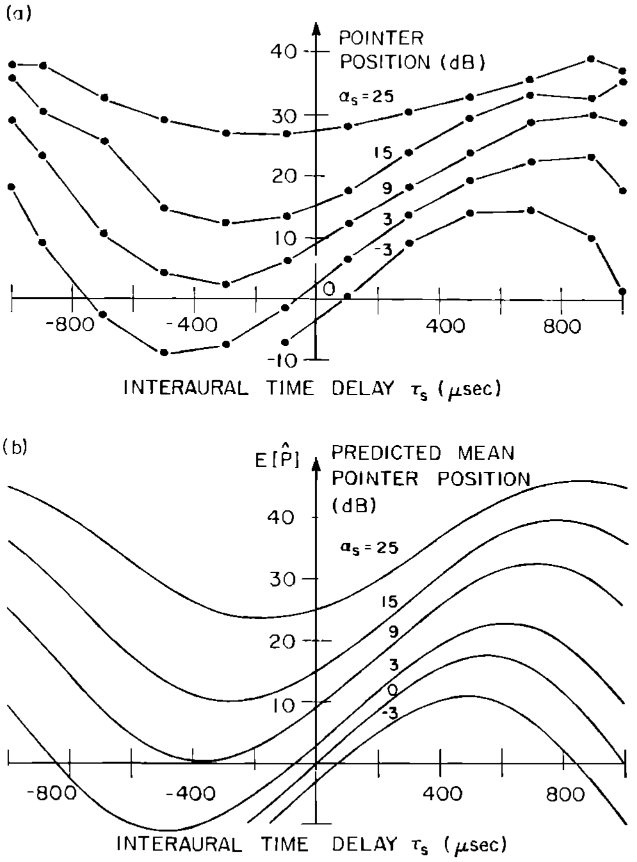

FIG. 3. (a) Experimental lateralization-matching results, showing the interaural intensity difference of a pointer tone required to match the position of a 500-Hz test tone as a function of the interaural intensity difference α_s and interaural time delay τ_s of the test tone [from Domnitz and Colburn (1977)]. (b) Theoretical predictions for the same stimuli, which were obtained using a Gaussian intensity function $L_I(\tau)$ with a value of 1778 μs for the width parameter W_I, and the function for the location along the τ-axis $M_I(\alpha_s)$ shown in Fig. 4. The timing function was obtained by averaging over all fiber pairs with characteristic frequencies between 190 and 3350 Hz.

tion judgments. We have attempted to consider experiments that are representative of all major types of lateralization-based tasks in the literature.

A. Lateralization-matching experiments

We first consider the ability of the model to describe data on lateralization-matching judgments. Figure 3(a) shows the average results of a series of such experiments by Domnitz and Colburn (1977) in which subjects varied the interaural intensity difference of a 500-Hz interaurally in-phase pointer tone to most closely match the position of a dichotic 500-Hz tone. Each set of connected points shows the dependence of the interaural intensity difference of the pointer tone on interaural time delay τ_s of the test tone with the intensity difference α_s of the test tone as a parameter. We refer to these data (and the corresponding predictions) as "position curves." Since the stimuli are 500-Hz tones, these position curves should be periodic in time delay with 2000-μs period. (We assume that the 500-ms duration and 50-ms rise–fall times of the tone bursts used in this experiment are sufficiently long that the amount of stimulus energy at frequencies other than 500-Hz is negligible.)

Our predictions for the same experiment are shown in Fig. 3(b), obtained with a value of 1778 μs for the width parameter W_I and with the function $M_I(\alpha)$ shown in Fig. 4. These values of $M_I(\alpha)$ and W_I were chosen to describe the lateralization data for tones generally and, in particular, to most closely describe the time delays at which the derivative of lateral position with respect to interaural time delay changes sign.[6]

We find the agreement between data and predictions satisfactory. In particular, we note that the following properties are common to both sets of curves.

(1) The position curves may be approximated by parallel straight lines in the region where both interaural time and interaural intensity differences are small.

(2) Time-intensity trading occurs in that a centered image is achieved by many combinations of interaural parameters with the louder side lagging in time.

(3) As the magnitude of the intensity difference increases, the effect of interaural time delay on position decreases. (In the limiting case of a monaural presentation, position is independent of interaural time delay.)

(4) The curves exhibit zero slope with respect to interaural time delay for certain delays. We call these time delays "cue-reversal points" because a small increase in the interaural time delay of a tone causes its subjective position to move toward the right for total interaural time delays between 0 μs and either cue-reversal point, and to the left otherwise. The time delays at which cue-reversal points are observed increase as the interaural intensity difference increases.

(5) For interaural intensity differences of 0 or ±3 dB the slopes of the position curves are greater about 1000 μs than about 0 μs time delay.

We briefly discuss the bases within the position-vari-

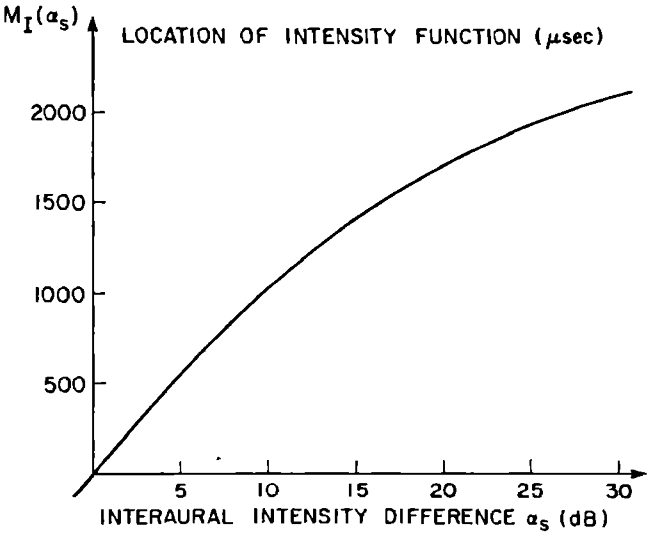

FIG. 4. Dependence of the location of the model's intensity function $M_I(\alpha_s)$ on α_s, the interaural intensity difference of the stimulus.

