

Lateralization and Detection of Low-Frequency Binaural Stimuli: Specification of the Extended Position-Variable Model

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ABSTRACT

This publication is a companion to a paper by Stern and Shear [R. M. Stern, Jr. and G. D. Shear, J. Acoust. Soc. Am. (1996, in press)] which extends the position-variable model to describe and predict binaural lateralization and detection phenomena at frequencies up to 1200 Hz. The most important modification made to the model is the development of a frequency-dependent form of a function referred to as $p(\tau|f_c)$ that describes the relative number of binaural coincidence detectors in the model as a function of their internal delay. The function $p(\tau|f_c)$ is fitted to describe the lateralization of pure tones with a fixed ITD over a range of frequencies, and to describe the ratio of N_0S_π to $N_\pi S_0$ binaural detection thresholds. In this publication we summarize the discussions leading up to the particular choice of the function $p(\tau|f_c)$ and other related parameters that are now part of the current formulation of the position-variable model. We also include in two appendices the complete set of equations that specify the position-variable model in its present form.

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INTRODUCTION

This publication is a companion to a paper by Stern and Shear (1996) that describes recent modifications to the position-variable model of binaural interaction (Colburn, 1973; Stern and Colburn, 1978, 1985). The modifications to the model extend the stimulus frequencies over which it can be applied to the range of 250 to 1200 Hz. The paper by Stern and Shear (1996) describes how the shape of $p(\tau|f_c)$, the function that specifies the distribution of internal delays in the model, affects the form of the model's lateralization and detection predictions. Stern and Shear (1996) describe several modifications to the model, including a new frequency-dependent form of $p(\tau|f_c)$, and they present and discuss several comparisons of the lateralization predictions of the revised model to the corresponding experimental data.

In this publication we summarize a series of discussions in Shear (1987) which describe the process by which the function $p(\tau|f_c)$ was modified to describe binaural lateralization and detection data over an extended range of frequencies. We also discuss some of the factors underlying the selection of ν , the order of the half-wave rectifier that is part of the model for auditory-nerve activity, and \mathbf{R}_{lat} and \mathbf{R}_{det} , the range of frequencies used to calculate predictions for lateralization and detection experiments. We include in two appendices the set of equations that completely specifies the revised model in its present form, so that interested researchers may develop similar predictions.¹

In modifying the position-variable model to extend its coverage beyond 500 Hz, our major goals have been twofold. First, we wanted to gain general insight into the dependence of the predictions on $p(\tau|f_c)$. Second, we sought to specify at least one set of the function $p(\tau|f_c)$ and the parameters ν , \mathbf{R}_{lat} , and \mathbf{R}_{det} that provides good agreement between predictions and two sets of experimental data: the lateralization of pure tones with a fixed interaural time delay (ITD) as a function of frequency (Schiano *et al.*, 1986), and the ratio of N_0S_π to $N_\pi S_0$ as summarized by Durlach and Colburn (1978).

We begin our discussion of modifications to the function $p(\tau|f_c)$ in Sec. I by describing a set of mathematical constraints that $p(\tau|f_c)$ must satisfy in order to describe the lateralization data of Schiano *et al.* (1986). Two sets of parametric definitions of $p(\tau|f_c)$ are proposed in Secs. II and III, and we examine how the values of the parameters must be chosen in order to comply with the constraints developed in Sec. I. This initial analysis of the impact of the choice of $p(\tau|f_c)$ used a value of ν that was arbitrarily set to 3, and we considered only those fiber pairs with CFs approximately equal to the target frequency in the lateralization and detection experiments. In Secs. IV and V we relax these constraints, and we briefly discuss the effect on the theoretical predictions of specific choices of the parameter ν and the frequency regions \mathbf{R}_{lat} and \mathbf{R}_{det} . Finally, the equations that characterize the position-variable model are summarized in the two Appendices.

¹A software package that implements the predictions of the model in Objective-C is also available from the first author.

I. LATERALIZATION CONSTRAINTS ON THE DISTRIBUTION OF FIBER PAIRS

Shear (1987) developed several constraints on the shape of $p(\tau|f_c)$ that are implied by the data of Schiano *et al.* (1986) which indicate that the lateral position of a pure tones of frequency f_0 is approximately a constant independent of frequency for values of f_0 less than 1200 Hz, and for ITDs less than $1/4f_0$. In this section we briefly summarize these constraints. The interested reader is encouraged to refer to Chapter 4 of Shear (1987) for a much more detailed discussion of these topics.

The first type of constraint on the nature of $p(\tau|f_c)$ is obtained if it is assumed that the binaural processor only uses those fiber pairs with CFs (nearly) equal to f_0 when computing the position estimate \hat{P} . For this choice of \mathbf{R}_{lat} , Shear (1987) demonstrated that any conditional distribution that can be expressed in the form

$$p(\tau|f_c) = [Cf_c] p_\theta(C\tau f_c), \quad \text{for } 300 \leq f_c \leq 1200 \text{ Hz}, \quad (1)$$

will produce a predicted lateral position for pure tones that is proportional to a fixed target ITD for all target frequencies, f_0 . In the above expression, C is the appropriate positive constant that causes $p_\theta(C\tau f_c)$ to be a valid probability density function, and $p_\theta(C\tau f_c)$ is an even pulse-shaped function of $C\tau f_c$ for a given value of f_c . This type of conditional density function is referred to as a *phase-based* distribution since it is simply a function of the product τf_c , which is dimensionally equivalent to phase. This function becomes "narrower" as f_c increases, consistent with intuition.

A more general constraint can be derived without any assumptions regarding the range of CFs considered. Specifically, Shear (1987) showed that the predicted lateralization of low-frequency tones will be directly proportional to their ITD (as in the trends of the data of Schiano *et al.*, 1986) if the following constraint on $p(\tau|f_c)$ is satisfied:

$$\frac{\partial \Phi_\tau(f|f_c)}{\partial f} \approx \frac{-C}{f}, \quad \text{for } 300 \leq f \leq 1200 \text{ Hz} \quad (2)$$

where $\Phi_\tau(f|f_c)$ is the Fourier transform (or characteristic function) of $p(\tau|f_c)$, and C is an arbitrary positive constant. This constraint implies that the characteristic function $\Phi_\tau(f|f_c)$ should be linearly related to the logarithm of f between 300 and 1200 Hz, which produces a weak dependence of the width of the function $p(\tau|f_c)$ on f_c . Functions that satisfy Eq. (2) are referred to as *log-based* distributions.

We now consider the lateralization and detection predictions produced by phase-based and log-based distributions.

