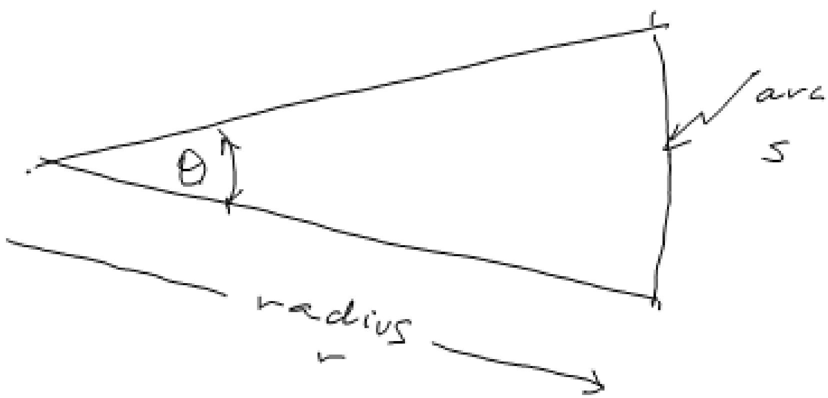
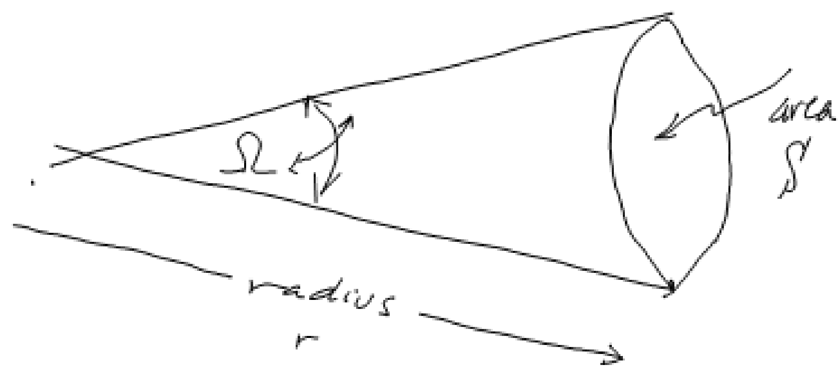
review: solid angle

in recent years I have been surprised to find that many students taking this course are not comfortable with the concept of "solid angle", so here is a brief review …

angle → transverse distance at a distance

solid angle → transverse area at a distance





$\theta = \frac{s}{r}$ → arc of circle

circle = 2π radian

$\Omega = \frac{S}{r^2}$ → cap of sphere

sphere = 4π steradian

luminance (as I use the term)

the light energy (usually already integrated over the spectral range of interest) per unit area (of the source) per unit solid angle (in the direction of interest) per unit time leaving the source surface

- watt/(m^2 steradian)
  - if not integrated over color then watt/(m^2 steradian Hz)
- fundamentally for self-luminous sources
- to keep terminology simple*, I will <u>also</u> use this word for the light ***leaving*** an *illuminated* surface

illumination (as I use the term)

- the light energy (usually already integrated over the spectral range of interest) per unit area (of a target) per unit solid angle (in the direction of the source) per unit time reaching the target surface
  - watt/(m^2 steradian) [or watt/(m^2 steradian Hz)]
  - fundamentally of interest for illuminated targets
  - to keep terminology simple, I will <u>also</u> use this word for the light ***reaching*** a sensor

how it “falls off with distance”

crucial to know who means what by “it”!