able model for these predictions in Appendix B, and the extent to which other models of subjective lateral position describe these data in Sec. III. The data and predictions in Fig. 3 are in general agreement with previous results by Moushegian and Jeffress (1959) and Sayers (1964), although there are some differences in the trial-by-trial results of the latter study as discussed in Domnitz and Colburn (1977). We have chosen to fit the free parameters of our model to the data of Domnitz and Colburn because these data were obtained with the widest range of stimulus conditions, and with direct monitoring of the acoustical stimulus delivered by the headphones.[7]

Figure 5 shows predictions for the standard deviation of the position estimate. The standard deviations are specified in the same units as $\hat{P}$, i.e., in terms of the interaural intensity difference of the matching acoustical pointer. These predictions are plotted for the same stimulus parameters and with the same values of W_I and $M_I(\alpha)$ as in Fig. 3. The main properties of the curves are that the standard deviation increases (i) as the interaural intensity difference of the stimulus increases and (ii) as the interaural time delay approaches half the stimulus period. For large intensity differences the curves are asymmetrical, with the standard deviation slightly greater when the interaural parameters are cancelling (louder signal lagging). This asymmetry is consistent with the subjective reports of image width by subjects performing interaural discrimination experiments (cf. Domnitz, 1973).

B. Centering experiments

Many experimental investigations of binaural lateralization have employed a technique by which subjects adjust the interaural time or intensity difference of a presented binaural sound to produce a centered preceptual image. In two such studies using pure tones, Elpern and Naunton (1964) and Young (1976) measured the interaural intensity difference required to center a stimulus presented with an interaural time delay. In Fig. 6 we compare some of the data from these experiments to the corresponding predictions of the position-variable model. The Young (1976) data plotted in Fig. 6 were obtained at frequencies of 400 and 600 Hz, and we consequently express all data and predictions in terms of interaural phase, rather than time delay. We have assumed that Young's subjects exhibit no consistent left-right bias in their lateralizations, and have averaged their data for symmetric phase differences (such as +90° and −90°).

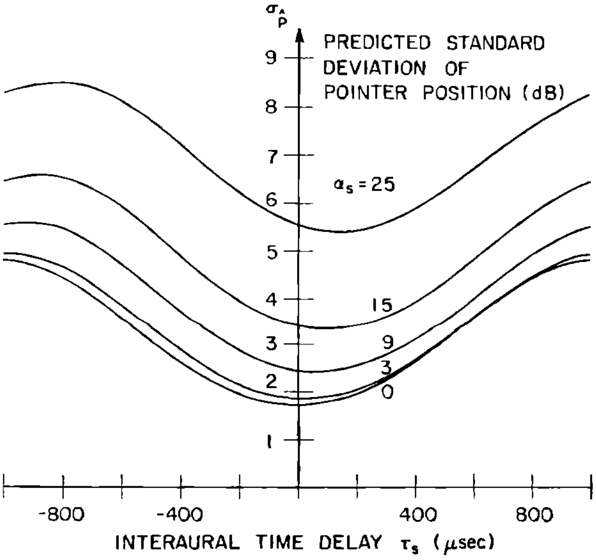

FIG. 5. $\sigma_{\hat{P}}$, the predicted standard deviation of the position variable $\hat{P}$, in terms of the interaural intensity difference of the pointer tone in dB, for the stimulus parameters used in Fig. 3.

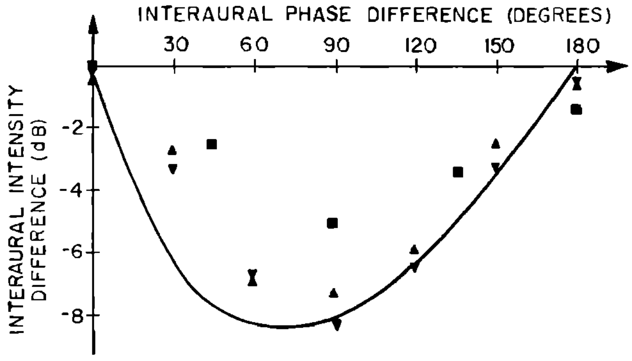

FIG. 6. Combinations of interaural phase differences and intensity differences producing centered binaural images. Data from Young (1976) at 70 dB SPL and 400 Hz (▼) and 600 Hz (▲), Elpern and Naunton (1964) at 40 dB SL and 500 Hz (■). Predictions of the model are indicated by the solid curve.

The predictions of the model are in reasonable qualitative agreement with the data. The phase difference for which the greatest intensity difference is required to center the image is approximately 90° in the data and approximately 75° in the predictions. These results are easily accounted for in terms of the data and predictions of Fig. 3. For example, it is seen in Fig. 3(b) that approximately 9-dB interaural intensity difference is needed to center a 500-Hz tone presented with −400-μs interaural time delay, while only 3 dB is needed if the tone is presented with a −75- or −860-μs delay.[8] The quantitative discrepancies between the predictions and data (and between the data of the two different experiments) may be the result of differences in the frequencies or overall intensities at which the stimuli were presented (400 and 600 Hz and 70 dB SPL for Young versus 500 Hz and 55 dB SPL for Domnitz and Colburn), differences between the subjects of the two centering experiments and those used in Domnitz and Colburn's (1977) experiment, or differences in the experimental paradigms.

C. Laterality-comparison experiments

We now consider the results of two studies (Yost *et al.*, 1975; Molino, 1974) in which subjects judged the relative laterality of tonal stimuli. In the first study (Yost *et al.*, 1975), subjects were presented two binaural stimuli during each trial and were required to indicate which of the two tones was perceived farther to the right. The first stimulus of each trial contained only an interaural time delay, and the second only an