II. LATERALIZATION AND DETECTION PREDICTIONS USING PHASE-BASED DISTRIBUTIONS

In order to demonstrate the ability (or inability) of phase-based distributions to describe both the lateralization and detection data of interest, we consider the frequency-dependent phased-based gaussian distribution $p_G(\tau|f_c)$

$$p_G(\tau|f_c) = \frac{1}{\sqrt{2\pi}\sigma(f_c)} \exp[-\tau^2/2\sigma^2(f_c)] \quad (3)$$

with

$$\sigma(f_c) \triangleq \begin{cases} C_\sigma/f_c^\gamma, & \text{for } f_c < 1200 \text{ Hz} \\ C_\sigma/1200^\gamma, & \text{for } f_c \geq 1200 \text{ Hz} \end{cases} \quad (4)$$

When γ equals one, the above distribution satisfies the "phase" constraint of Eq. (1). The parameter γ is introduced in order to examine how the model's predictions are affected by the rate at which $p(\tau|f_c)$ "narrows" as f_c increases.

We compared predictions of the position-variable model using $p_G(\tau|f_c)$ to the data of Schiano *et al.* (1986) with values of γ between 0.9 and 1.2, and $\sigma(f_c)$ defined according to Eq. (4). We found that a value of 1.2 for γ provides the best description of the actual lateralization data. As the value of γ is decreased to 0.9, the ability of the model to describe the data at frequencies below 1200 Hz diminishes. Predictions obtained using γ equal to 1.0 (the truly phase-based distribution) are in fact approximately constant for frequencies below 1000 Hz, but the actual observed lateral position varies somewhat with the frequency of the tone. Unfortunately, while values of γ less than 1.0 are required to provide a good description of the masking-level difference for binaural detection thresholds in the $N_\pi S_0$ versus $N_0 S_\pi$ configuration, values greater than 1.0 are necessary to provide a good description of the lateralization phenomena (Shear, 1987). In other words, it is not possible to specify a single density function of the form given by Eq. (3) that will describe the available psychophysical evidence for the two experiments considered. Similar observations were made for other phase-based forms of $p(\tau|f_c)$ such as a double-sided exponential with a "plateau", which resembles the shape of the original $p(\tau|f_c)$ specified by Colburn (1977).

Phase-based distributions for $p(\tau|f_c)$ have been part of other models of binaural interaction. For example, the model of Lindemann (1986) uses a phase-based distribution, but he has considered neither the lateralization nor the detection data discussed in Shear (1987) and Stern and Shear (1996). In light of our results, we believe that Lindemann's model would have difficulty describing at least some of these lateralization and detection data, and the inclusion into the Lindemann model of a log-based density function such as those described in Sec. III below may be necessary.

III. LATERALIZATION AND DETECTION PREDICTIONS USING LOG-BASED DISTRIBUTIONS

We now consider log-based functions, and specifically the problem of finding a function $p(\tau|f_c)$ that has a Fourier transform that exhibits the properties described by Eq. (2) over the range of frequencies of interest. Several such characteristic functions exist. The function $\Phi_L(f)$ below was selected because it has a corresponding density function $p_L(\tau)$ that is analytic in τ .

$$\Phi_L(f) = \frac{1}{2 \ln(k_h/k_l)} \ln[(f^2 + k_h^2)/(f^2 + k_l^2)] \quad (5)$$

The density function $p_L(\tau)$, the inverse transform of $\Phi_L(f)$ is

$$p_L(\tau) = \frac{1}{2 \ln(k_h/k_l)} \frac{e^{-2\pi k_l |\tau|} - e^{-2\pi k_h |\tau|}}{|\tau|} \quad (6)$$

where k_l and k_h are constants which can be selected in order to best describe the lateralization and detection data. To best fit the data while satisfying Eq. (2), k_l should generally be less than 200 sec^{-1} and k_h should generally be greater than 1200 sec^{-1} .

We found empirically that using Eq. (6) for the function $p(\tau|f_c)$, the parameter values of $k_l = 50 \text{ sec}^{-1}$ and $k_h = 4000 \text{ sec}^{-1}$ jointly minimized the discrepancies between predictions and data for both sets of data considered (Shear, 1987). We found that for frequencies below approximately 1200 Hz there is fairly good correspondence between the model's predictions and the lateralization results of Schiano *et al.* (1986). However, Shear (1987) also found that $p_L(\tau)$ had too many fiber pairs with internal delays near zero to properly describe the detection data.

In order to provide a better fit to the detection data without adversely affecting the ability of the model to describe the lateralization data, two additional modifications were implemented: (1) the function $p_L(\tau)$ was "clipped" so that it is constant for small $|\tau|$, and (2) the parameters k_l and k_h were allowed to depend on CF. The resulting modified density function defined below, called $p_{LF}(\tau|f_c)$, was found to best describe both the detection and lateralization data.

$$p_{LF}(\tau|f_c) = \begin{cases} C_{LF}(f_c), & \text{for } |\tau| \leq 200 \mu\text{s} \\ C_{LF}(f_c) (e^{-2\pi k_l(f_c)|\tau|} - e^{-2\pi k_h|\tau|}) / |\tau|, & \text{otherwise} \end{cases} \quad (7)$$

where

$$k_l(f_c) = \begin{cases} 0.1 f_c^{1.1}, & \text{for } f_c \leq 1200 \text{ Hz} \\ 0.1 (1200)^{1.1}, & \text{for } f_c > 1200 \text{ Hz} \end{cases}, \quad (8)$$

The parameter k_h in Eqs. (7) and (8) is set equal to 3000 sec^{-1} , and $C_{LF}(f_c)$ is chosen so that $p_{LF}(\tau|f_c)$ is a valid density function. This is the function used to produce the predictions described in Stern and Shear (1996) and it is sketched in Fig. 5 of that paper. Although $k_l(f_c)$ is nearly proportional to f_c for frequencies less than 1200 Hz, $p_{LF}(\tau|f_c)$ is not a phase-based distribution because k_h is independent of CF and because $p_{LF}(\tau|f_c)$ is constant for $|\tau|$ less than 200 μsec .

In developing detection predictions using the function $p_{LF}(\tau|f_c)$, we originally considered two types of decision statistics for reasons described by Shear (1987). The first statistic, referred to as \hat{Q}_o , considers the optimal weighting (for a particular stimulus configuration and target-to-masker ratio) of the outputs of coincidence-counting units over a range of CFs. The second statistic, called \hat{Q}_c , develops binaural predictions by summing the outputs of coincidence-counting units over a range of CFs that is centered at the target frequency. These decision statistics are formally defined in Appendix A. The detection predictions described in Fig. 6 of Stern and Shear (1996) were obtained using \hat{Q}_o . Predictions using both \hat{Q}_o and \hat{Q}_c are included in Shear (1987).

