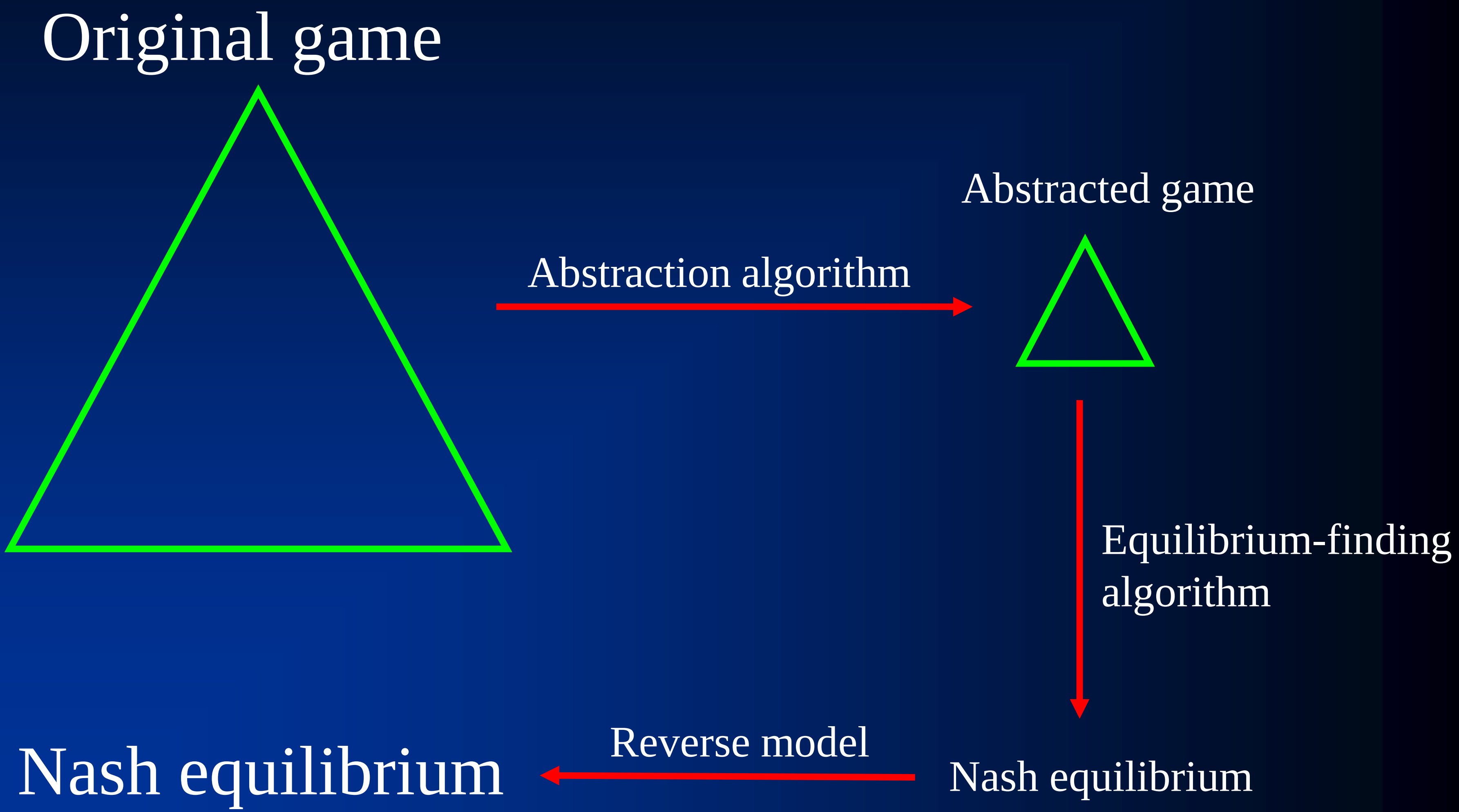
State-of-the-Art
Practical Game Abstraction

Tuomas Sandholm

Automated game abstraction

[Gilpin & Sandholm, EC-06/*J. of the ACM* 2007, AAAI-06…]

Now used basically by all competitive Texas Hold'em programs



Foreshadowed by Shi & Littman 01 and Billings et al., IJCAI-03

Lossless game abstraction

[Gilpin & Sandholm, EC-06, *J. of the ACM* 2007]

Information filters

- Observation: We can make games smaller by filtering the information a player receives