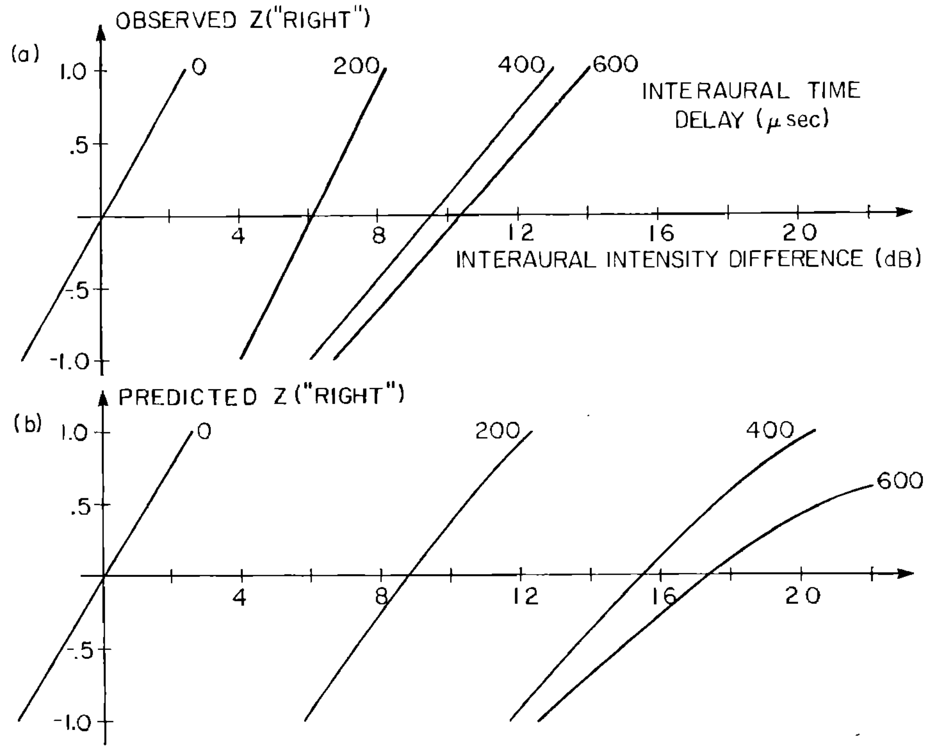

FIG. 7. (a) Observed and (b) predicted z scores for percentage of lateralization judgments toward the right side when a 55-dB 500-Hz tone presented with an interaural time delay is compared with a second tone presented with an interaural intensity difference. Data averaged over four observers, from Yost, Tanis, Nielson, and Bergert (1975).

interaural intensity difference. The results for 55 dB SPL and 500 Hz are plotted in Fig. 7(a) in the form of z scores for percentage of judgments to the right as a function of the interaural intensity difference of the second tone, with the interaural time delay of the first tone as a parameter. The second study (Molino, 1974) is more complex; it was concerned with the extent to which externalized sound sources could be simulated with headphones, and consisted of parallel studies using stimuli presented both in a free field, and with headphones. We consider only the headphone experiment at this time. The stimuli contained combinations of interaural time delay and intensity difference that would be measured at the ears in response to a tonal point source in a free field at a given azimuth angle. Subjects were fixed with an arm extended toward one of four reference azimuths, and indicated the relative laterality of presented dichotic tones with respect to the position of the arm. Z scores for percentage of judgments toward the right for the 500-Hz case are plotted as a function of the simulated azimuth angle in Fig. 8.

Predictions of the model are compared to the 55-dB SPL data of the Yost *et al.* experiment in Fig. 7 and to the 500-Hz data of the Molino experiment in Fig. 8. The predicted z scores were obtained using the expression

$$\hat{z}(\text{“right”}) = \frac{k(E[\hat{P}] - E[\hat{P}_{\rm ref}])}{\frac{1}{2}(\sigma_{\hat{P}} + \sigma_{\hat{P}_{\rm ref}})}, \qquad (3)$$

where in the Yost *et al.* experiment $E[\hat{P}]$, $E[\hat{P}_{\rm ref}]$, $\sigma_{\hat{P}}$, and $\sigma_{\hat{P}_{\rm ref}}$ are the means and standard deviations of the position estimates for the two stimulus tones. In the Molino experiment, $\hat{P}$ and $\hat{P}_{\rm ref}$ are the position estimates for the presented tone and a tone containing the interaural parameters of the " reference azimuth" toward which the subject pointed. The constant k is fitted to most closely describe the 0-μs data in the Yost *et al.* experiment and the 0°-azimuth data in the Molino experiment. Since the curves of Figs. 7 and 8 are plotted with the fitted parameter k, comparisons of predictions to data are discussed in terms of ratios of predicted z scores rather than the absolute scores themselves. This is done because predicted absolute sensitivity is dependent on such model parameters as the number of fiber pairs used and the criteria for obtaining a coincidence in interaural event times, which are either loosely or arbitrarily specified by the model.

The data and predictions of the Yost *et al.* (1975) ex-

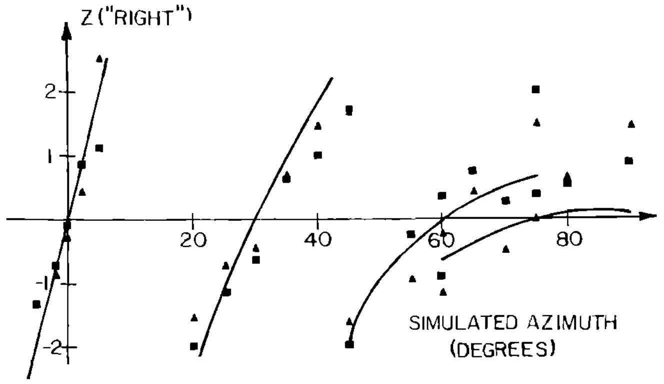

FIG. 8. Observed and predicted z scores for percentage of lateralization judgments toward the right side with respect to reference azimuths of 0°, 30°, 60°, and 75°. 500-Hz stimuli were presented through headphones with interaural time and intensity differences that simulated the free field conditions for a given azimuth. Data from Molino (1974) for Subjects N. L. (▲) and M. S. (■). Predictions of the model are indicated by the solid curves.

periment can be characterized by two major observations. First, the intensity difference producing the closest match in subjective lateral position to a given interaural time delay (which is indicated by the $z=0$ intercept for each curve) increases less rapidly as a function of time delay as the time delay is increased.[9] This can be expected from the predictions of Fig. 3(b) in that the slope of the 0-dB position curve decreases with increasing interaural time delay.[10] The second trend of Fig. 7 is that the variability of the position judgments (which is inversely related to the slope of the performance curves for a given time delay) increases as the time delay of the first tone increases. This trend is consistent with the increase in the standard deviation of $\hat{P}$ of the first tone (which is increasing in time delay) and the second tone (which is increasing in intensity difference) as seen in Fig. 5.