consider illumination

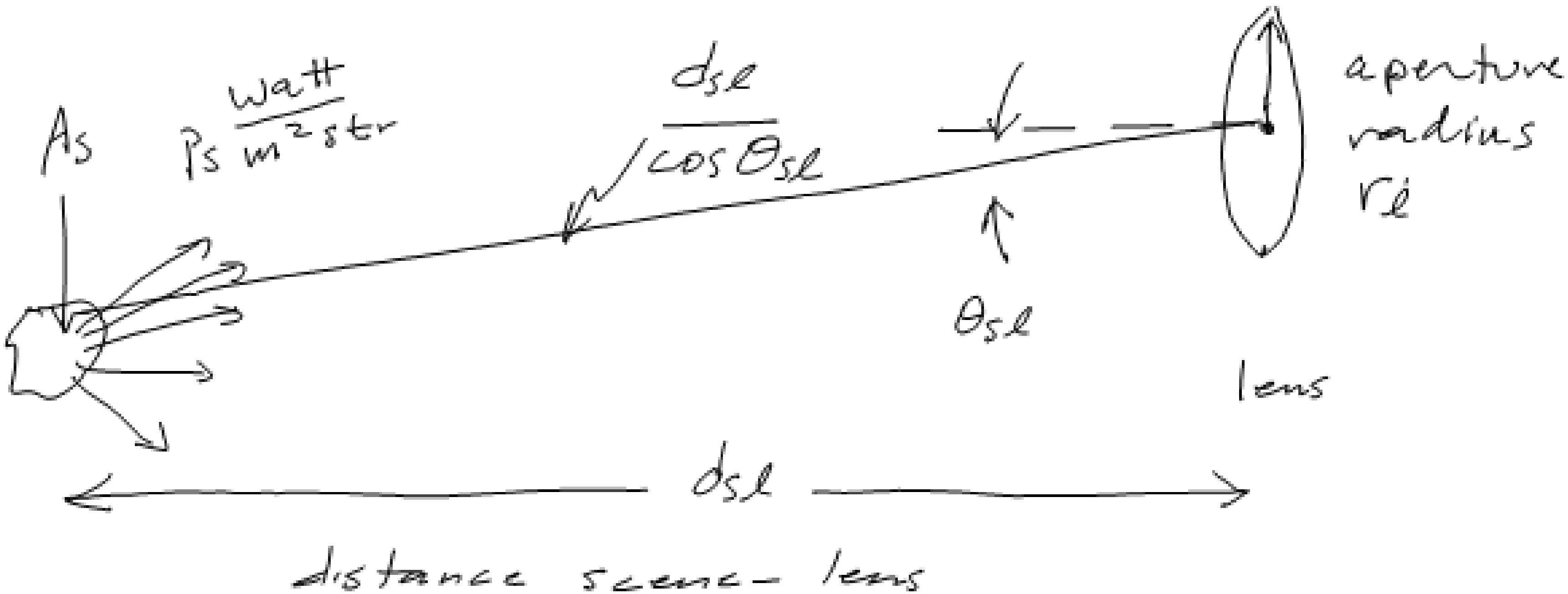
- from an idealized “point source” of light the energy per unit area (normal to the direction of the point source) per unit time falling on a target (or a sensor) falls off as $1/\text{distance}^2$
- but a real source is an area, never a point
  - if very close, it doesn’t fall off at all with distance
  - if line-like and not too close it falls off as distance^{-1}

from scene to lens

- consider a small area A_S of a scene emitting p_S watt/(m^2 steradian) in the spectral range of interest in the direction of the lens
- the power collected by lens P_L is then

$$P_L = A_S\, p_S\, \pi\, r_L^2 \cos\theta_{SL} / (d_{SL}/\cos\theta_{SL})^2$$

- this power is delivered by the lens to the sensor – but to what area of the sensor?
- (of course, this treatment ignores all the actual losses to scattering, absorption, etc)



power reaching lens

projected area of lens

$$P_\ell = A_s p_s \frac{(\pi r_\ell^2 \cos\theta_{s\ell})}{\dfrac{d_{s\ell}^2}{\cos^2\theta_{s\ell}}}$$

area of source

square of distance from this part of the scene to the lens

from lens to sensor (“detector” D)