For both types of density functions considered [$p_G(\tau|f_c)$ and $p_{LF}(\tau|f_c)$] lateralization predictions for tones above 1200 Hz generally indicate a larger displacement of the binaural image from center than do the data. While it is possible to develop (fairly elaborate) forms of $p(\tau|f_c)$ that can more accurately describe the sharp shift in image position above 1200 Hz, we feel that a more plausible cause for this discrepancy between predictions and data is the lowpass filter function of the auditory-nerve model, $G(f)$. We have found that lateralization predictions for tones at high frequencies are strongly dependent on the shape of this function, which was specified to describe the loss of synchrony exhibited by nerve fibers in cats. Since the human auditory system is sensitive to a smaller range of frequencies than the auditory system of cats, it is reasonable to assume that the shape of $G(f)$ in humans and/or its stopband might also be more compressed with respect to frequency. Such a lowpass filter function (with a sharper stop band) would better describe the lateralization data at frequencies above 1300 Hz. While we believe that these observations warrant further consideration of how the lowpass filter should be specified, we do not attempt such an investigation in this work. It is sufficient to note that the current model, at the very least, predicts the general trends of the data at high frequencies.

IV. DEPENDENCE OF THEORETICAL PREDICTIONS ON THE ORDER OF THE RECTIFIER

We summarize in this section some observations concerning the ways in which predictions of the extended position-variable model are affected by the model parameter ν , the assumed power of the half-wave rectifier in the model for auditory-nerve activity. We include here only the major results of these studies, and the reader is referred to Shear (1987) for further details.

We had found that the ν^{th} -law half-wave rectifier of the auditory-nerve model should be of order 1, 2, or 3 to best describe the maximum synchronization index auditory-nerve fibers responding to low frequency tones, as measured by Johnson (1980). For the lateralization experiment of Schiano *et al.* (1986), values of ν between 1 and 3 all provide a reasonably accurate description of the data. Since it is generally accepted that the peripheral transformation is somewhat expansive (*cf.* Kiang *et al.*, 1965; Kiang, 1968; Johnson, 1980), we reject the use of a half-wave linear rectifier. Discrepancies between predictions and data can become significant for values of ν greater than 5.

The predicted difference between $N_\pi S_0$ and $N_0 S_\pi$ detection thresholds is more sensitive than the predictions for the lateralization experiment to the specific value chosen for the rectifier power ν , and the nature of these dependencies is affected by the choice of decision statistic, \hat{Q}_o versus \hat{Q}_c . For the optimal detection variable \hat{Q}_o , we found that an increase in the rectifier power results in an increase in the predicted difference between the $N_\pi S_0$ and $N_0 S_\pi$ detection thresholds (especially at low frequencies). This is caused by the fact that for small values of ν , the differences which occur in the "valleys" of the interaural cross-correlation functions are almost as significant as the differences which occur at the peaks. On the other hand, larger values of ν cause the peaks to dominate the detection process, causing predicted $N_0 S_\pi$ performance to improve relative to $N_\pi S_0$ performance. If the simpler decision statistic \hat{Q}_c is used for the predictions, an increase in the rectifier power results in a *decrease* in the predicted difference between the $N_\pi S_0$ and $N_0 S_\pi$ detection thresholds. It is not obvious at present why \hat{Q}_c should behave

oppositely to \hat{Q}_o in this respect. For most of the density functions examined, setting v equal to 3 yielded similar predictions using either \hat{Q}_c or \hat{Q}_o .

Since we are not aware of any evidence that suggests which definition of \hat{Q} should be adopted, it is convenient to designate a value of v such as 3 that yields similar predictions for both \hat{Q}_o and \hat{Q}_c [although an arbitrary decision was made to use \hat{Q}_o in the detection predictions of Stern and Shear (1996)]. This value also provides reasonably accurate predictions for the data of Schiano *et al.* (1986), and it is consistent with the physiological data of Johnson (1980).

V. DEPENDENCE OF THEORETICAL PREDICTIONS ON THE RANGE OF CHARACTERISTIC FREQUENCIES

In this section we describe how the major results considered depend on \mathbf{R}_{det} or \mathbf{R}_{lat} , the range of CFs over which predictions are evaluated for lateralization or detection experiments. Again we include here only the major results of these studies, and we refer the reader to Shear (1987) for further details.

It was found in general that for definitions of $p(\tau|f_c)$ that exhibit only a moderate dependence on CF (such as the log-based distributions), predictions for the lateralization data of Schiano *et al.* (1986) are fairly insensitive to the specification of \mathbf{R}_{lat} . However, predictions obtained using distributions that are strongly dependent on CF (such as the phase-based distributions) can be significantly affected by the range of CFs considered, and the predictions described the lateralization data only when \mathbf{R}_{lat} consists only of a narrow range of CFs about the frequency of the target tone. This is of little concern to us because the distribution that was eventually adopted for subsequent theoretical predictions, $p_{LF}(\tau|f_c)$, is log-based. In Stern and Shear (1996) we assume that only fiber pairs receiving inputs from "active" auditory-nerve fibers from each of the two ears are used by the lateralization mechanism. This is reasonable since little or no timing information can be extracted from other regions of the correlation display.

Unlike \mathbf{R}_{lat} , \mathbf{R}_{det} is well specified by the defining assumptions of the model if we assume that this range should be chosen to provide optimal performance for the given task. If the optimal decision statistic \hat{Q}_o is used, this range theoretically includes all outputs of the binaural displayer since the central processor simply ignores those units which provide no useful information. However, we have found that only those fiber pairs with CFs within ± 75 Hz of the target frequency contribute significantly to improving detection performance, since peripheral filtering causes the effective target-to-masker ratio to decrease sharply for fiber pairs with CFs not close to the target frequency. Similar observations have been made regarding the optimal range for \hat{Q}_c , with one exception: performance actually begins to degrade if \mathbf{R}_{det} is made too broad. This is due to the fact that all outputs of the binaural displayer are weighed equally in the formation of \hat{Q}_c and that only those units with CFs near the target frequency are actually useful.

We have also found that detection threshold computations can be expedited by considering only CFs that are nearly equal to the target frequency. Relative detection threshold predictions obtained in this fashion differ by only a fraction of one dB from predictions obtained using \mathbf{R}_{det} , the optimal range of frequencies. We consider this approximation to be acceptable because this discrepancy is much less than the measured standard deviation of the data (~ 2 dB).