The data for the two subjects in the Molino (1974) experiment at 500 Hz are shown in Fig. 8 along with the corresponding theoretical predictions, for reference azimuths of 0°, 30°, 60°, and 75°. The predictions adequately describe the data, although less accurately for the 75° reference condition. Predicted and observed lateralization acuity (which is proportional to the slope of the performance curves for each reference azimuth) decrease as the reference azimuth increases from 0° to 90°. This trend in the predictions of the model occurs for at least two reasons. First, the combinations of interaural time and intensity differences of the presented stimuli approach the cue-reversal points on the position curves for azimuth angles between 60° and 90°, so there is little change in $E[\hat{P}]$ with changes in azimuth for these stimuli. (The time and intensity differences used in the experiment are tabulated in Molino, 1974. The simulated azimuths between 60° and 90° contain an interaural intensity difference of 4 dB and interaural time delays between 490 and 650 μs.) Second, the predicted standard deviation of $\hat{P}$ increases as the azimuth approaches 90° because the presented interaural time delays and intensity differences both increase in magnitude.

III. COMPARISONS WITH PREVIOUS MODELS

We now compare the position-variable model to two other models of binaural interaction that generate predictions of subjective lateral position, the lateralization model of Hafter (1971) and the crosscorrelation model of Sayers and Cherry (1957). The count-comparison model of van Bergeijk (1962) is not considered at this time because it is not sufficiently specified to enable a critical comparison to the data of Domnitz and Colburn (1977).

A. The lateralization model

The lateralization model (Hafter and Carrier, 1970; Hafter, 1971; Hafter and Carrier, 1972) relates interaural discrimination and binaural detection performance to changes in the lateralization of a binaural image. [Similar comparisons using the position-variable model have been made (Stern, 1976) and will be discussed in later publications.] The description of position used thus far in applying the lateralization model to the discrimination and detection data, however, correctly describes only a limited portion of the lateralization-matching results of Domnitz and Colburn (1977). In particular, Hafter and Carrier have assumed a linear dependence of predicted position on the interaural time and intensity of the stimulus, and a constant standard deviation of the position variable. These assumptions are consistent with the lateralization-matching data and the predictions of the position-variable model for tonal stimuli with small interaural differences (as seen in Figs. 3 and 5), but such a description cannot be applied to a more general class of stimuli without modification. Some of the important subjective phenomena that are only observed with larger interaural differences are the cue-reversal points in the lateralization-matching results, and the decreased sensitivity of subjective lateral position to interaural time delay for large intensity differences [Fig. 3(a)]. The results of the centering and laterality-comparison experiments discussed in this paper are also not predicted by the lateralization model using its current description of lateral position.

Hafter and Carrier (1972) have also suggested that the linear dependence of position on interaural time and intensity differences (for interaural differences of small magnitude) implies a peripheral intensity-to-time conversion at the site where motion of the basilar membrane is transduced to activity along the auditory nerve. We list three difficulties with this and all peripheral time-intensity trading hypotheses. First, the vertical as well as horizontal translation of the lateralization-matching curves in Fig. 3(a) with changes in the interaural intensity difference of the stimulus are inconsistent with a peripheral intensity-to-time conversion, as noted by Sayers and Toole (1964). Second, a peripheral trading hypothesis is incompatible with the differences in form of time and amplitude just noticeable differences (jnds) measured by Domnitz and Colburn (1977); in particular, very poor time-discrimination performance is observed for certain combinations of interaural time and intensity differences of tonal stimuli, while the amplitude-discrimination results exhibit much less variation with respect to the interaural differences of the stimuli. Finally, the phase lag of the synchronized response of auditory-nerve fibers to low-frequency tones relative to the stimulus phase does not appear to support a peripheral intensity-to-time conversion hypothesis. No significant level-dependent phase shifts have been found in auditory-nerve fibers that are most sensitive to the stimulus frequency for stimuli of moderate intensity (Anderson *et al.*, 1969; Kiang *et al.*, 1977). The phase shifts with level found in fibers tuned above and below this frequency are of opposite polarity, so any averaging of these two sets of fibers would tend to decrease the overall phase effect of changes of stimulus intensity. These phase measurements do not preclude the possibility of a simple conversion of intensity to time delay at a site more central than the auditory nerve, of course, but they are inconsistent with the specific threshold-based mechanism proposed by Hafter and Carrier (1972).

B. The crosscorrelation model

The only model of lateral position besides our model that can generate position predictions for tonal stimuli with all possible combinations of interaural time and intensity differences is the crosscorrelation model of Sayers and Cherry (1957). Interaural timing information is obtained by computing the running crosscorrelation function of the acoustical stimulus, added on each side to a number that is proportional to stimulus intensity. The crosscorrelation function is then weighted so that only small internal delays influence the results. Interaural intensity differences are introduced by multiplicatively weighting the crosscorrelation function separately for positive and negative values of the argument in proportion to the intensity of the stimuli in the left and right ears. The resulting weighted, running crosscorrelation function is then time averaged over the duration of the stimulus. Position judgments are made by integrating the time-averaged, weighted function separately over positive and negative values of the argument of the crosscorrelation function, and taking as an estimate of lateral position the ratio of the difference of these two integrals to their sum.

In both the crosscorrelation model and the position-variable model, interaural timing information is given by a modified crosscorrelation function, which is weighted to give greater emphasis to functional values at arguments of smaller magnitude. [In the position-variable model this weighting occurs because of the distribution of fiber-pair delay times, as specified by the function $p(\tau)$ in Fig. 2(b).] The two models differ in the way in which interaural intensity differences affect the display of interaural timing information, and in the algorithm for computing the position estimate from the resulting function. As stated above, the two halves of the "timing function" in the crosscorrelation model are multiplied by different constants (for positive and negative values of the argument τ) with magnitudes that depend on the interaural intensity difference of the stimulus, while the timing function of the position model is multiplied by a pulse-shaped weighting function whose location along the τ axis depends on the intensity difference. The position estimate of our model is obtained by computing the centroid of the position function $L_P(\tau)$, while the crosscorrelation model's measure of laterality is the difference divided by the sum of the areas of the two halves of the intensity-weighted crosscorrelation functions. The most important structural differences between the position model and the crosscorrelation model, therefore, are in the assumptions specifying transformations after the display of interaural timing information.

