Depth-Limited Endgame solving, and *Pluribus*, the state of the art for multi-player no-limit Texas hold’em

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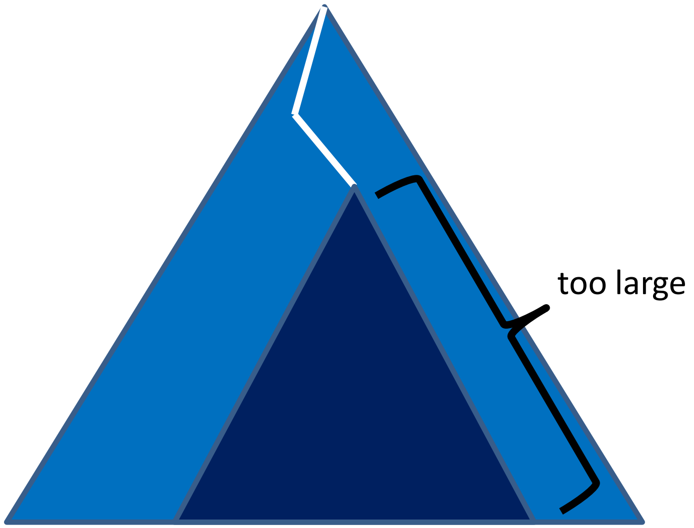
CS 15-888

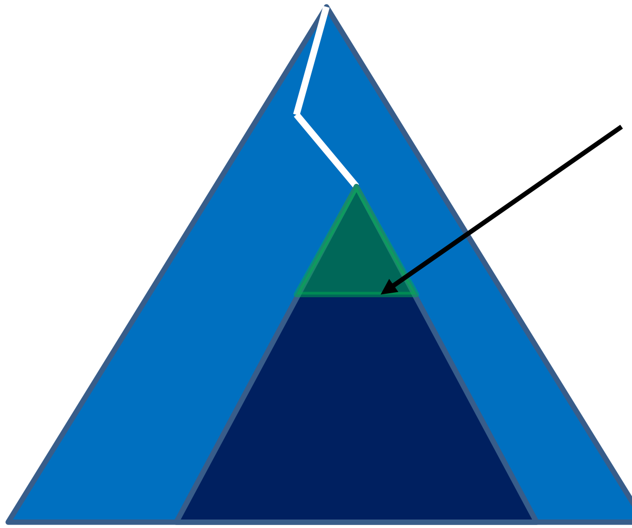
Depth-limited subgame solving

[Brown, Sandholm & Amos, *NeurIPS*-18; Brown & Sandholm, *Science* 2019]

Perfect-information games and single-agent search



Perfect-information games and single-agent search

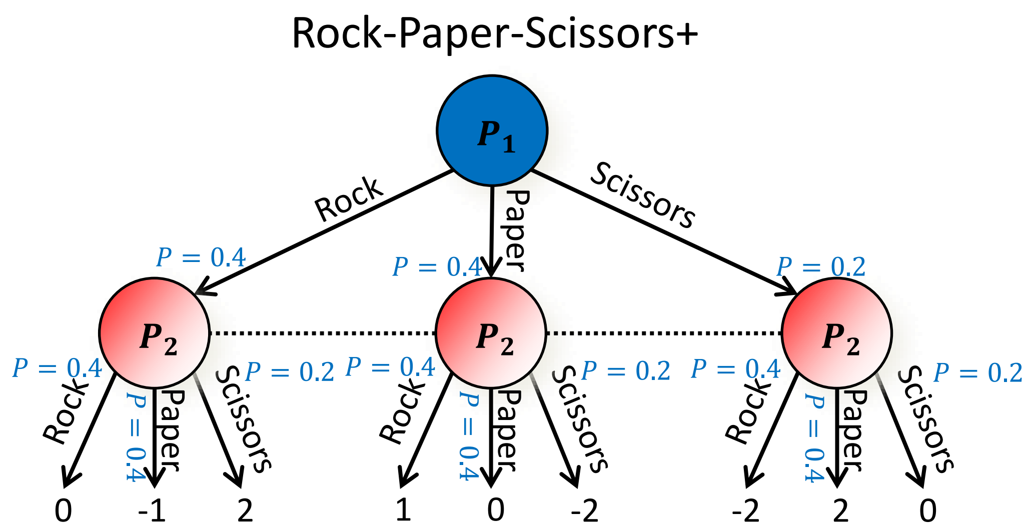


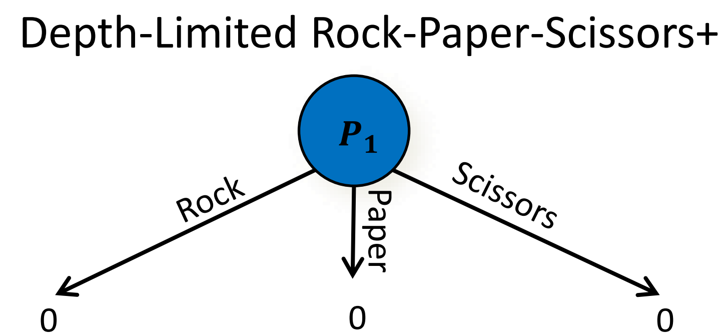
Value substituted at leaf node is estimate of both players playing perfectly thereafter

If estimate is perfect, limited-lookahead search plays an equilibrium strategy

But state values are not well defined in imperfect-information games!

