Carnegie Mellon

School of Computer Science

Deep Reinforcement Learning and Control

Asynchronous RL

CMU 10703

Katerina Fragkiadaki

Non-stationary data problem for Deep RL

- Stability of training neural networks requires the **gradient updates to be de-correlated**
- This is not the case if data arrives **sequentially**
- Gradient updates computed from some part of the space can cause the value (Q) function approximator to **oscillate**
- Our solution so far has been: **Experience buffers** where experience tuples are mixed and sampled from. Resulting sampled batches are more stationary that the ones encountered online (without buffer)
- This limits deep RL to **off-policy** methods, since data from an older policy are used to update the weights of the value approximator.

Asynchronous Deep RL

Asynchronous Methods for Deep Reinforcement Learning

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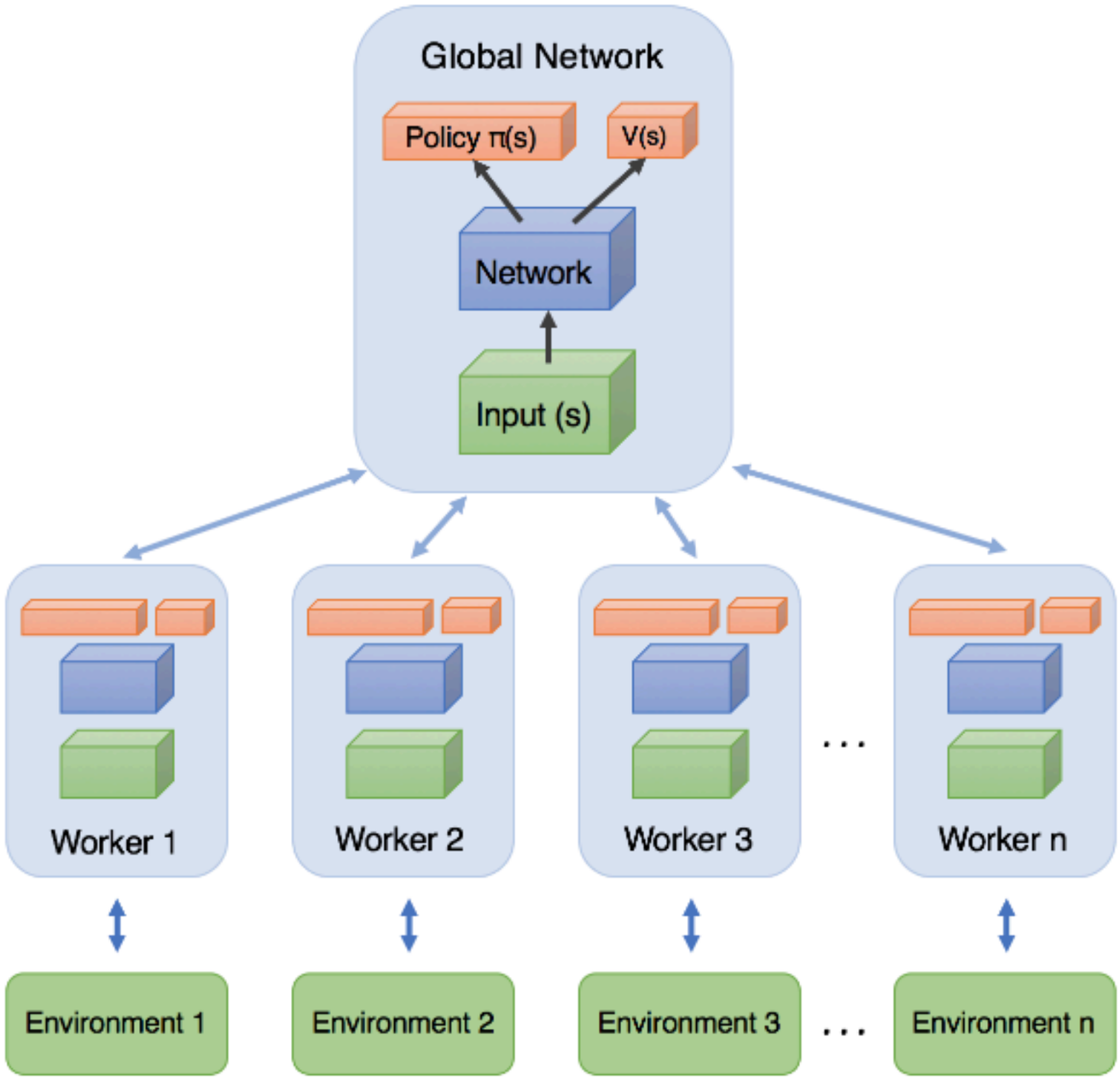
- Alternative: parallelize the collection of experience and stabilize training **without experience buffers**!