While Sayers and Cherry (1957) applied the crosscorrelation model only to experiments involving left-right sidedness judgments, predictions for the lateralization experiments of Sayers (1964) and Domnitz and Colburn (1977) are easily obtained. We have found (Stern, 1976, Chap. VII) that the position curves for 500-Hz tones predicted by the crosscorrelation model exhibit cue-reversal points occurring at ± 500-μs interaural time delay for all interaural intensity differences, in contrast to the systematic variation of cue-reversal points with intensity difference seen in the data of Sayers (1964) as well as Domnitz and Colburn (1977). The predictions of the position model, therefore, describe the data more accurately than those of the crosscorrelation model.[11]

In an effort to assess the relative significance of some of the similarities and differences of the crosscorrelation and position-variable models, we have investigated the predictions of a model that incorporates the intensity-weighting and position-computing mechanisms proposed by Sayers and Cherry (1957), but uses the timing function $L_T(\tau)$ of our model in place of the running crosscorrelation function (Stern, 1976, Chap. VII). Specifically, we multiplied each half of the position-variable model's timing function separately by constants related to the interaural intensity difference of the stimulus, integrated the products, and computed the ratio of their difference divided by their sum. We found that these position predictions also differ from the data of Sayers (1964) and Domnitz and Colburn (1977) in that the locations of the predicted cue-reversal points vary much less with intensity difference than the data. In addition, the slope of the lateralization-matching curves of this model for an equal-amplitude stimulus is greater at 0 μs than at 1000 μs, contrary to the data.

We conclude from these observations that the assumptions used to specify the intensity-weighting mechanism and the position-calculating algorithm of the position-variable model predict the lateralization-matching phenomena more accurately than those of Sayers and Cherry's crosscorrelation model.

IV. DISCUSSION AND SUMMARY

In this paper we have presented a model that obtains the random position variable $\hat{P}$ from simple operations on auditory-nerve activity, and we have compared its predictions to the results of various experimental studies. The model makes use of the description of auditory-nerve response and the display of interaural timing information presented in previous papers of this series (Colburn, 1973, 1977a). The essential new features of the position-variable model presented in this paper are the assumptions concerning the combination of interaural time and intensity information and computation of the position estimate from the resulting function. We believe that a peripheral intensity-to-time conversion (or time-intensity "trade") does not describe all available lateralization data, and have instead proposed a more central intensity-dependent weighting of the display of interaural timing information. The position estimate produced by the model is monotonically related to the center of mass of the resulting function.

The position-variable model describes the results of subjective lateralization experiments by Domnitz and Colburn (1977) and Sayers (1964) more accurately than the crosscorrelation model of Sayers and Cherry (1957), and over a wider range of stimulus parameters than the description used in evaluating the lateralization model (Hafter, 1971; Hafter and Carrier, 1972). All three models produce similar position predictions for interaural time and intensity differences of small magnitude. The position-variable model also predicts the major results of centering studies by Elpern and Naunton (1964) and Young (1976) and the laterality-comparison results of Molino (1974) and Yost *et al.* (1975) without further assumptions.

The only parameters of the position-variable model that are specifically fitted to subjective lateralization data are the width W_I of the intensity-weighting function, the function $M_I(\alpha)$ describing its location along the internal-delay axis as the interaural intensity difference of the stimulus is varied, and the range of characteristic frequencies over which computations are obtained. While we have assumed in the present paper that W_I is independent of interaural intensity difference, and that $M_I(\alpha)$ and W_I are independent of the frequency of the stimulus, the generality of these assumptions remains to be determined for other simple stimuli, such as tones of other frequencies, clicks, and broadband noise. We do not expect the relatively simple intensity-weighting function used in this paper to remain valid for more complex binaural stimuli with separately lateralizable components.

A monotonic transformation is implicit in converting the centroid of the position function [Eq. (2)] expressed in μs into the interaural intensity difference of a matching pointer tone in decibels. As we have noted, this transformation does not affect comparisons of the model to the lateralization-matching data of Domnitz and Colburn (1977). This transformation also has no effect on predictions of the position-variable model for interaural discrimination and binaural detection experiments.

In future papers we plan to describe the predictions of the model for interaural discrimination and binaural detection experiments, assuming that performance is dependent on changes in subjective lateral position of the binaural image and limited by the variability of the predicted position estimates.

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APPENDIX A: QUANTITATIVE DESCRIPTION OF THE POSITION-VARIABLE MODEL

In this Appendix we briefly present the expressions used to obtain the means and standard deviations for the predicted position estimates for 500-Hz tones.

A. Timing function

The detailed assumptions used to describe auditory-nerve activity are given in Colburn (1973). These assumptions are intended to describe the *informational content* of a typical fiber, rather than the detailed behavior of a particular unit. It is also worth noting that the specific constants in the auditory-nerve descriptions are selected to describe physiological data, and should therefore not be considered free parameters when applied to the psychoacoustical results discussed in the present paper.

We assume that a particular fiber is synchronized to a low-frequency stimulus tone if the tone is sufficiently intense and the separation between the characteristic frequency of the fiber and the frequency of the tone is sufficiently small, as specified in Colburn (1973) and modified in Colburn (1977b).[12] Given a 500-Hz tone stimulus $A\cos[2\pi 500(t-\tau)]$, we assume the rate function specifying the instantaneous probability of firing for synchronized fibers to be

$$r_m(t)=a\exp\{(20)^{1/2}\cos[2\pi 500(t-\tau)]\}\ , \tag{A1}$$

where the parameter a is chosen to make the time-averaged predicted firing rate for a synchronized fiber 200/s. Fibers that are not synchronized to the stimulus are referred to as firing spontaneously, and are assumed to exhibit rate functions $r_m(t)=50/\mathrm{s}$.

Since we assume that the auditory-nerve fibers are described by statistically independent Poisson processes, and that the rate functions describing these processes are of slow variation over the time interval (10 μs) within which events from each fiber of a pair must be observed to a record a coincidence, the expected number of coincidences recorded by the mth fiber pair is approximately given by

$$E[L_m]=T_W\int_0^{T_S} r_{L_m}(t-\tau_m)r_{R_m}(t)dt \tag{A2}$$

where T_W is the time interval for coincidence of the fiber pair,[13] T_S is the duration of the stimulus tone, and $r_{L_m}(t)$ and $r_{R_m}(t)$ are the rate functions associated with the two fibers of the pair.