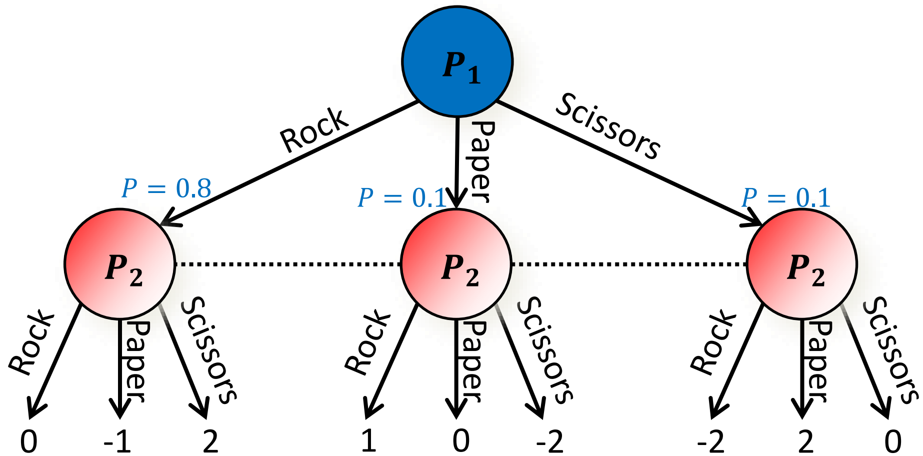
Depth-limited solving



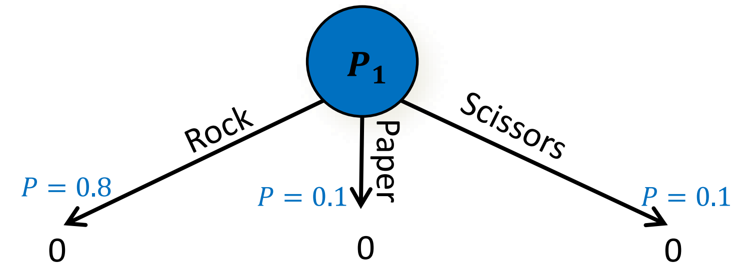


Depth-limited solving

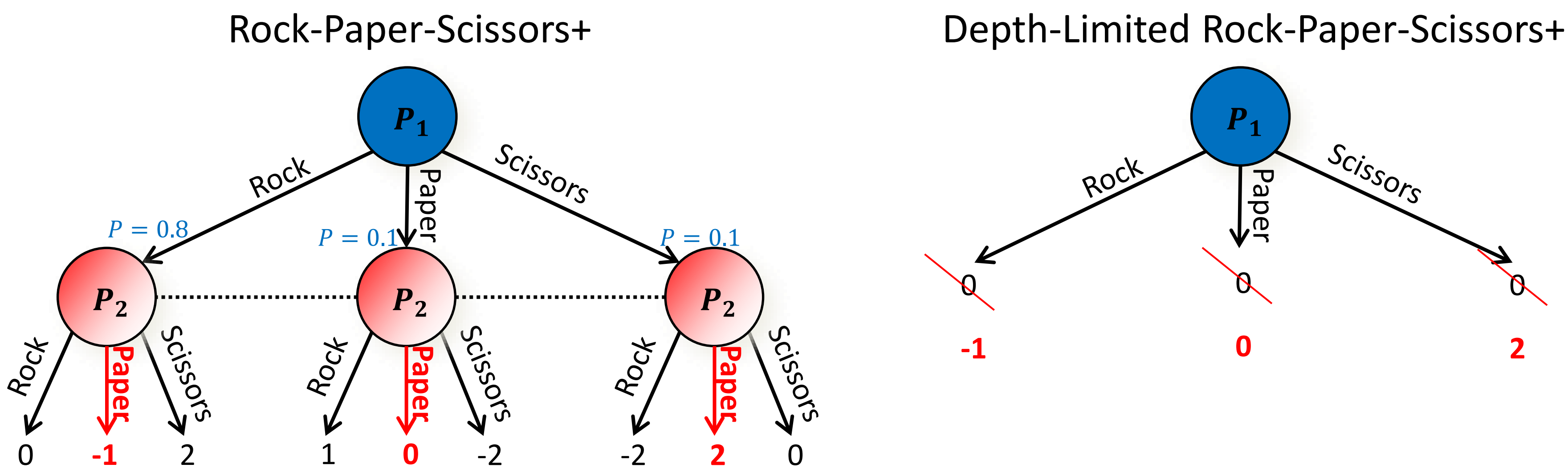
Rock-Paper-Scissors+



Depth-Limited Rock-Paper-Scissors+

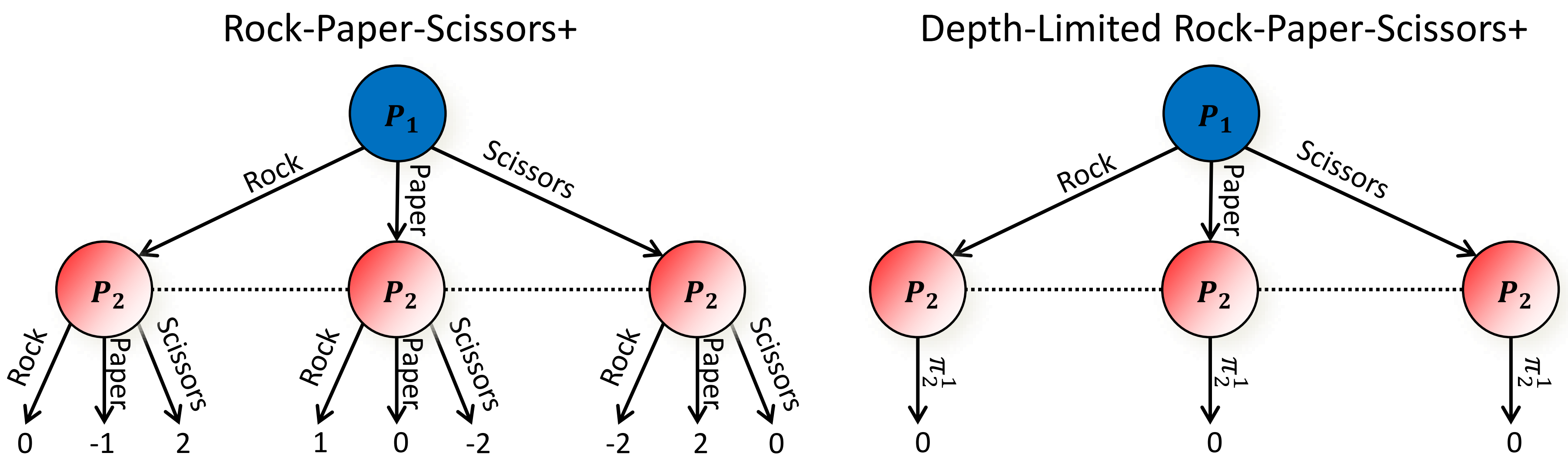


Depth-limited solving



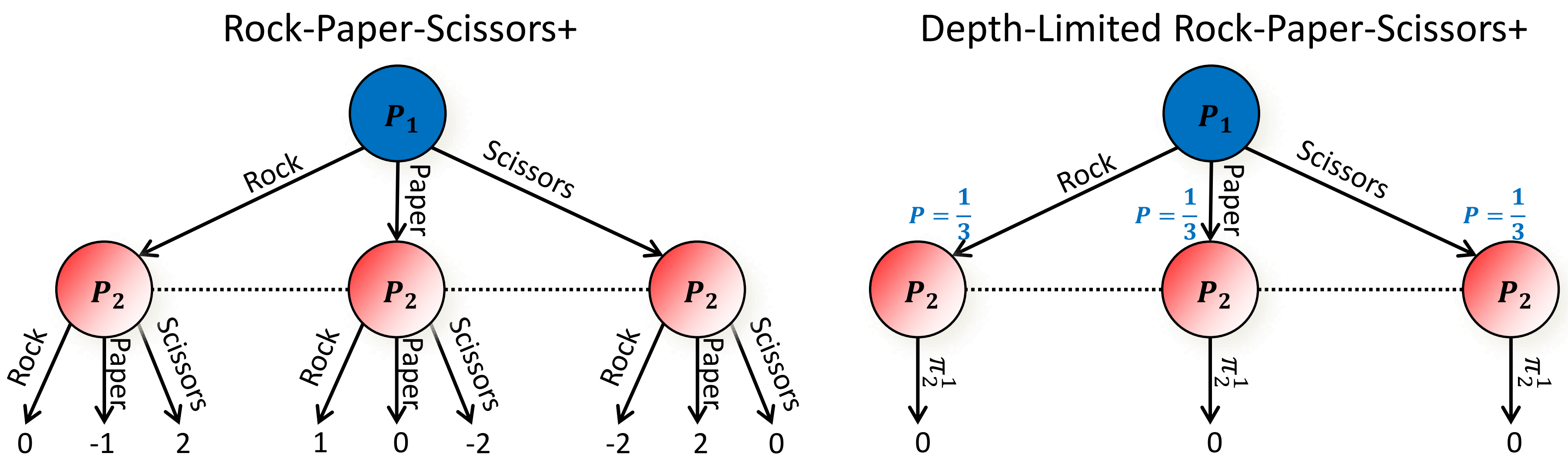
How to tackle this issue?

Depth-limited solving



- At leaf nodes, allow player one final action choosing among multiple *policies* for the remaining game