- **Multiple threads of experience**, one per agent, each exploring in different part of the environment contributing experience tuples

- **Different exploration strategies** (e.g., various \epsilon values) in different threads increase diversity

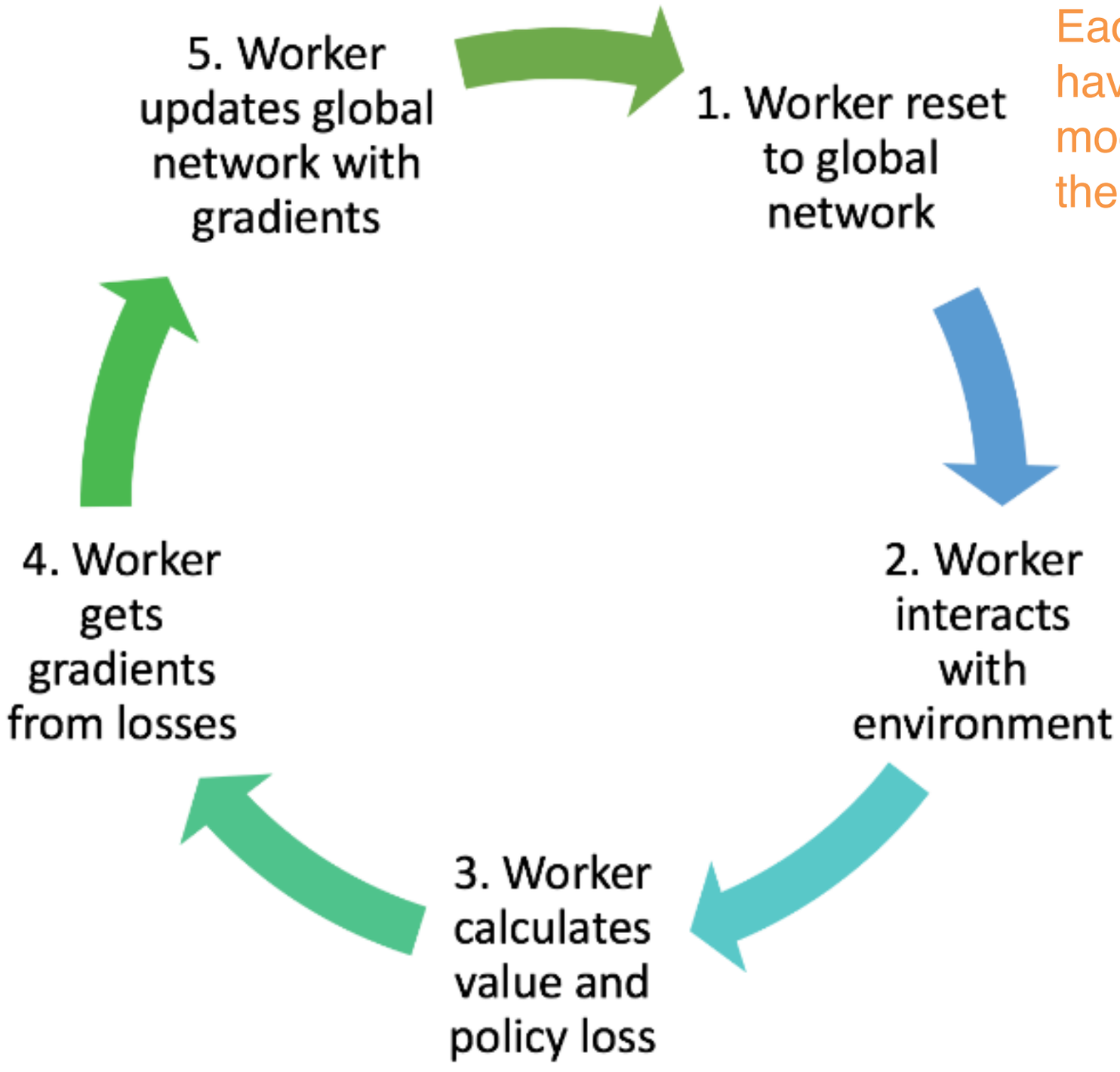
- It can be applied to **both on policy and off policy** methods, applied it to SARSA, DQN, and advantage actor-critic