- Instead of observing a specific signal exactly, a player instead observes a **filtered set** of signals
  - *E.g.*, receiving signal {A♠,A♣,A♥,A♦} instead of A♥

Signal tree

- Each edge corresponds to the revelation of some signal by nature to at least one player
- Our abstraction algorithm operates on it
  - Doesn't load full game into memory

Isomorphic relation

- Captures the notion of strategic symmetry between nodes
- Defined recursively:
  - Two leaves in signal tree are isomorphic if for each action history in the game, the payoff vectors (one payoff per player) are the same
  - Two internal nodes in signal tree are isomorphic if their children are isomorphic
    - *Challenge*: permutations of children
    - *Solution*: custom perfect matching algorithm between children of the two nodes such that only isomorphic children are matched

Abstraction transformation

- Merges two isomorphic nodes that are siblings

- **Theorem.** *If a strategy profile is a Nash equilibrium in the abstracted (smaller) game, then its interpretation in the original game is a Nash equilibrium*

GameShrink algorithm

- Bottom-up pass: Run DP to mark isomorphic pairs of nodes in signal tree
- Top-down pass: Starting from top of signal tree, perform the transformation for siblings where applicable
- **Theorem.** Conducts all these transformations
  - $\tilde{O}(n^2)$, where n is #nodes in *signal tree*
  - Usually highly *sublinear* in game tree size

Solved Rhode Island Hold'em poker

- AI challenge problem [Shi & Littman 01]
  - 3.1 billion nodes in game tree
- Without abstraction, LP has 91,224,226 rows and columns => unsolvable
- *GameShrink* runs in one second
- After that, LP has 1,237,238 rows and columns (50,428,638 non-zeros)
- Solved the LP
  - CPLEX *barrier* method took 8 days & 25 GB RAM
- Exact Nash equilibrium
- Largest incomplete-info game solved by then by over 4 orders of magnitude

Lossy game abstraction

Example game for the rest of this lecture: Texas hold'em poker

- Nature deals 2 cards to each player
- Round of betting
- Nature deals 3 shared cards
- Round of betting
- Nature deals 1 shared card
- Round of betting
- Nature deals 1 shared card
- Round of betting

- 2-player Limit has ~10^{18} nodes
- 2-player No-Limit has ~10^{165} nodes
- Losslessly abstracted game too big to solve
  => abstract more
  => lossy

First abstraction algorithm applied to Texas hold'em [Gilpin & Sandholm, AAAI-06]

- *GameShrink* can be made to abstract more by not requiring a perfect matching => lossy
  - for speed of the matching we used a faster matching heuristic: $|wins_{node1}-wins_{node2}| + |losses_{node1}-losses_{node2}| < k$
  - Greedy => lopsided abstractions

Better and more scalable approach for lossy abstraction than GameShrink: [Gilpin & Sandholm, AAMAS-07]

- Operates in signal tree of **one** player's signals & common signals at a time (i.e., no longer in signal tree of both player's signals)
  - This'll be the case also in the state-of-the-art algorithm described later
- "Clustering + IP":
  - For every betting round *i*, tell the algorithm how many buckets K_i it is allowed to generate
    - This determines the size of the abstraction, and should be set based on the available computational resources for the equilibrium computation
  - For the first betting round, run ***k_1*-means clustering** to bucket the nodes
  - In each later round *i*, run an **IP** to determine how many children each parent should be allowed to have so the total number of children doesn't exceed K_i
    - The value of allowing a parent to have *k* children is done by running *k*-means clustering for different values of *k* under each parent before running the IP