- Step 1: Solve subgame with current set of P_2 leaf-node policies
- Step 2: Calculate a P_2 best response
- Step 3: Add P_2 best response to set of leaf-node policies
- Repeat

Depth-limited solving



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Depth-limited solving

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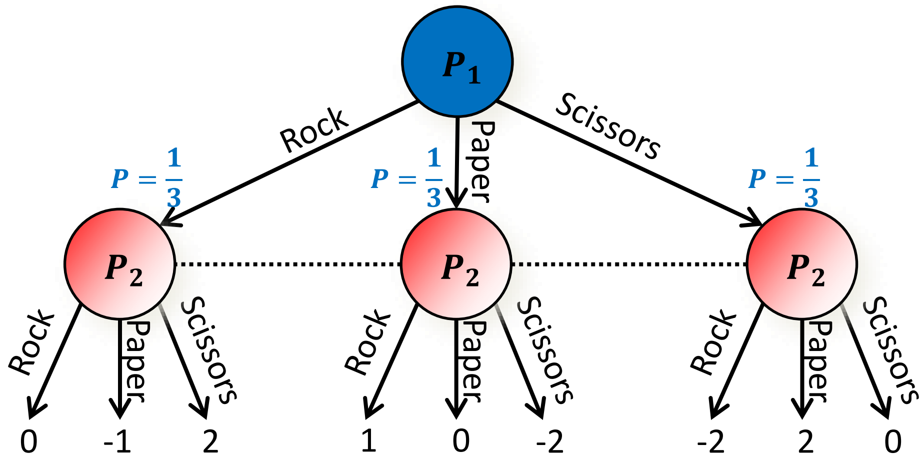
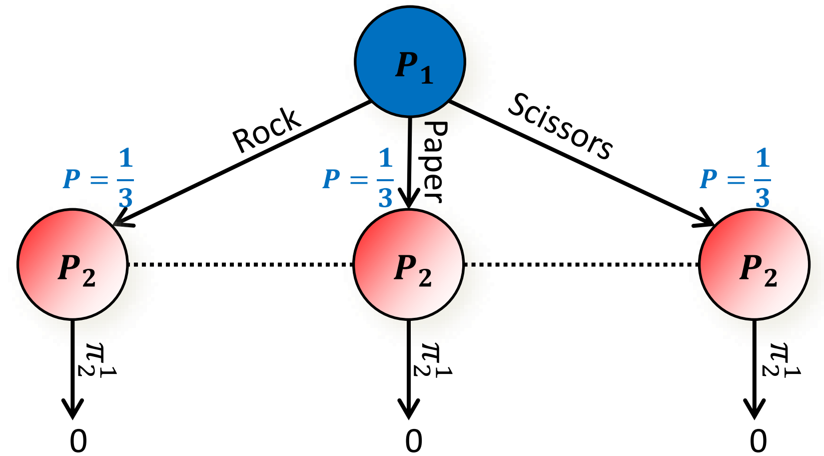