VI. SUMMARY

In this publication we described several modifications to the position-variable model (Stern and Colburn, 1978) that enable it to describe binaural lateralization and detection phenomena over a much wider range of frequencies. The most important of these modifications concerned the function $p(\tau|f_c)$ which describes the assumed distribution of internal delays in the model. We showed that the function $p(\tau|f_c)$ must be carefully chosen to enable the model to describe both the lateralization data of Schiano *et al.* (1986) and the observed ratio of N_0S_π to $N_\pi S_0$ binaural detection thresholds. We then introduced two types of classes of specifications for $p(\tau|f_c)$ that could describe the lateralization data. The first class of distribution is "phase-based" in that $p(\tau|f_c)$ becomes narrower as f_c increases, and in effect is a function of internal phase difference rather than internal time delay. The second form of $p(\tau|f_c)$ is called "log-based" because it is derived from a constraint on the log of the Fourier transform of $p(\tau)$.

Using phase-based distributions, we found that although satisfactory descriptions of the lateralization data could be obtained, predictions for the ratios of $N_\pi S_0$ vs. $N_0 S_\pi$ detection thresholds did not exhibit as strong a dependence on target frequency as is observed in the data. Attempts to improve predictions for these detection data degraded the ability of the model to describe the lateralization phenomena.

On the other hand, we were able to specify the log-based distribution $p_{LF}(\tau|f_c)$ that allows the model to describe both the lateralization and detection phenomena. This function is similar in form to the original function proposed by Colburn (1977) except that (1) the tails of $p_{LF}(\tau|f_c)$ decay more rapidly with respect to τ than the tails of the original function, and (2) the rate at which these tails decay is dependent on CF.

The experimental data considered do not provide a strong indication of the order of the half-wave rectifier that is most likely to be "correct". Of the values considered, we prefer using a half-cubic rectifier (*i.e.* $v=3$), but this is mainly an issue of convenience.

Using $p_{LF}(\tau|f_c)$, predictions of the extended position-variable model for the lateralization of tones are fairly insensitive to \mathbf{R}_{lat} , the range of CFs considered by the central processor. Predictions for the relative detection threshold data considered are also somewhat insensitive to the range of CFs considered. In particular, predictions obtained considering only fibers for which the CF is approximately equal to the target frequency are almost identical to predictions obtained using the optimal combination of CFs.

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Appendix A

SUMMARY OF THE MODIFIED POSITION-VARIABLE MODEL

We provide in this Appendix a brief summary of the equations specifying the position-variable model used for the predictions described in this paper, which is a modification of the model described by Colburn (1973, 1977) and Stern and Colburn (1978). A more detailed description of the modified model may be found in Shear (1987). We first describe the model for auditory-nerve activity. The model for central binaural processing is described in the second section of this Appendix, and a summary of the methods by which lateralization and detection predictions are obtained from the model is included in final two sections.

A. The Model of Auditory-Nerve activity

Following the work of Colburn (1973, 1977) and others, we describe the firing patterns of individual auditory-nerve fibers as sample functions from nonhomogeneous Poisson processes (Parzen, 1962). Each auditory nerve is assumed to be composed of 3×10^4 fibers, each characterized by a pair of numbers, the characteristic frequency f_{cm} , and the sensitivity constant K_m . Characteristic frequencies are spaced uniformly on a logarithmic scale between 20 Hz and 20 kHz and sensitivity constants of fibers with characteristic frequency f_{cm} are spaced uniformly on a logarithmic scale over a range of 40 dB so that the curve described by the minimum values of K_m as a function of f_{cm} has the same shape as the threshold of hearing curve for tones (Kiang *et al.* 1965, p. 89). Theoretical predictions were obtained using a piecewise approximation $\zeta(f_{cm})$ to the threshold-of-hearing curve given by

$$\begin{aligned} \zeta(f_{cm}) &= 4.5 + 44.846 \log(500/f_{cm}), & \text{for } f_{cm} < 500 \text{ Hz} \\ \zeta(f_{cm}) &= 4.5 - 14.9847 \log(f_{cm}/500), & \text{for } 500 \leq f_{cm} < 1000 \text{ Hz} \\ \zeta(f_{cm}) &= 0, & \text{for } 1000 \leq f_{cm} < 2500 \text{ Hz} \\ \zeta(f_{cm}) &= 28.7044 \log(f_{cm}/2500), & \text{for } f_{cm} \geq 2500 \text{ Hz} \end{aligned} \quad (\text{A.1})$$

where f_{cm} is in Hz and $\zeta(f_{cm})$ is in dB SPL. This function is sketched in Fig. A-1.

With each characteristic frequency f_{cm} , there is associated a filter with impulse response $h_m(t)$. We specify $h_m(t)$ through $H_m(f)$, the magnitude of the corresponding frequency response, and $\theta_m(f)$, the minimum-phase characteristic consistent with $H_m(f)$. As in Colburn (1973) and Stern and Colburn (1978), $H_m(f)$ is given by

$$H_m(f) = \begin{cases} (f/f_{cm})^{\alpha(f_{cm})}, & \text{for } 0 \leq f \leq f_{cm} \\ (f_{cm}/f)^{2\alpha(f_{cm})}, & \text{for } f > f_{cm} \end{cases} \quad (\text{A.2})$$

where $\alpha(f_{cm})$ is specified by the equation

$$\alpha(f_{cm}) = \begin{cases} 4, & \text{for } 0 \leq f_{cm} \leq 800 \text{ Hz} \\ 4(f_{cm}/800), & \text{for } f_{cm} > 800 \text{ Hz} \end{cases} \quad (\text{A.3})$$

Intensity functions $r_m(t)$ for an arbitrary stimulus $x(t)$ are specified by the relations

$$r_m(t) = \begin{cases} a_m R_v [h_m(t) * x(t)] * g(t), & \text{for } \mathbf{MS}[h_m(t) * x(t)] > K_m \\ 50, & \text{otherwise} \end{cases} \quad (\text{A.4})$$

where \mathbf{MS} is a short-time mean-square operator, a_m is chosen so that the time-average of $r_m(t)$ is 200 per second. In the expression above, $R_v[z]$ is the v^{th} -law half-wave rectifier defined by Eq. (1) of Stern and Colburn (1992), and the impulse response $g(t)$ is the inverse Fourier transform of a low-pass filter which has a frequency response with magnitude $G(f)$ described by Eq. (2) of that paper.