lens equation: $1/d_{SL} + 1/d_{LD} = 1/f$

for simplicity, assume $d_{LD} << d_{SL}$ so $d_{LD} \approx f$

image area A_i corresponding to scene area A_S

is then given by ratios $A_i/f^2 = A_S/d_{SL}^2$

so the power per unit area on the sensor is

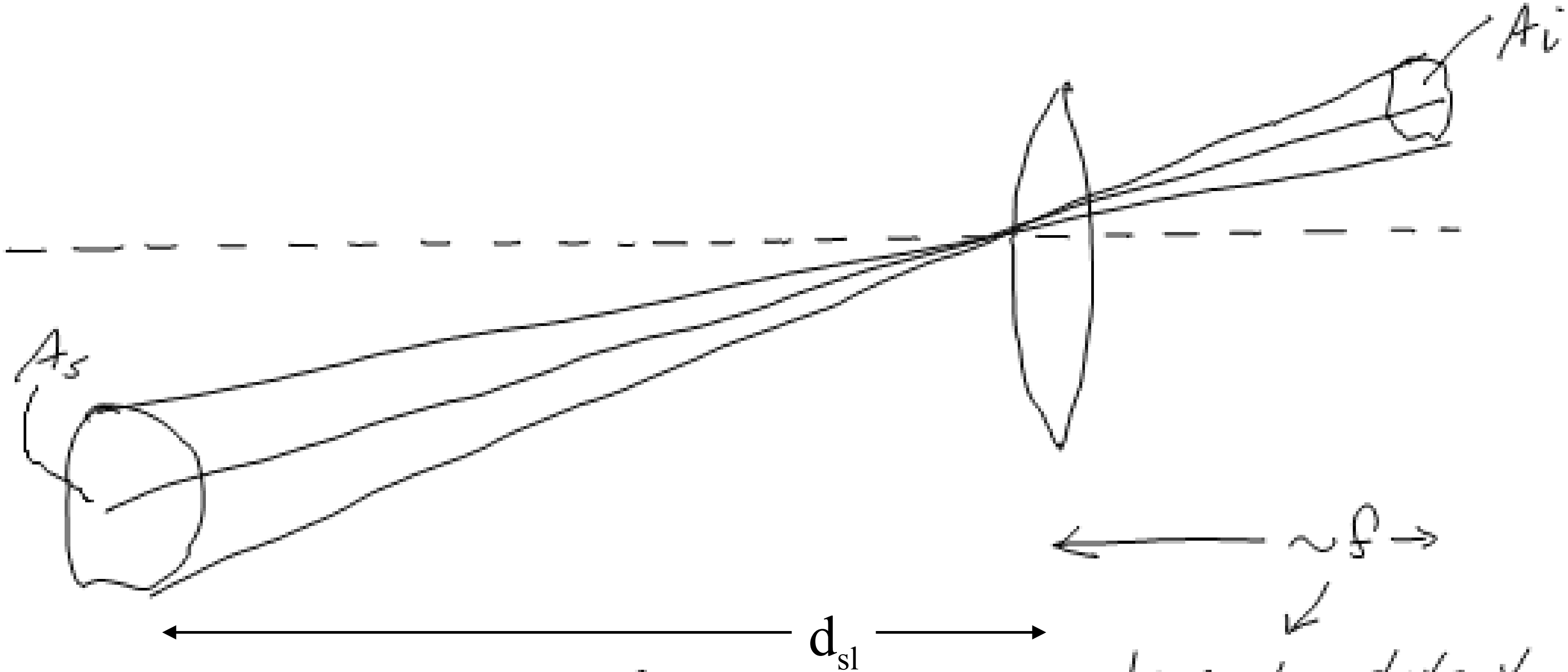
$$p_D = (A_S p_S \pi r_L^2 \cos^3\theta_{SL}/d_{SL}^2)/((A_S/\cos\theta_{SL}) f^2/d_{SL}^2)$$

$$= \pi\, p_S \cos^4\theta_{SL}\, (r_L/f)^2$$

$$= (\pi/4)\, p_S \cos^4\theta_{LD} / \text{f-number}^2$$

so scene-to-camera distance doesn’t matter!

but it says image gets dimmer as $\cos^4\theta_{LD}$ *...*



$$\frac{A_i}{A_s} = \frac{f^2}{d_{sl}^{\,2}}$$

lens-to-detector distance ≈ focal length if scene is far away

so what will the ultimate signal be?

we found $p_D \sim p_S$ / f-number^2

what does that tell us about the signal we can expect to see from the sensor?

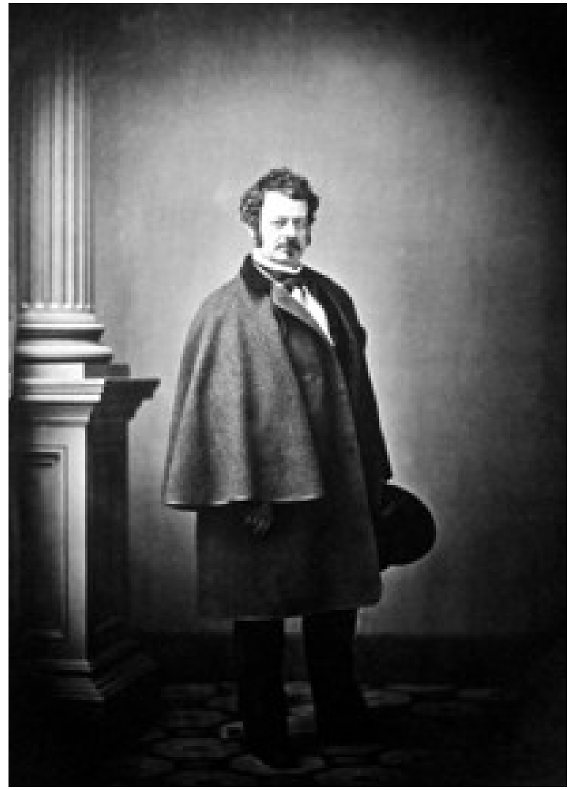
it depends on the sensor!

- typical sensor output (CCD signal voltage, photographic film blackness) is proportional to p_d times exposure time
  - (independent of pixel area for CCD, but not for film!)
- others might deliver, e.g., output current proportional to p_d times pixel area (but independent of exposure time!)

“sensing” is the preceding fundamentals …

… “sensors” are these still-open details

but it says image gets dimmer as $\cos^4\theta_{LD}$ …



… and if you look at OLD photographs it does!

assignment

5) For a scene illuminated by typical Pittsburgh sunlight (how many watts/m ?) estimate the