Depth-Limited Rock-Paper-Scissors+



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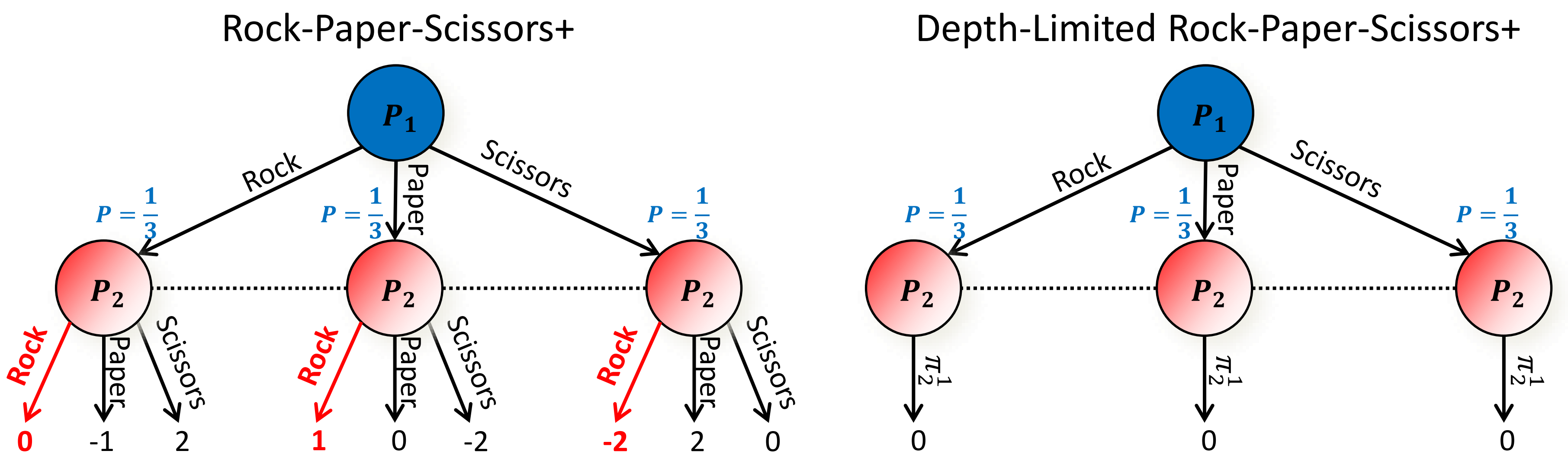

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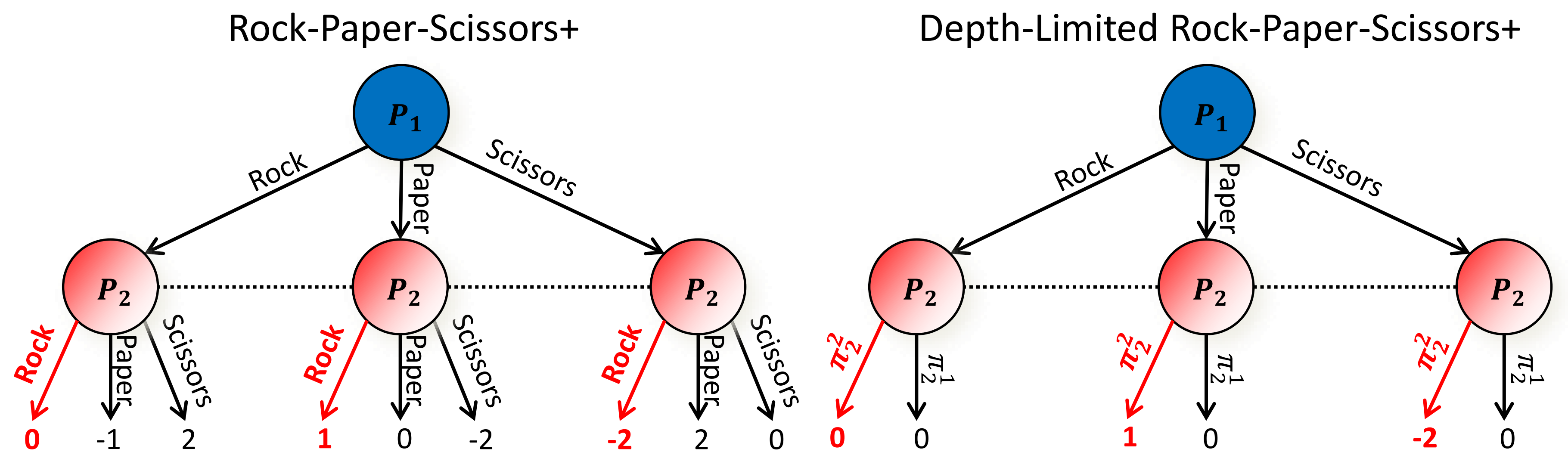

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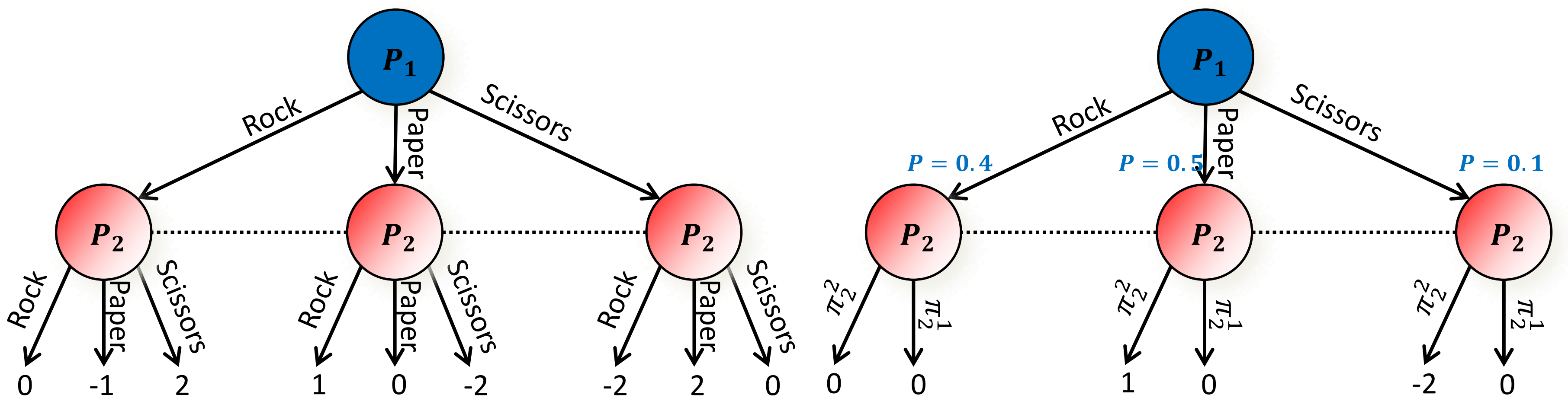
Depth-limited solving

Rock-Paper-Scissors+ Depth-Limited Rock-Paper-Scissors+



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Also other ways to generate continuation strategies for the opponent.

Theorem. Converges to Nash equilibrium in 2-player 0-sum games.
In practice, reaches very low exploitability in a small number of iterations.
Can be used with the safe, recursive subgame solving.