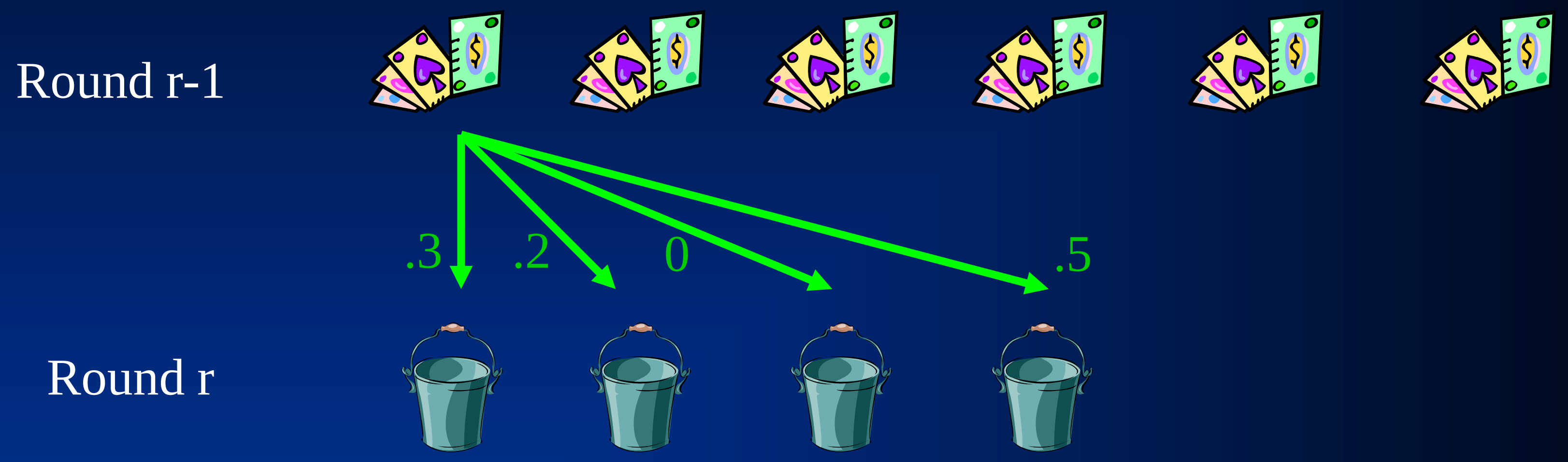
Potential-aware abstraction

- All prior abstraction algorithms had probability of winning (assuming no more betting) as the similarity metric
  – Doesn't capture *potential*

- Potential not only positive or negative, but "multidimensional"

- We developed an abstraction algorithm that captures potential … [Gilpin, Sandholm & Sørensen, AAAI-07; Gilpin & Sandholm, AAAI-08]

Bottom-up pass to determine abstraction for round 1



- Clustering using L_1 norm
  - Predetermined number of clusters, depending on size of abstraction we are shooting for

- In the last (4th) round, there is no more potential => we use probability of winning (assuming rollout) as similarity metric

Determining abstraction for round 2

- For each 1^{st}-round bucket i:
  - Make a bottom-up pass to determine 3^{rd}-round buckets, considering only hands compatible with i
  - For $k_i = 1, 2, \ldots, \max$
    - Cluster the 2^{nd}-round hands into k_i clusters
      - based on each hand's histogram over 3^{rd}-round buckets
- IP to decide how many children each 1^{st}-round bucket may have, subject to $\Sigma_i k_i \leq K_2$
  - Error metric for each bucket is the sum of L_2 distances of the hands from the bucket's centroid
  - Total error to minimize is the sum of the buckets' errors
    - weighted by the probability of reaching the bucket

Determining abstraction for round 3

- Done analogously to how we did round 2

Determining abstraction for round 4

- Done analogously, except that now there is no potential left, so clustering is done based on probability of winning (assuming rollout)

- Now the potential-aware abstraction has been computed!

Important ideas for practical lossy abstraction 2007-13

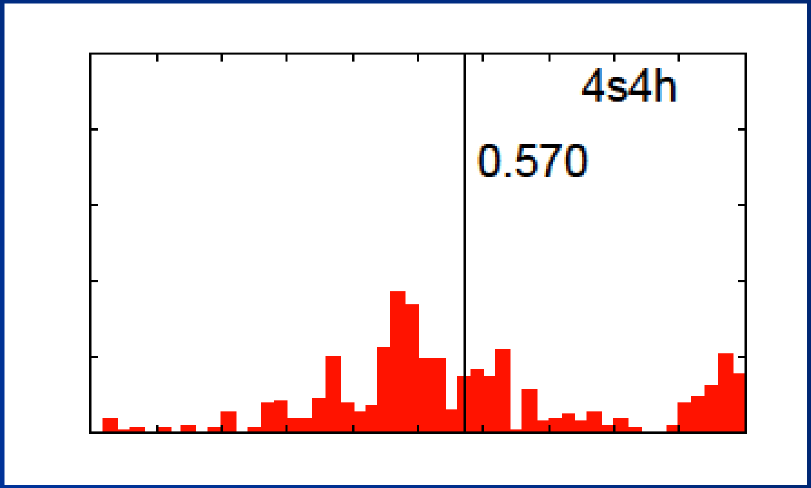
- Integer programming [Gilpin & Sandholm, AAMAS-07]
- Potential-aware [Gilpin, Sandholm & Sørensen, AAAI-07; Gilpin & Sandholm, AAAI-08]
- Imperfect recall [Waugh et al., SARA-09. Johanson et al., AAMAS-13]