For 500-Hz tones in the left and right ears described by

$$s_L(t)=(A/\alpha_s)\cos[2\pi 500(t-\tau_s)]$$

and

$$s_R(t)=A\cos(2\pi 500t),$$

respectively,[14] if T_S is an integral multiple of the 2-ms stimulus period, and if both fibers of a pair are synchronized to the stimulus, the expected number of coincidences observed by that fiber pair is given by

$$\begin{aligned}E[L_m(\tau_m)]&=a^2T_ST_W\\&\quad\times I_0((40)^{1/2}\{1+\cos[2\pi 500(\tau_s+\tau_m)]\}^{1/2})\\&\equiv \mathfrak{L}_2(\tau_m),\end{aligned} \tag{A3a}$$

where τ_m is the internal delay of the fiber pair and $I_0[x]$ is the tabulated zeroth-order modified Bessel function of the first kind. The identity

$$I_0[(x_1^2+x_2^2)^{1/2}]=\frac{1}{\pi}\int_0^{\pi}[\exp(x_1\cos\theta+x_2\sin\theta)]d\theta$$

TABLE I. Typical values of η_i, the fraction of fiber pairs with characteristic frequencies between 190 and 3350 Hz for which i fibers are synchronized to a 500-Hz binaural tone, as a function of $|\alpha|$, the magnitude of interaural intensity difference in dB at which the tone is presented. The more intense signal is assumed to be at 55 dB SPL, and consequently η_0 is equal to 0.635 in all cases.

$\|\alpha\|$	η_2	η_1
0	0.365	0.000
3	0.336	0.029
9	0.277	0.088
12	0.243	0.122
15	0.212	0.152
20	0.163	0.202
25	0.120	0.245

was used to obtain Eq. (A3a). If only one fiber of the pair is synchronized to the stimulus tone,

$$E[L_m] = T_S\, T_W(200)(50) \equiv \mathcal{L}_1, \tag{A3b}$$

and if neither fiber of the pair is synchronized to the stimulus,

$$E[L_m] = T_S\, T_W(50)(50) \equiv \mathcal{L}_0. \tag{A3c}$$

The fraction of fibers that are synchronized to a given 500-Hz binaural tone depends on the intensities of the signals to the two ears. Our values for these fractions are computed using the assumptions for auditory-nerve fiber thresholds and frequency characteristics stated in Colburn (1973, 1977b), and assuming that the more intense 500-Hz signal is presented at 55 dB SPL. Typical values of η_i, the fraction of fiber pairs between 190 and 3350 Hz with i fibers synchronized ($i = 0, 1,$ or 2) are given in Table I as a function of the magnitude of the interaural intensity difference of the stimulus.

The expected value of the timing function representing the total number of coincidences as a function of internal delay is given by

$$E[L_T(\tau)] = N p(\tau)\left[\eta_2 \mathcal{L}_2(\tau) + \eta_1 \mathcal{L}_1 + \eta_0 \mathcal{L}_0\right], \tag{A4}$$

where N equals 12 500, the total number of fiber pairs with characteristic frequencies between 190 and 3350 Hz. $p(\tau)$ is the probability density function specifying the distribution of internal delays, which may be found in Colburn [1977a, Eq. (3)] and is plotted in Fig. 2(b) of this paper.

We assume that the random process describing the number of coincidences observed by a fiber pair is also Poisson. This approximation will be satisfied if the time window over which coincidences are measured is sufficiently small that the probability of an event on one fiber in a pair coinciding with more than one event on the other is negligible. Under these conditions,

$$\text{Variance}[L_T(\tau)] = E[L_T(\tau)].$$

B. Position calculations

The mean and variance of the position estimate are obtained under the approximation

$$\hat{P} = \frac{\int_{-\infty}^{\infty} \tau L_P(\tau)\,d\tau}{\int_{-\infty}^{\infty} L_P(\tau)\,d\tau} \simeq \frac{\int_{-\infty}^{\infty} \tau L_P(\tau)\,d\tau}{E\left[\int_{-\infty}^{\infty} L_P(\tau)\,d\tau\right]} = \frac{\int_{-\infty}^{\infty} \tau L_I(\tau) L_T(\tau)\,d\tau}{\int_{-\infty}^{\infty} L_I(\tau) E[L_T(\tau)]\,d\tau}, \tag{A5}$$

where the intensity function $L_I(\tau)$ is given by

$$L_I(\tau) = \frac{1}{(2\pi)^{1/2} W_I} \exp\left(\frac{-[\tau - M_I(\alpha_s)]^2}{2W_I^2}\right) \tag{A6}$$

with $W_I = 1778$ μs and $M_I(\alpha_s)$ shown in Fig. 5. The approximation is reasonably accurate because

$$\text{Standard deviation}\left[\int_{-\infty}^{\infty} L_P(\tau)\,d\tau\right] \ll E\left[\int_{-\infty}^{\infty} L_P(\tau)\,d\tau\right]$$

for all tonal stimuli considered. If this approximation fails for other stimuli it can be incorporated into the model as an assumption.

The means and variance of the position estimate are easily obtained, using the expressions

$$E[\hat{P}] = \frac{\int_{-\infty}^{\infty} \tau E[L_P(\tau)]\,d\tau}{\int_{-\infty}^{\infty} E[L_P(\tau)]\,d\tau} \tag{A7}$$

and

$$\text{Variance}[\hat{P}] = \frac{\int_{-\infty}^{\infty} \tau^2 L_I(\tau)\, E[L_P(\tau)]\,d\tau}{\left\{\int_{-\infty}^{\infty} E[L_P(\tau)]\,d\tau\right\}^2}\,. \tag{A8}$$

The actual calculations were obtained by replacing the integrals by finite sums, e.g.,

$$E[P] = \frac{\sum_{i=-M}^{M} (i\tau_I)\, E[L_P(i\tau_I)]}{\sum_{i=-M}^{M} E[L_P(i\tau_I)]}\,. \tag{A9}$$

In general, the computations were obtained using a value of 50 μs for τ_I and 4000 μs for $M\tau_I$. It was found that further increases in $M\tau_I$ or decreases in τ_I do not affect interaural discrimination or binaural detection predictions.