Multi-player games

- Multi-player poker is a recognized game theory, AI, and OR milestone that has been open for decades
- Most popular variant in the world: 6-player no-limit Texas hold'em
- Superhuman player, *Pluribus*, for this using the depth-limited recursive subgame solving [Brown & Sandholm *Science* 2019]
  - Each player was picking continuation policies at leaves
- 1st superhuman player in any game beyond 2-player 0-sum games

Safe depth-limited solving starting later than the root

- In imperfect-information games, “subgames” are not independent
- However, techniques from *Libratus*’s endgame solving can be applied, but now the endgames are midgames that end in continuation strategy choices
  - Have a blueprint strategy for the whole game
    - E.g., via abstraction+equilibrium computation, Deep CFR [Brown, Lerer, Gross & Sandholm, *ICML*-19c], or manual
  - When determining our strategy for an endgame, **give opponent the choice of model:** blueprint or endgame model [Burch *et al*., *AAAI*-14; Jackson, *AAAI*-14; Moravcik *et al*., *AAAI*-16; Brown & Sandholm, *NIPS*-17; Moravcik *et al*., *Science* 2017; Brown & Sandholm, *Science* 2018]
    - Want to solve for our endgame strategy such that opponent isn’t better off choosing endgame model for any private type she may have => Theorem: safe
    - Allow opponent to get back in the endgame the gifts she has given so far => Theorem: safe [Brown & Sandholm NIPS-17 Best Paper; *Science* 2018]
- Can apply this recursively
  - Can include the action that the opponent made
  - Can use finer abstraction when endgame starts closer to end of the game
  - Theorem: Safe [Brown & Sandholm, *NIPS*-17 Best Paper; *Science* 2018]

Head-to-head performance in 2-player no-limit Texas hold'em

[Brown, Sandholm & Amos, *NeurIPS*-18e]

- Baby Tartanian8
  [2016 champion]
  - 2 million core hours
  - 18 TB of memory
- Slumbot
  [2018 champion]
  - 250,000 core hours
  - 2 TB of memory
- Modicum
  - 700 core hours
  - 16 GB of memory
  - Plays in real time with a 4-core CPU in 20 seconds per hand

	Baby Tartanian8	Slumbot
Modicum (no real-time reasoning)	-57 ± 13	-11 ± 8
Modicum (just one continuation strategy)	-10 ± 8	-1 ± 15
Modicum (just a few continuation strategies)	6 ± 5	11 ± 9

Unit: milli-big-blinds / game

Key takeaways from this segment

- Planning is important in imperfect-information games, but different
- In real-time planning, you must consider how the opponent can adapt to changes in your strategy
  - Except in perfect-information games and single-agent setting
- States don’t have well-defined values in imperfect-info games
- Our depth-limited solving algorithm:
  - Is sound
  - Enabled 2nd-best AI for heads-up no-limit Texas hold’em poker to be developed on a 4-core CPU with 16 GB of RAM

MULTI-PLAYER GAMES

Multi-player games

- All prior superhuman AI game-playing milestones have been in 2-player games:
  - **Checkers**: *Chinook* 1994
  - **Othello**: *Logistello* 1997
  - **Chess**: *Deep Blue* 1997
  - **2-player limit Texas hold'em**: *Polaris* 2008
  - **Go**: *AlphaGo* 2016
  - **2-player no-limit Texas hold'em**: *Libratus* 2017
  - **Starcraft II**: *AlphaStar* 2019 and **DOTA 2**: *OpenAI Five* 2019 (if they are superhuman)

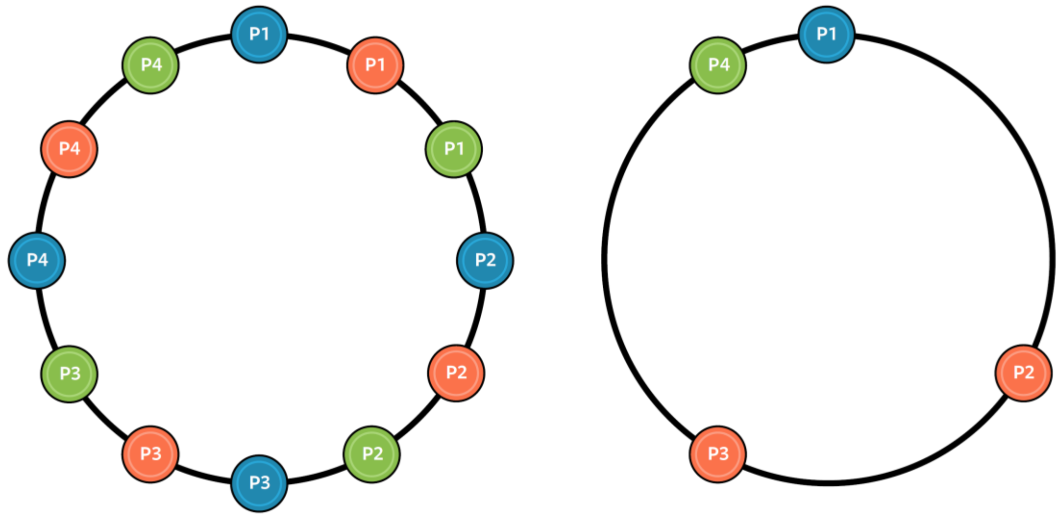
- Our research led to techniques that enabled us to develop a superhuman AI for multi-player no-limit Texas hold'em …

Multi-player poker

- Recognized AI, game theory, and OR milestone that has been open for decades
- Most popular variant in the world: 6-player no-limit Texas hold'em
- We developed a superhuman AI, *Pluribus*, for this game [Brown & Sandholm, *Science* 2019]
  - Science Breakthrough of the Year runner-up, 2019