Most of the assumptions summarized in this Appendix were also used in developing predictions for the model that were described in previous papers (e.g. Stern and Colburn, 1978, 1985).

B. The Model of Binaural Processing

The model for binaural processing features a display of binaural information containing a network of units that, effectively, estimates the interaural cross-correlation function of binaural stimuli after peripheral frequency analysis. Specifically, each unit is assumed to record coincidences in firing times (within 10 μs) from auditory-nerve units of comparable characteristic frequency from the two ears, after a small fixed internal interaural time delay τ_m . The values of τ_m are distributed over all fiber pairs independently of all parameters other than characteristic frequency, and their distribution is specified by the conditional density function $p(\tau|f_c)$, which is an even, pulse-shaped function of τ for a given value of f_c . The derivation of an appropriate function $p(\tau|f_c)$ is discussed in Sec. I of this publication. For a particular stimulus the expression $L_m(\tau_m, f_{c_m})$ refers to the number of coincidences observed by the m^{th} fiber pair with internal delay τ_m and characteristic frequency f_{c_m} .

Following Colburn (1973), the variance of the displayer output L_m is assumed to be

$$\text{Var}[L_m] = E\{ \text{Var}[L_m|x(t)] \}, \quad (\text{A.5})$$

where $\text{Var}[L_m|x(t)]$ denotes the conditional variance of L_m given the stimulus waveforms $x_R(t)$ and $x_L(t)$. In other words, the variance in the coincidence counts is assumed to be dominated by the contribution of the Poisson process that models the auditory-nerve activity, and it is assumed that the contribution of the variability of the stimulus to $\text{Var}[L_m]$ can be ignored. While we regard this assumption to be adequate for the present calculations, experimental data by Siegel and Colburn (1983) indicate that the variance due to the stimulus can play a role in determining overall performance in binaural detection experiments.

If the stimulus is assumed to be of duration T_S and the final expectation is taken over the binaural inputs $x(t)$, we obtain

$$E[L_m] = E[\text{Var}\{ L_m|x(t) \}] \approx T_w \int_0^{T_S} E[r_{Lm}(z - \tau_m) r_{Rm}(z)] dz \equiv T_w T_S R_{RLm}(\tau_m) \quad (\text{A.6})$$

where $R_{RLm}(\tau_m)$ is the time-averaged (or ensemble-averaged) interaural crosscorrelation function of the deterministic (or stochastic) stimulus after it is passed through the band-pass filter, nonlinear rectifier, and lowpass filter of the model for auditory-nerve activity. The expectations in the above expressions are

taken with respect to the neural point processes.²

If the fiber pair is "doubly active" (*i.e.* each fiber is firing at a rate above the spontaneous rate), the form of $R_{RLm}(\tau_m)$ will depend on the type of stimulus employed, as well as on the characteristic frequency f_{cm} and interaural delay τ_m . In Appendix B, we derive expressions for this correlation function when the binaural input is a pure tone, a Gaussian noise, the sum of a tone and noise, and amplitude-modulated tones.

C. Estimates of Lateral Position

The predicted position estimate of the position-variable model is the centroid along the internal-delay axis of the number of coincidence counts, L_m

$$\hat{P} \approx \frac{\sum_{m \in \mathbf{Z}_{lat}} \tau_m L_m}{\sum_{m \in \mathbf{Z}_{lat}} L_m} \quad (\text{A.7})$$

where \mathbf{Z}_{lat} is defined to be the indices for the set of fiber pairs that are used for lateralization calculations.³

In order to compute the expected value of \hat{P} , we make the following approximation,

$$\hat{P} \approx \frac{\sum_{m \in \mathbf{Z}_{lat}} \tau_m L_m}{E\left[\sum_{m \in \mathbf{Z}_{lat}} L_m\right]} \quad (\text{A.8})$$

which is justified by the observation that the standard deviation of the term in the denominator of Eq. (A.8) is much smaller than its mean for the stimuli considered. Thus, the mean of the position variable is given by

$$E[\hat{P}] \approx \frac{E\left[\sum_{m \in \mathbf{Z}_{lat}} \tau_m L_m\right]}{E\left[\sum_{m \in \mathbf{Z}_{lat}} L_m\right]} = \frac{\sum_{m \in \mathbf{Z}_{lat}} \tau_m E[L_m]}{\sum_{m \in \mathbf{Z}_{lat}} E[L_m]} \quad (\text{A.9})$$

In the above expressions, it is assumed that we are using some particular realization of the binaural displayer with fixed, known values of the characterizing parameters f_{cm} , τ_m , and the sensitivity constants

²The above expressions for the mean and variance of L_m should actually be considered accurate only to within some arbitrary multiplicative constant. This is due to the dependence of these expressions on the specific assumptions regarding various properties of the coincidence counters including the shape of the coincidence window. While this degree of inaccuracy does not affect most calculations of interest, it does prevent us from generating meaningful predictions for absolute detection thresholds.

³The original position-variable model (Stern and Colburn, 1978) included an additional weighting function for characterizing the effects of the interaural intensity difference (IID) of the stimulus. The present paper is concerned only with stimuli with zero IID, and the effects of the intensity-weighting function are ignored at present. As discussed in Stern and Shear (1996), the intensity-weighting function will be re-incorporated into the model when the model is extended to describe broadband stimuli presented with IIDs.

for the left and right fibers, K_{Lm} and K_{Rm} . In actuality we do not know their values, and it is necessary to compute the expectation over these parameters. Thus, using continuous approximations to the summations, and using the functions $p(f_c)$ and $p(\tau|f_c)$ to indicate the distributions of CFs and internal delays, the mean of the position estimate can also be given by

$$E[\hat{P}] = \frac{\int_{\mathbf{R}_{lat}} p(f_c) \int_{-\infty}^{\infty} \tau L(\tau, f_c) p(\tau|f_c) d\tau df_c}{\int_{\mathbf{R}_{lat}} p(f_c) \int_{-\infty}^{\infty} L(\tau, f_c) p(\tau|f_c) d\tau df_c}, \quad (\text{A.10})$$

with \mathbf{R}_{lat} defined to be the range of CFs over which lateralization predictions are calculated, and

$$L(\tau, f_c) \stackrel{\Delta}{=} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[L_m | \tau_m = \tau, f_{cm} = f_c, K_{Lm} = K_L, K_{Rm} = K_R] p(k_L|f_c) p(k_R|f_c) dK_L dK_R \quad (\text{A.11})$$

As defined above, $p(f_c)$ is uniformly distributed with respect to log frequency over a range of 20 Hz to 20 kHz, and $p(k_L|f_c)$ and $p(k_R|f_c)$ are uniformly distributed with respect to intensity in dB over a range of 40 dB.