APPENDIX B: BASES WITHIN THE MODEL FOR PREDICTED DEPENDENCIES

In this Appendix we briefly consider the bases within the model for some of the features of the predicted means and standard deviations of the position estimate $\hat{P}$ seen in Figs. 3(b) and 5. These predictions completely specify the model's performance for the subjective lateralization experiments shown in Figs. 6–8, as well as predicted performance in interaural discrimination and binaural detection experiments when it is assumed that $\hat{P}$ contains the only information available to the decision maker (Stern, 1976).

To the extent that the mean predicted position estimates shown in Fig. 3(b) may be approximated by sinusoidal functions of interaural time delay, they are characterized by the location of the cue-reversal points and the difference between maximum and minimum values of the curves for each intensity difference (corresponding to the phase-amplitude specification of a sine wave with fixed frequency). We discuss each of these aspects of the predictions separately.

The phenomenon of cue reversals and the change in location of the cue-reversal points as a function of intensity difference are easily understood in terms of the structure of the model. In estimating position, the relative number of coincidences as a function of time delay $L_m(\tau)$, which resembles the crosscorrelation of the presented binaural stimulus tone and is shown in Fig. 2(a), is weighted by the product of two pulse-shaped functions, the distribution of the internal delays of the fiber pairs $p(\tau)$, and the intensity function $L_I(\tau)$. A tone presented with zero intensity difference and a small interaural time delay will produce a position function that consists largely of a single pulse, centered about the interaural time delay of the stimulus. As the interaural time delay is increased, the primary lobe of the position function will move toward the right. As the time delay continues to increase, a secondary lobe will move in from the left and become increasingly more prominent, as seen in the timing function of Fig. 2(c). The cue-reversal point occurs approximately at the time delay for which the difference between the areas of the two position-function pulses (one growing on the left and one shrinking on the right) changes more rapidly than the rate at which both pulses are moving along the τ axis. The time delay at which the cue-reversal point occurs depends on the location of the product of $p(\tau)$ and $L_I(\tau)$ along the τ axis. As the interaural intensity difference of the stimulus increases, $L_I(\tau)$ moves toward the right, causing the mode of the product of $p(\tau)$ and $L_I(\tau)$ to also move toward the right, which in turn causes the cue-reversal point to occur at a more positive value of τ.

The difference between the maximum and minimum values of the predicted position curves for each intensity difference decreases as the intensity to one ear is decreased because the timing function includes a greater fraction of fiber pairs with at least one fiber firing spontaneously, as indicated in Sec. II.

The predicted standard deviation of the position estimate increases with increasing time delay and intensity difference of the stimulus, as seen in Fig. 5. This increase occurs because computation of the centroid requires a multiplication of the weighted coincidence-counter output by the factor τ. This causes the contribution of each fiber pair to be scaled by the factor τ in the mean and τ^2 in the variance of the position estimate, as specified in Eqs. (A7) and (A8). As the interaural intensity difference increases in magnitude the weighting function $L_I(\tau)$ preemphasizes the fiber pairs with large magnitudes of τ. The variance of the event-time coincidences of these units is then scaled by the relatively large τ^2, causing the standard deviation of the position estimate to increase. Similarly, the increase in standard deviation with increasing magnitude of time delay is seen because the primary mode of the position function occurs at larger values of internal delay τ, and is multiplied by τ^2 in computing the variance.

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[1] In Fig. 1 we show separate monaural processors as mechanisms for generating the intensity-based weighting function, but the theoretical predictions presented in this paper do not depend on the particular mechanism by which interaural intensity information is obtained.

[2] The curves in Fig. 2 are continuous-function approximates, since the coincidence counters of the model are finite in number, with discrete delays.

[3] In general, the range of characteristic frequencies over which we obtain our predictions affects the percentage of auditory-nerve fibers in the model that exhibit a synchronized response to a binaural tonal stimulus. The particular range of 190–3350 Hz for 500-Hz stimuli was chosen to be broad, so that the dependence of predicted position on interaural time delay would decrease as the magnitude of the interaural intensity difference of the stimulus increases (as seen in Fig. 3). A broader range of characteristic frequencies, however, would have significantly degraded predicted performance in interaural time-discrimination experiments (Stern, 1976). A different range of characteristic frequencies would be used to obtain predictions for stimuli at frequencies other than 500 Hz.

[4] We assume that the pulse-shaped intensity function $L_I(\tau)$ is Gaussian in shape. We have found that the particular shape assumed for $L_I(\tau)$ does not greatly affect the predictions of the model, although the exact choice of $M_I(\alpha_S)$ and W_I needed to fit the data in Fig. 3 depends on the specific shape selected.

[5] We refer to combinations of interaural time and intensity differences in which the more intense signal is leading in time as "reinforcing" because if the magnitude of the interaural time delay is less than about 300 μs (for 500-Hz tones), each interaural difference presented by itself would tend to move the perceptual image toward the same side of the head. Similarly, a "canceling" combination of the interaural parameters is one in which the more intense signal is lagging in time.