2-player 0-sum vs. multi-player games

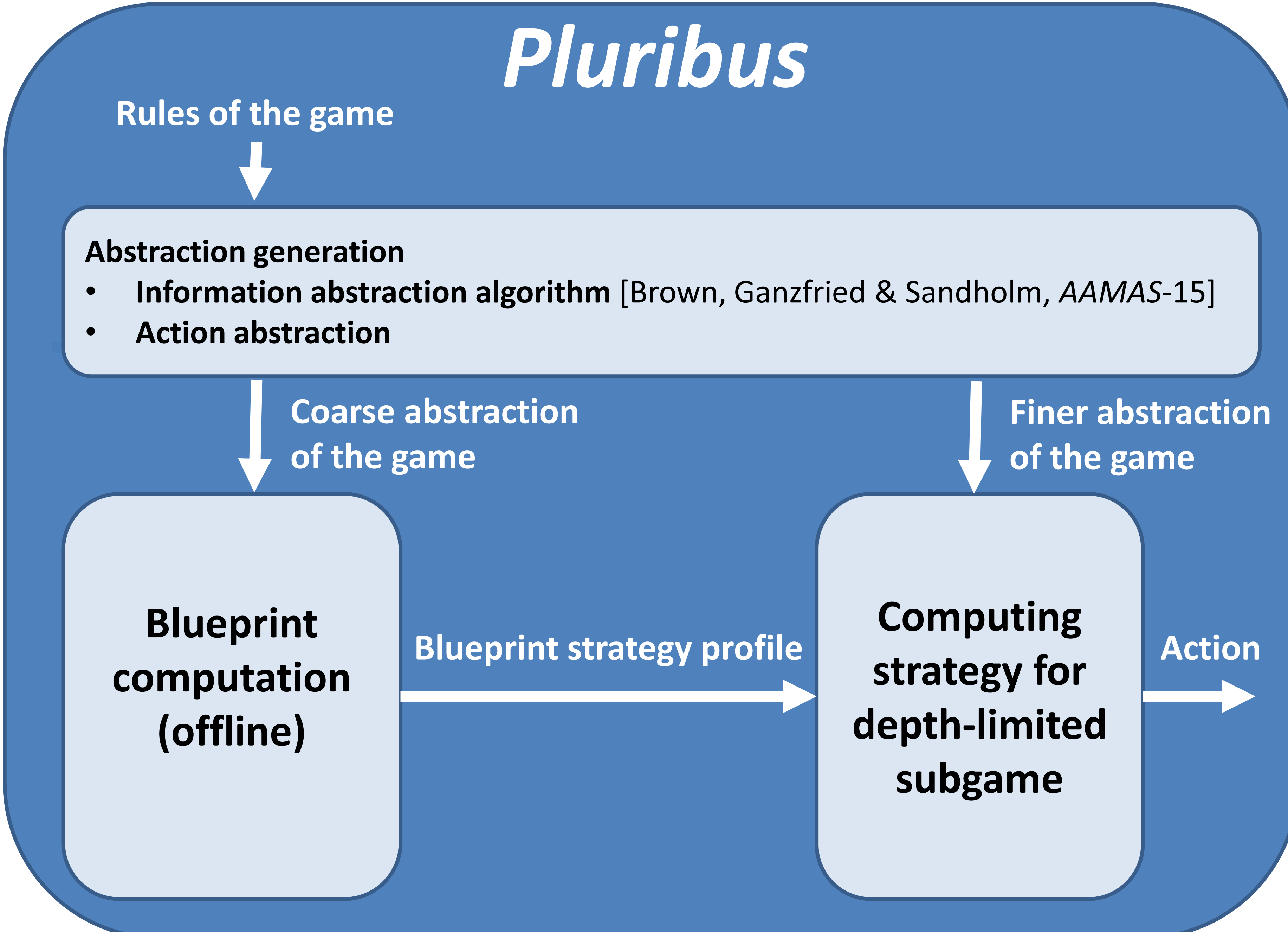
- All prior superhuman AI game milestones have been in 2-player 0-sum games
- Multi-player games have additional issues (even in normal form):
  - Playing a Nash equilibrium is not safe