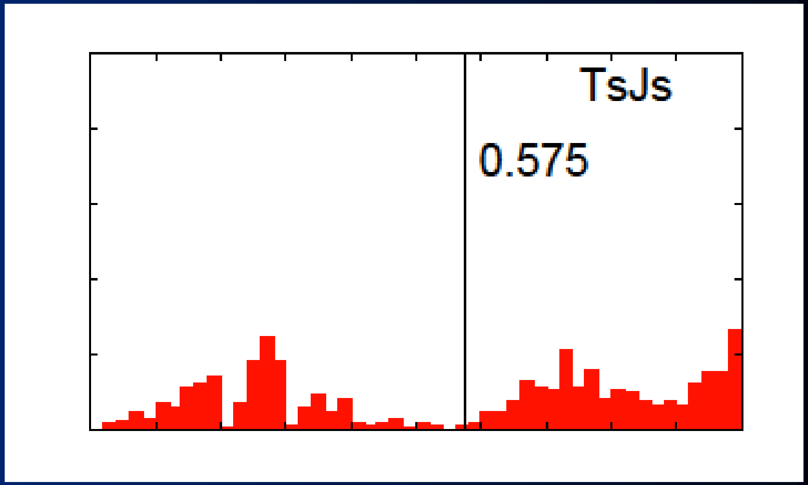
STATE OF THE ART:
Potential-Aware Imperfect-Recall Abstraction
with Earth Mover's Distance in Imperfect-Information Games

[Ganzfried & Sandholm, AAAI-14]

Expected Hand Strength (EHS)

- EHS (aka equity) is the probability of winning (plus ½ x probability of tying)
  - against a uniform random draw of private cards for the opponent,
  - assuming a uniform random rollout of the remaining public cards
- Early poker abstraction approaches used EHS (or EHS exponentiated to some power) to cluster hands [e.g., Billings et al., IJCAI-03; Gilpin & Sandholm, AAAI-06; Zinkevich et al., NIPS-07; Waugh et al., SARA-09]
- EHS fails to account for the **distribution** of hand strength
  - 4s4h and TsJs have very similar EHS (0.575 and 0.570), but 44 frequently has EHS in [0.4,0.6] and rarely in [0.7,0.9], while the reverse is true for TsJs





Distribution-aware abstraction

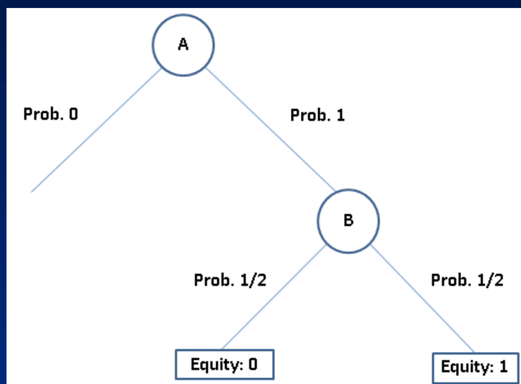
- Takes into account the full distribution of hand strength. Uses earth-mover's distance (EMD) as distance metric between histograms
  - EMD: "minimum cost of turning one pile into the other, where cost is amount of dirt moved times the distance by which it is moved"
- EMD can be computed in linear time for 1D setting, but more challenging in higher dimensions

- Prior best approach used distribution-aware abstraction with imperfect recall for flop and turn rounds. The histograms were over equities after all public cards are dealt (assuming uniform random hand for opponent) [Johanson et al., AAMAS-13]