[6] We describe in this footnote our procedure for fitting M_I and W_I. In general, we arbitrarily selected a value of W_I, and chose $M_I(\alpha)$ such that combinations of interaural time and intensity that produced the same matching pointer intensity difference in the Domnitz and Colburn (1977) data were predicted by the model to exhibit the same mean position, and then repeated this procedure for several values of W_I. For example, we selected $M_I(3)$ by finding (in the data) the time delay τ_0 such that tones with interaural time and intensity differences $(0, 3)$ and $(\tau_0, 0)$ were matched by pointer tones of equal intensity difference, where (τ, α) denotes a stimulus interaural time delay of τ and interaural intensity difference of α. $M_I(3)$ was then chosen so that $\hat{P}(0,3)=\hat{P}(\tau_0,0)$. [($\hat{P}(\tau_0,0)$ was already known because $M_I(0)=0$, by symmetry considerations.] For other values of α_s, $M_I(\alpha_s)$ was similarly obtained by finding the time delay of the equal-amplitude tone that empirically matched the position of a pointer tone with intensity difference α_s, and choosing $M_I(\alpha_s)$ so that the model predicted identical expected values of $\hat{P}$ for these two tones. $M_I(\alpha)$ could be obtained in this fashion for values of α less than or equal to 9 dB in magnitude. For intensity differences of greater magnitude we chose $M_I(\alpha)$ such that combinations of interaural time and intensity differences $(0, \alpha)$ that caused lateral position matches identical to tones with the interaural parameters $(\tau_0, 9)$ gave rise to equal position estimates. This procedure was repeated for several values of W_I, and we selected the one that most closely described the overall dependence on interaural intensity difference of the predicted time delays for which the derivative of $\hat{P}$ with respect to time delay changes sign (the "cue reversal points"). This method of fitting $M_I(\alpha)$ made no specific assumptions about the nature of the monotonic relationship between $\hat{P}$ and the interaural intensity difference of the pointer tone in the matching experiment, but proved to be quite tedious to use. Furthermore, it was quickly noted that this method predicted position estimates that exhibited the relationship $\hat{P}(0,\alpha)\simeq k\alpha$ for all α of interest. In later revisions of the model we chose values of the function M_I such that $\hat{P}(0,\alpha)=k\alpha$ for all α, in the interest of expediency.

[7] The individual lateralization judgments of Domnitz and Colburn's (1977) subjects for 1000 μs time delay and 0-dB intensity difference were clustered near the midline, while those

of Sayers' (1964) subjects tended to exhibit a trimodal distribution. One possible reason for this discrepancy is that Domnitz and Colburn's subjects were instructed to average their judgments over successive presentations, while Sayers' subjects made their judgments on the basis of single stimulus presentation. The individual-trial results of these two studies may also differ because the stimuli presented in the Domnitz and Colburn experiment were monitored by microphones within the headphone cavity in order to detect and electrically compensate for run-to-run acoustic imbalances caused by variability in the fit of the headphone to the head, as described by Domnitz (1975). This procedure is especially important in comparing the results of these two studies because subjective lateral position is far more dependent on the precise interaural differences of a stimulus presented with time delay at or near 1000 μs than for delays near 0 μs.

The use of the lateralization results of Domnitz and Colburn, including the cue-reversal phenomenon, is also attractive because the results provide a natural explanation for the reversal in direction of the lateralization cue described by subjects in interaural time-discrimination experiments (Domnitz, 1973; Domnitz and Colburn, 1977) when the stimuli are presented at time delays near 1000 μs. We plan to compare the predictions of our model to these data (Stern, 1976) in a later publication.

[8] The second crossing of the 3-dB curve at -860 μs results from the reversal of lateralization observed at time delays near ±1000 μs and does not imply a second time-intensity trading ratio of 287 μs/dB. We consider the trading-ratio parameter to be meaningful primarily for the small interaural time and intensity differences for which the position curves of Fig. 3 may be approximated by parallel straight lines.

[9] The quantitative difference between the $z=0$ intercepts of the data and predictions in Fig. 7, are the results of intersubject differences in time-intensity trading ratios, and do not indicate a deficiency of the model. For example, the 200-μs data and predictions imply an average trading ratio of about 33 μs/dB for the subjects used by Yost *et al.* (1975). Our predictions describe the behavior of Domnitz and Colburn's (1977) subjects who, on the average, exhibit a trading ratio of less than 25 μs/dB.

[10] In discussing their discrimination results Yost *et al.* (1975) imply that, as the interaural time delay of a 500-Hz tone is increased from 0 to 1000 μs, the image of that tone will move steadily from the center of the head to one side. Unfortunately, this assertion is not tested by their own discrimination experiments because the time delays selected were all smaller in magnitude than that of the cue-reversal point for 0-dB tones. We feel that if this experiment were performed under carefully monitored conditions using time delays between 600 and 1000 μs the intensity difference giving rise to chance lateralization judgments would decrease with increasing time delay, consistent with the perceptural reversals in time and experiments described by Domnitz (1973) and Domnitz and Colburn (1977), and the trial-by-trial data from Domnitz and Colburn's lateralization-matching experiments.

[11] Voelcker (1961), using a different set of *a priori* considerations and a decision-theory approach, constructed a model that generates lateralization estimates that are also the sum of a term based on interaural intensity difference and a term representing the weighted crosscorrelation of the stimulus, which is the form of the lateralization predictions of the Sayers and Cherry model. Therefore, the predictions of this model differ from the data of Sayers (1964) and Domnitz and Colburn (1977) in the same ways.

[12] It was assumed in Colburn (1973) on the basis of reports in Kiang (1968) that the thresholds of each fiber are spaced uniformly over a range of 20 dB with the graph of minimum thresholds as a function of frequency similar to a threshold-of-hearing curve. The average rating of firing and the synchrony of the firings were described as continuous functions of intensity and showed increases over a range of about 20 dB. In Colburn (1977a, 1977b) and the present paper, we have simplified the description of the average rate and synchrony as functions of intensity to be either the unstimulated values (50/s and 0, respectively) or the maximum values (200/s and $(20)^{1/2}$ for 500-Hz tones) with the transition occurring at a level related to the threshold of each fiber. Since the transition is now abrupt, we assume that the individual fiber thresholds range over 40 dB at each characteristic frequency instead of 20 dB as before. In both cases the total average rate of firing from all fibers with a given characteristic frequency increases with intensity over a range of 40 dB. All intensity variations in the simplified model are now contained in the number of "stimulated" fibers. It is argued in Colburn (1977b) that these simplifications do not alter the rate and synchrony predictions for ensembles of fibers.

[13] The parameter T_W is equal to the expression $\int_{-\infty}^{\infty} f(x)\,dx$ in Colburn (1977b).

[14] Our notation for the interaural time and intensity differences of the stimulus tone is based on that adopted by Domnitz and Colburn (1977), but differs slightly from the notational conventions used in previous papers of this series. Specifically, our variables τ_s and α_s correspond to the variables T and A in Domnitz and Colburn (1977) and to $-\tau$, $-\tau_s$, or $-\phi_s/2\pi f_0$ and α in Colburn (1977a, 1977b) and Colburn and Latimer (1978).

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