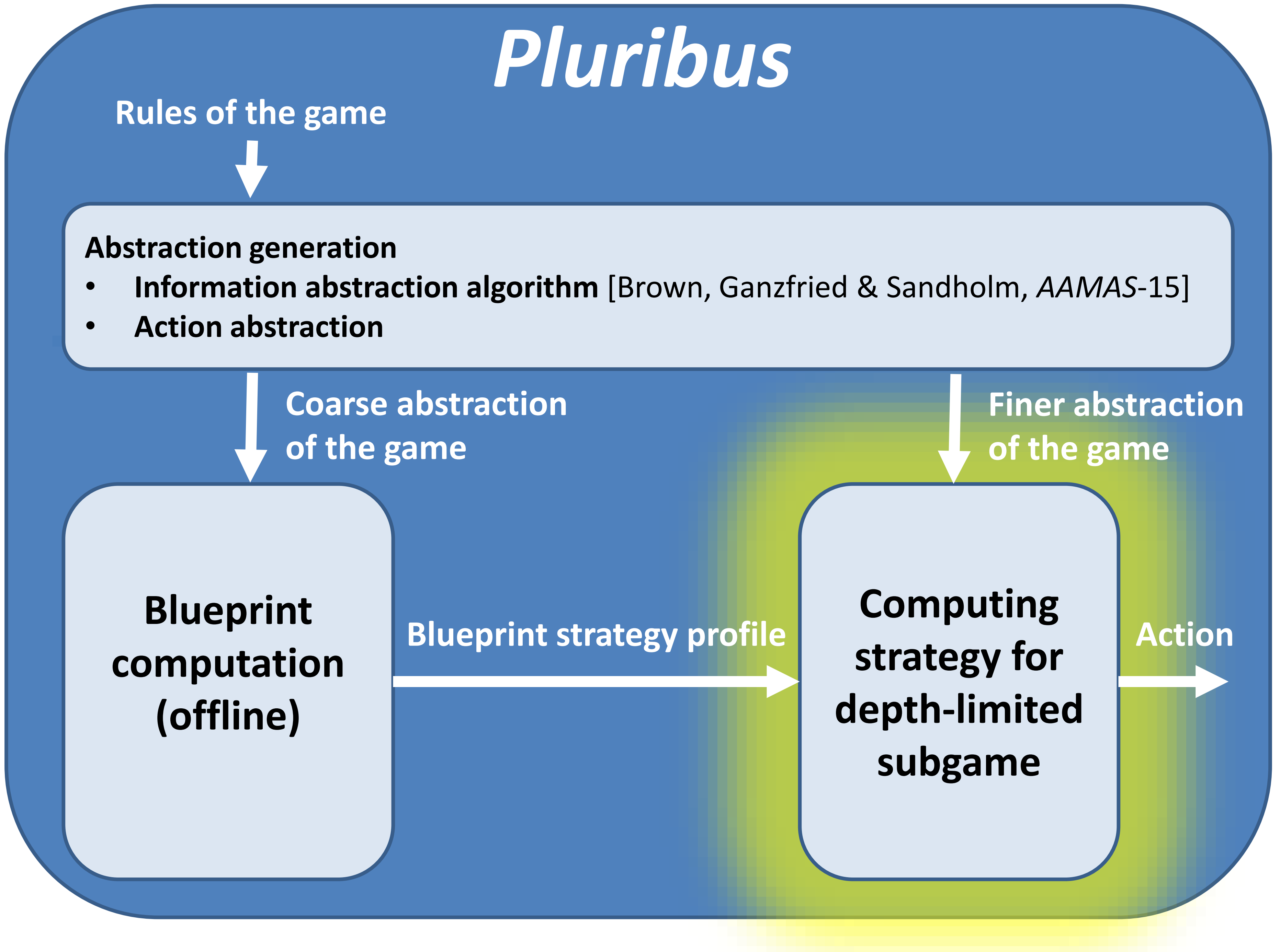


  - Finding even an approximate Nash equilibrium is hard
    - In theory [Daskalakis *et al*. 2009; Chen *et al*. 2009; Rubinstein 2018]
    - In practice, fastest complete algorithm only scales to 3-5 players and 3-5 strategies per player [Berg & Sandholm, *AAAI*-17]
- *Pluribus* finds superhuman strategies with a novel set of algorithms
  - No guarantee that the solution is a Nash equilibrium (beyond 2-player 0-sum games)

How does *Pluribus* work?

- Developed and runs on a single server, no GPUs
- Doesn’t use any data
- Doesn’t adapt to the opponent
- Offline blueprint computation and real-time depth-limited search


Pluribus
Rules of the game
Abstraction generation
• Information abstraction algorithm [Brown, Ganzfried & Sandholm, AAMAS-15]
• Action abstraction
Coarse abstraction of the game
Finer abstraction of the game
Blueprint computation (offline)
Blueprint strategy profile
Computing strategy for depth-limited subgame
Action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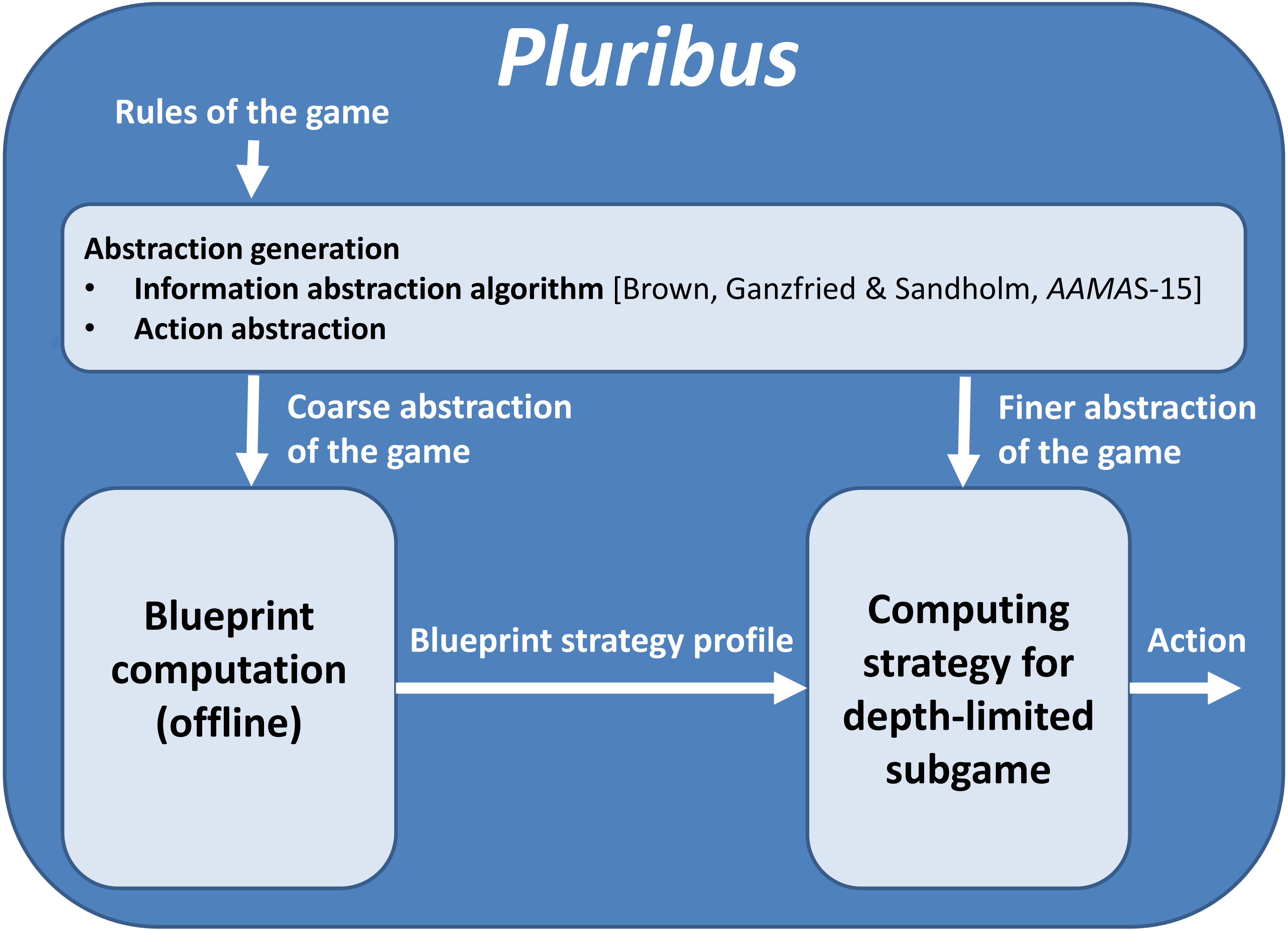
Pluribus's new form of depth-limited search for imperfect-information games