The remaining expectation is computed with respect to the stimulus and the auditory-nerve model, and it is computed as in Eq. (A.6). The above expression can be simplified by noting that the sensitivity constants only determine whether or not a fiber is active. There are three distinct cases to be considered: both fibers active, only one active, and neither active. Thus, $L(\tau, f_c)$ can be expressed as,

$$L(\tau, f_c) = \eta_2(f_c) L_2(\tau, f_c) + \eta_1(f_c) L_1 + \eta_0(f_c) L_0 \quad (\text{A.12})$$

where

$$L_i(\tau, f_c) \stackrel{\Delta}{=} E[L_m | \tau_m = \tau, f_{cm} = f_c, \text{" } i \text{ fibers are active" }] \text{ for } i=0, 1, 2 \quad (\text{A.13})$$

and $\eta_i(f_c)$ is defined to be the fraction of fiber pairs with characteristic frequency f_c that have i active fibers ($i=0, 1, 2$) for a particular stimulus.

If either fiber in a pair is firing spontaneously, the two intensity functions are statistically independent and the interaural crosscorrelation $R_{RLm}(\tau_m)$ is simply the product of the two mean firing rates. Thus, $L_1 = T_w T_S(50)(200)$ and $L_0 = T_w T_S(50)(50)$, independently of the stimulus properties (and the characteristics of the fiber pair). On the other hand, the function $L_2(\tau, f_c)$ does depend on the type of input involved.

D. Calculation of Detection-Threshold Predictions

Predictions for binaural detection thresholds are based on the values of one of two decision statistics: an "optimal" decision statistic \hat{Q}_o , and \hat{Q}_c a "constant" decision statistic. Specifically, we define \hat{Q}_o to be

$$\hat{Q}_o = \sum_{m \in \mathbf{Z}_{task}} c_m L_m \quad (\text{A.14})$$

where each coefficient c_m is chosen to yield optimal performance in the detection task, and its value may depend on the characteristics of the m^{th} fiber pair and the stimulus.

\hat{Q}_c is simply the sum of all coincidence counts from the fibers considered,

$$\hat{Q}_c = C \sum_{m \in \mathbf{Z}_{task}} L_m \quad (\text{A.15})$$

Again we assume that all fibers of a given CF are either used or not used in calculating the decision statistics, so the set of indices \mathbf{Z}_{task} may be replaced by a set of characteristic frequencies \mathbf{R}_{task} such that

$$\mathbf{Z}_{task} = \{ m \mid m = 1 \text{ to } M \text{ and } f_{cm} \in \mathbf{R}_{task} \},$$

In order to calculate predicted performance for detection experiments, we assume that a symmetric, two-interval, two-alternative-forced-choice (2I-2AFC) paradigm is used. Detection threshold is achieved when the decision statistic \hat{Q}_o or \hat{Q}_c is reduced by more than its intrinsic standard deviation as the target is added to the masker. Specifically, the detection threshold is defined to be the target-to-masker ratio for which the performance index Q_d has unit value, where Q_d is defined by

$$Q_d = \frac{(E[\hat{Q} \mid \text{Target Plus Masker}] - E[\hat{Q} \mid \text{Masker Alone}])^2}{\text{Var}\{\hat{Q}\}}, \quad (\text{A.16})$$

where \hat{Q} equals either \hat{Q}_o or \hat{Q}_c .

For the optimal statistic \hat{Q}_o it is argued in Shear (1987) that

$$Q_d = \int_{\mathbf{R}_{det}} \int_{-\infty}^{\infty} \frac{(L_2(\tau, f_c \mid \text{Target Plus Masker}) - L_2(\tau, f_c \mid \text{Masker Alone}))^2}{\sqrt{L_2(\tau, f_c \mid \text{Target Plus Masker}) L_2(\tau, f_c \mid \text{Masker Alone})}} p(\tau, f_c) d\tau df_c \quad (\text{A.17})$$

where $L_2(\tau, f_c \mid \text{Target Plus Masker})$ is the mean of L_m given (1) that the m^{th} fiber pair is doubly active, (2) that the target is presented as well as the masker, (3) that $\tau_m = \tau$, and (4) that $f_{cm} = f_c$. $L_2(\tau, f_c \mid \text{Masker Alone})$ is similarly defined.

For the "constant" statistic \hat{Q}_c it is argued in Shear (1987) that

$$Q_d = \frac{\left[\int_{\mathbf{R}_{det}} \int_{-\infty}^{\infty} L_2(\tau, f_c \mid \text{Target Plus Masker}) p(\tau, f_c) d\tau df_c - \int_{\mathbf{R}_{det}} \int_{-\infty}^{\infty} L_2(\tau, f_c \mid \text{Masker Alone}) p(\tau, f_c) d\tau df_c \right]^2}{\sqrt{\int_{\mathbf{R}_{det}} \int_{-\infty}^{\infty} L_2(\tau, f_c \mid \text{Target Plus Masker}) p(\tau, f_c) d\tau df_c} \sqrt{\int_{\mathbf{R}_{det}} \int_{-\infty}^{\infty} L_2(\tau, f_c \mid \text{Masker Alone}) p(\tau, f_c) d\tau df_c}} \quad (\text{A.18})$$

where $L_2(\tau, f_c \mid \text{Target Plus Masker})$ and $L_2(\tau, f_c \mid \text{Masker Alone})$ are the conditional means of L_m , as defined in Equation (A.11), but further conditioned on whether the target is present or absent in the stimulus presentation.

Appendix B

Response of the Binaural Displayer to Tones, Noise, Combinations of Tones and Noise, and Amplitude-Modulated Tones

In this appendix we present expressions for the interaural correlation function $L_2(\tau, f_c)$ when the input stimulus is an additive combination of a tone and Gaussian noise.

A. Response to a Tone with Additive Noise

Consider the binaural stimulus given by

$$\begin{aligned} x_L(t) &= n_L(t) + A \cos 2\pi f_0 t \\ x_R(t) &= n_R(t) + A \cos 2\pi f_0 (t - \tau_s), \end{aligned} \tag{B.1}$$

where the noise process $n_L(t)$ has the one-sided spectral density function $N(f)$. The noise component to the right ear $n_R(t)$ is obtained by time-delaying and phase-shifting $n_L(t)$ by τ_n and ϕ_n , respectively.