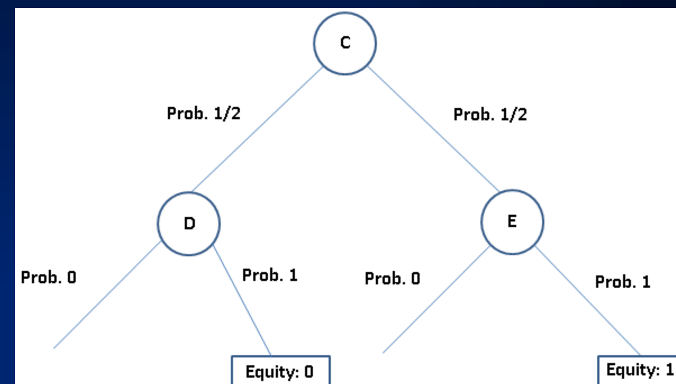
Potential-aware abstraction

- Hands can have very similar distributions over strength at the end, but realize the equity at different ways/rates
- **Potential-aware abstraction** [Gilpin, Sandholm & Soerensen, AAAI-07] considers all future rounds, not just final round
- In distribution-aware abstraction, histograms are over cardinal equities
- In potential-aware abstraction, histograms are over non-ordinal next-round states
  => must compute EMD in higher-dimensional space

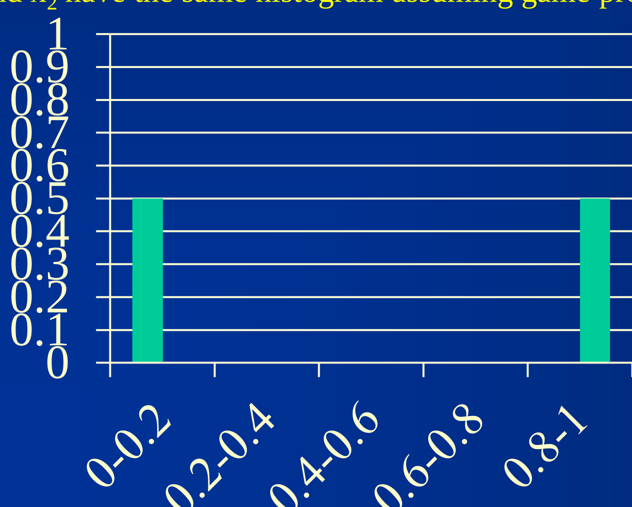
Private signal x_1



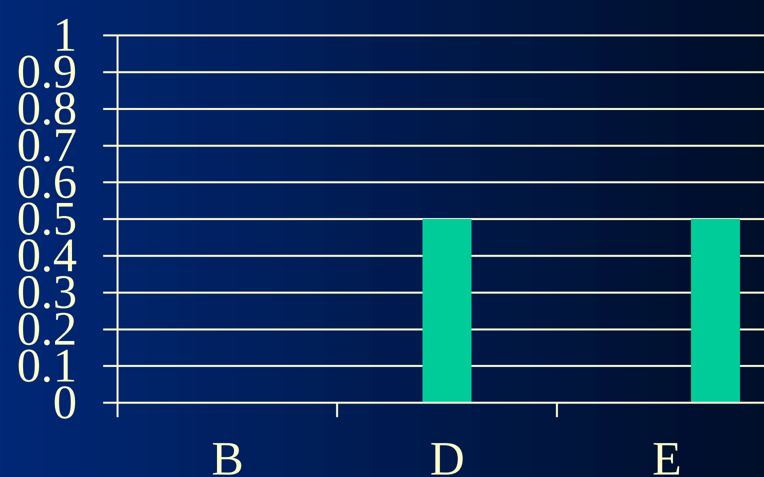
Private signal x_2



x_1 and x_2 have the same histogram assuming game proceeds to the end



Histogram for private signal x_2 at round 1 over non-ordinal information states at round 2



Algorithm for potential-aware imperfect-recall abstraction with EMD

- Perform bottom-up pass of the tree, using histograms over distributions of clusters at next round
  - EMD is now in multi-dimensional space, where the ground distance is assumed to be the (next-round) EMD between the corresponding cluster means
- Best implementation of EMD is far too slow for Texas Hold'em. We developed a fast custom heuristic for approximating it in this setting
- Using our algorithm to compute the abstraction for the flop round, we beat best prior abstraction algorithm
- Notes:
  - No need to perform multiple bottom up passes like in potential-aware abstraction before, due to imperfect recall
  - No need for IP, due to imperfect recall

Conclusions

- Domain-independent techniques
- Automated lossless information abstraction: exactly solved 3-billion-node game
- Lossy information abstraction is key to tackling large games like Texas Hold'em. Main progress 2007-2013: integer programming, potential-aware, imperfect recall
- State of the art from our 2014 paper:
  - First information abstraction algorithm that combines potential aware and imperfect recall
- Future research
  - Applying these techniques to other domains