- All players (not just opponents) pick from k continuation strategies at leaves
- Search starts before current situation (beginning of current betting round)
  - Mitigates exploitability of unsafe search while keeping its advantages
  - Our player's strategy is kept fixed for the moves already taken
  - As in *Libratus*, opponents' actual actions are added to subgame model before the subgame is solved
    => no need to reverse map actions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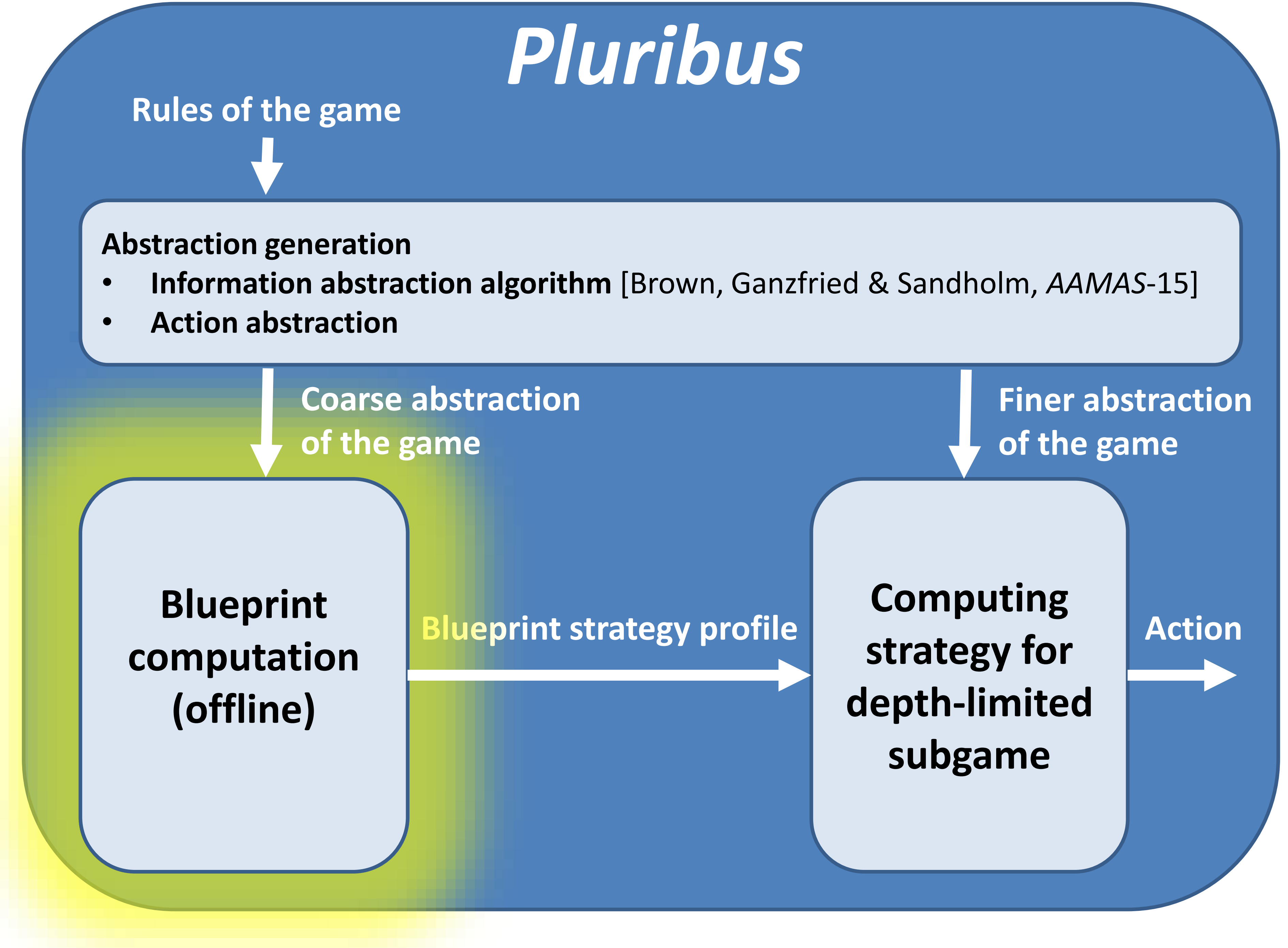
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Pluribus’s new equilibrium-finding algorithm

- Used for blueprint computation and for solving depth-limited subgames
- Significant improvement over MCCFR [Lanctot *et al.*, *NeurIPS*-09]
- Uses fastest sampling-based equilibrium-finding algorithm for zero-sum games: ***linear CFR*** [Brown & Sandholm, *AAAI*-19 Distinguished Paper Honorable Mention]
  - *Pluribus* uses linear weighting for both regrets and for averaging the strategies
  - => “Linear MCCFR”
- New form of dynamic pruning in early part of the run
  - Not in last two steps of the game
- Saving memory: sequences allocated in RAM only if encountered

At play time, *Pluribus*:

- Runs on a regular computer using
  - 2 CPUs
  - Less than 128 GB RAM
  - No GPUs
- Plays twice as fast as human pros (20 sec / hand)

Performance against top human pros

- AIVAT [Burch *et al*., *AAAI*-18] was used in the evaluation for variance reduction

- **Experiment 1:** 1 human pro, 5 copies of *Pluribus*
  - Independent copies of *Pluribus*; didn't know even seat of others
  - Each of Chris Ferguson and Darren Elias played 5,000 hands (also, monetary incentive to play as well as they can)
  - *Pluribus* beat each opponent with statistical significance
  - In a later identical experiment, *Pluribus* also beat Linus Loeliger

- **Experiment 2:** 5 human pros, 1 *Pluribus*
  - 10,000 hands
  - For each 6-player session, 5 humans were selected based on availability from 13 human pros
    - Each had won over $1M playing poker, many had won over $10M
    - Linus Loeliger, Jimmy Chou, Seth Davies, Michael Gagliano, Anthony Gregg, Dong Kim, Jason Les, Daniel McAulay, Nick Petrangelo, Sean Ruane, Trevor Savage, Jake Toole
  - $50,000 divided among human pros to incentivize them to play as well as they can
  - *Pluribus* won with statistical significance (p=0.028)

Improvement of *Pluribus* with training time

- 64-core server, 512 GB RAM, no GPUs
- ~$150 at cloud prices