When considering the response of the m^{th} fiber pair to this input, it is helpful to define the following quantities.

$$\begin{aligned} N_m(f) &= H_m^2(f) N(f) \\ \sigma_{nm}^2 &= \int_0^\infty H_m^2(f) N(f) df \\ R_{nm}(\tau) &= F^{-1} \{ N_m(f) \exp(-j2\pi f \tau_n - j\phi_n) \} \\ \sigma_{sm}^2 &= \frac{H_m^2(f_0) A^2}{2} \\ \sigma_m^2 &= \sigma_{sm}^2 + \sigma_{nm}^2 \\ g(\tau_m) &= F^{-1} \{ G(f) \} \end{aligned}$$

where $F^{-1} \{ \}$ represents the inverse Fourier transform operation (accounting for the fact that $N_m(f)$ is one sided).

If both fibers are active, then the interaural correlation function $R_{RLm}(\tau_m)$ [as defined in Equation (A6)] will depend on the values f_{cm} and τ_m , as well as on the properties of the binaural stimulus. In order to determine this value, we must compute the crosscorrelation of the outputs of the v^{th} -law half-wave

rectifiers from each ear. Using the results of Davenport and Root (1958, pp. 277-308) we obtain⁴

$$R_{RLm}(\tau_m) = \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{\varepsilon_i h_{ik}^2}{k!} R_{nm}^k(\tau_m) \cos [2\pi i f_0 (\tau_m - \tau_s)] * g(\tau_m) * g(-\tau_m) \quad (B.2)$$

where ε_i is the Neumann factor $\varepsilon_0 = 1$, $\varepsilon_i = 2$ ($i = 1, 2, \dots$),

$$h_{ik} = \frac{200 (\sigma_{sm}^2 / \sigma_{nm}^2)^{i/2} \Gamma(1 + \frac{v}{2}) 2^{k/2}}{i! \Gamma[1 - (i+k-v)/2] \sigma_{nm}^k} {}_1F_1\left(\frac{i+k-v}{2}; i+1; -\frac{\sigma_{sm}^2}{\sigma_{nm}^2}\right), \quad (B.3)$$

and ${}_1F_1(a; c; z)$ is the confluent hypergeometric function defined by the series

$${}_1F_1(a; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k z^k}{(c)_k k!} = 1 + \frac{az}{c} + \frac{a(a+1)z^2}{c(c+1)2!} + \dots \quad (B.4)$$

The expression for the cross-correlation given above reduces to the simple power series

$$R_{RLm}(\tau_m) = 200^2 \left\{ 1 + \sum_{k=1}^{\infty} \frac{2^k \Gamma^2(1 + \frac{v}{2})}{k! \Gamma^2[1 - (k-v)/2]} [R_{nm}(\tau_m) / \sigma_{nm}^2]^k \right\} * g(\tau_m) * g(\tau_m) \quad (B.5)$$

when there is no tone present (*i.e.*, σ_{sm}^2 equals zero).

In both cases,

$$L_2(\tau, f_c) = R_{RLm}(\tau_m) T_S T_W \quad (B.6)$$

where T_W is the width of the coincidence window and T_S is the duration of the stimulus.

B. Response to a Pure Tone

Equation (B.3) does not converge when the stimulus is a pure tone (*i.e.*, σ_{nm}^2 equals zero). This stimulus configuration must be considered as a special case.

As was already mentioned, the mean of L_m is a constant when either of the input fibers to the coincidence counter is inactive. If we consider a single fiber pair and assume that the input tone is sufficiently intense to activate both fibers, it is apparent that the automatic gain control element causes the intensity function for each fiber to be independent of the characteristic frequency of the fiber. In addition, each intensity function is periodic with fundamental frequency f_0 . These observations suggest the following means of expressing the crosscorrelation $R_{RLm}(\tau_m)$,

$$R_{RLm}(\tau_m) = (200)^2 \left\{ 1 + 2 \sum_{n=1}^{\infty} S_n^2 G^2(nf_0) \cos [2\pi n f_0 (\tau_m - \tau_s)] \right\} \quad (B.7)$$

where S_n is the magnitude of the n^{th} coefficient of the Fourier series of the output of the rectifier normalized by the mean firing rate (200 per second). The coefficients $\{S_n\}$ depend on the order of the

⁴Davenport and Root (1958) derive an expression for the autocorrelation of the output of the rectifier. However, since the intensities of the component stimuli of interest are identical in each ear, these results can be applied directly to the crosscorrelation considered.

half-wave rectifier being used. Table B-1 gives the values of the first eight coefficients when v equals 1, 2, and 3. The more expansive the rectifier is (*i.e.*, the larger v is), the larger the coefficients of the second and third order terms are.

S_n^2			
n	$v=1$	$v=2$	$v=3$
1	0.61685028	0.72050619	0.78070113
2	0.11111111	0.25	0.36
3	0.0	0.02882025	0.08674457
4	0.00444444	0.0	0.00734694
5	0.0	0.00058817	0.0
6	0.00081633	0.0	0.00009070
7	0.0	0.00006535	0.0
8	0.00043403	0.0	0.00000675

Table B-1: Normalized coefficients of the Fourier series of the output of a v^{th} -law half-wave rectifier when the input is a tone.

C. Response to Amplitude-Modulated Tones

Unlike the previous cases, the response to amplitude-modulated tones was determined computationally rather than analytically.

The stimuli for these experiments are of the form

$$\begin{aligned}
 & x_L(t) = A(1 + m \cos(2\pi f_m t)) \cos(2\pi f_c t) \\
 \text{and} \\
 & x_R(t) = A(1 + m \cos(2\pi f_m(t - \tau_d))) \cos(2\pi f_c(t - \tau_c))
 \end{aligned} \tag{B.8}$$

or, equivalently,

$$\begin{aligned}
 & x_L(t) = A \cos(2\pi f_c t) + \frac{mA}{2} \cos((2\pi f_c + 2\pi f_m)t) + \frac{mA}{2} \cos((2\pi f_c - 2\pi f_m)t) \\
 \text{and} \\
 & x_R(t) = A \cos(2\pi f_c(t - \tau_c)) + \frac{mA}{2} \cos\left((2\pi f_c + 2\pi f_m)\left(t - \frac{(2\pi f_c \tau_c + 2\pi f_m \tau_d)}{(2\pi f_c + 2\pi f_m)}\right)\right) \\
 & \quad + \frac{mA}{2} \cos\left((2\pi f_c - 2\pi f_m)\left(t - \frac{(2\pi f_c \tau_c - 2\pi f_m \tau_d)}{(2\pi f_c - 2\pi f_m)}\right)\right)
 \end{aligned} \tag{B.9}$$

where f_c is the carrier frequency and f_m is the modulator frequency, and τ_c and τ_d are the interaural carrier and modulator delays respectively. (A *waveform* delay τ_w is obtained by setting both τ_c and τ_d equal to τ_w .)

The correlation operation was implemented using discrete Fourier transforms (DFTs; Oppenheim and Schaffer, 1989). This requires that both the carrier and modulation frequencies be integer multiples of the quantity $1/NT_i$ Hz, where N is the size of the DFT and T_i is the sampling time of the discrete-time

approximation to the continuous-time signal. Hence the quantities f_c and f_d are approximated by the values \tilde{f}_c and \tilde{f}_d , the closest integer multiples of $1/NT_i$ to f_c and f_d . In general we use parameter values of $N=4096$ and $T_i=.025$ ms, so frequencies are quantized to integer multiples of approximately 9.8 Hz.

The outputs of the peripheral linear bandpass filters are characterized by the equations

$$y_L(t) = A H_m(\tilde{f}_c) \cos(2\pi\tilde{f}_c t) + \frac{mA}{2} H_m(\tilde{f}_c + \tilde{f}_m) \cos((2\pi(\tilde{f}_c + \tilde{f}_m))t) + \frac{mA}{2} H_m(\tilde{f}_c - \tilde{f}_m) \cos((2\pi(\tilde{f}_c - \tilde{f}_m))t)$$

and

$$y_R(t) = A H_m(\tilde{f}_c) \cos(2\pi\tilde{f}_c(t - \tau_c)) + \frac{mA}{2} H_m(\tilde{f}_c + \tilde{f}_m) \cos((2\pi(\tilde{f}_c + \tilde{f}_m)) (t - \frac{(2\pi(\tilde{f}_c \tau_c + \tilde{f}_m) \tau_d)}{(2\pi\tilde{f}_c + 2\pi\tilde{f}_m)}))$$

$$+ \frac{mA}{2} H_m(\tilde{f}_c - \tilde{f}_m) \cos((2\pi(\tilde{f}_c - \tilde{f}_m)) (t - \frac{(2\pi\tilde{f}_c \tau_c - 2\pi\tilde{f}_m \tau_d)}{(2\pi\tilde{f}_c - 2\pi\tilde{f}_m)})) \quad (B.10)$$

The effects of the rectifier and lowpass filter are expressed by the equations

$$z_L(t) = a_L R_v[y_L(t)] * g(t)$$

$$z_R(t) = a_R R_v[y_R(t)] * g(t) \quad (B.11)$$

where $R_v[\cdot]$ is the half-wave v^{th} -law rectifier defined in Stern and Shear (1992), $g(t)$ is the impulse response of the lowpass filter of the model for auditory-nerve activity defined in Stern and Shear (1992), and the normalizing constants a_L and a_R are chosen such that

$$\int_0^{T_S} z_L(t) dt = \int_0^{T_S} z_R(t) dt = 200 T_S \quad (B.12)$$

Using DFTs, the function $R_{RLm}(\tau_m)$ is obtained by performing the circular convolution of $z_L(t)$ and $z_R(-t)$:

$$R_{RLm}(\tau_m) = z_L(t) \otimes z_R(-t) \quad (B.13)$$

D. Implementation Notes on the Generation of Predictions

In this section we describe the details of the computer algorithms used to generate the predictions presented in this report.

All integrals were computed as discrete Riemann sums with a time step of 0.025 ms and a log-frequency step of $0.01 \log_{10}(\text{Hz})$. The limits on the time summations were between ± 8 ms for detection experiments and ± 12.75 ms for lateralization experiments.

For detection experiments, threshold was declared when the square root of Q_d was between 0.975 and 1.025.

In order to determine samples of $R_{nm}(\tau)$ (as defined at the beginning of this Appendix), a 4096-point inverse discrete Fourier transform was used. Similarly, lowpass filtering was accomplished by means of a 4096-point discrete Fourier transform.

The infinite summations of Equation (B.2) were truncated to include only terms 0 through 14. For pure tones, the Fourier series of Equation (B.7) was truncated to include only terms 0 through 8. (Values of these coefficients for v equals 1, 2, and 3 are given in Table B-1.) Terms of the confluent hypergeometric

function given by Equation (B.4) were computed and summed until the absolute value of a given term divided by the summation at that point was less than 10^{-5} .

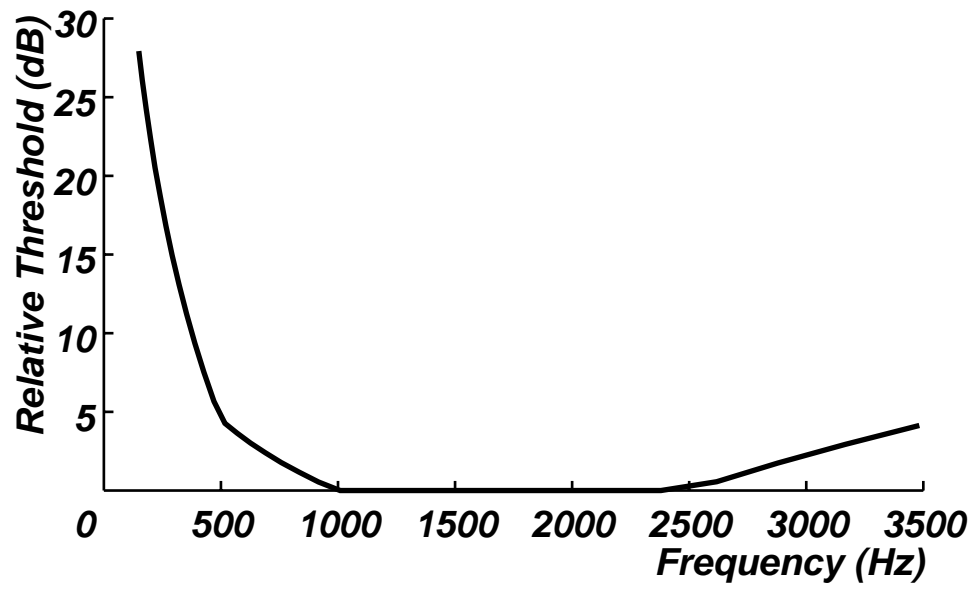


Figure A-1. The threshold-of-hearing curve $\zeta(f_{cm})$ plotted as a function of f_{cm} .

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