Distributed RL



Distributed Asynchronous RL

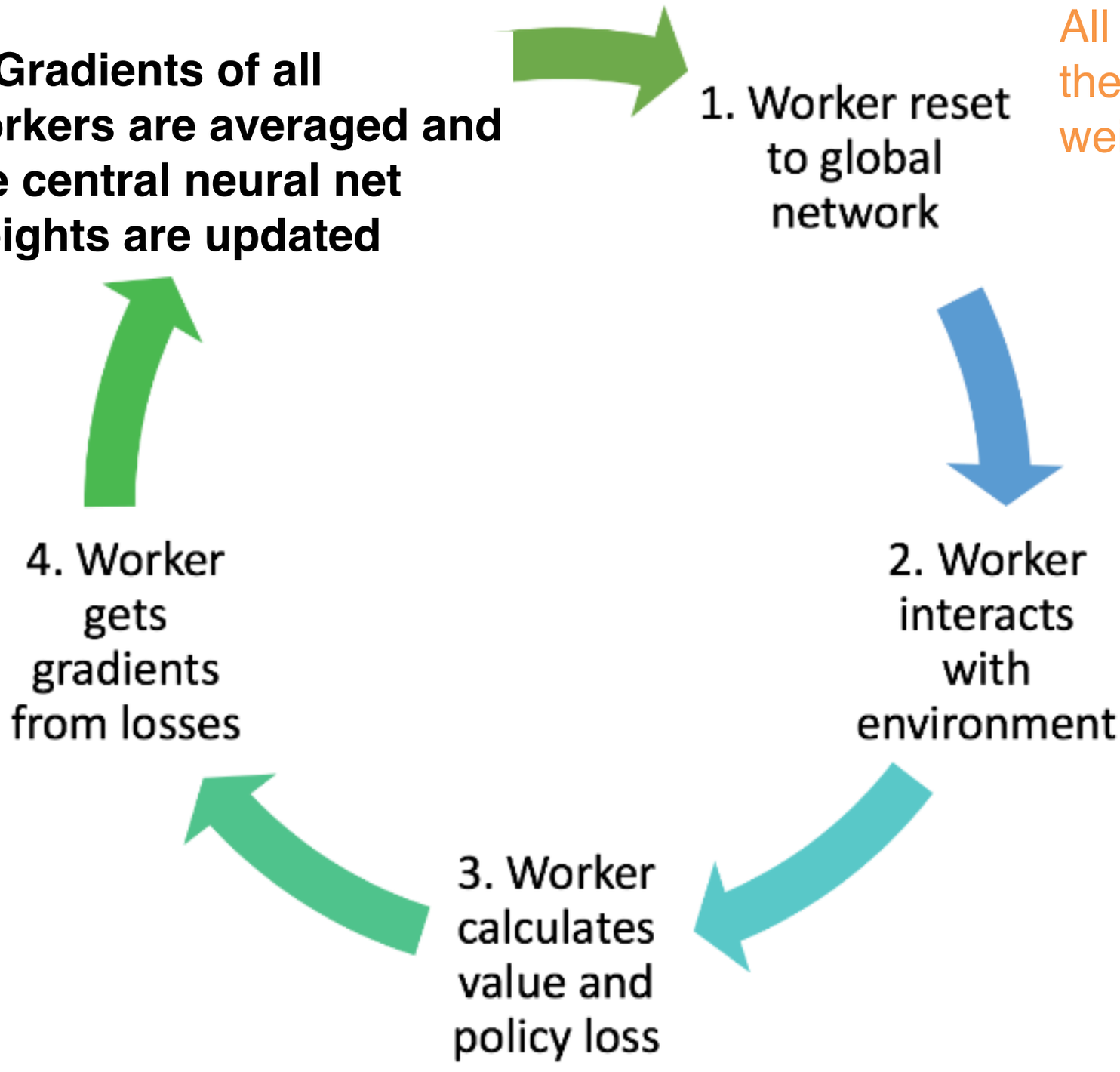
No locking



Each worker may have slightly modified version of the policy/critic

The actor critic trained in such asynchronous way is knows as A3C

Distributed Synchronous RL



All worker may have the same actor/critic weights

The actor critic trained in such synchronous way is knows as A2C

Algorithm S3 Asynchronous advantage actor-critic - pseudocode for each actor-learner thread.

// Assume global shared parameter vectors θ *and* θ_v *and global shared counter* $T = 0$
// Assume thread-specific parameter vectors θ' *and* θ'_v
Initialize thread step counter $t \leftarrow 1$
repeat
  Reset gradients: $d\theta \leftarrow 0$ and $d\theta_v \leftarrow 0$.
  Synchronize thread-specific parameters $\theta' = \theta$ and $\theta'_v = \theta_v$
  $t_{start} = t$
  Get state s_t
  repeat
    Perform a_t according to policy $\pi(a_t|s_t;\theta')$
    Receive reward r_t and new state s_{t+1}
    $t \leftarrow t + 1$
    $T \leftarrow T + 1$
  until terminal s_t **or** $t - t_{start} == t_{max}$
  $R = \begin{cases} 0 & \text{for terminal } s_t \\ V(s_t, \theta'_v) & \text{for non-terminal } s_t \text{// Bootstrap from last state} \end{cases}$
  for $i \in \{t-1, \ldots, t_{start}\}$ **do**
    $R \leftarrow r_i + \gamma R$
    Accumulate gradients wrt θ': $d\theta \leftarrow d\theta + \nabla_{\theta'} \log \pi(a_i|s_i;\theta')(R - V(s_i;\theta'_v))$
    Accumulate gradients wrt θ'_v: $d\theta_v \leftarrow d\theta_v + \partial \left(R - V(s_i;\theta'_v)\right)^2 / \partial \theta'_v$
  end for
  Perform asynchronous update of θ using $d\theta$ and of θ_v using $d\theta_v$.
until $T > T_{max}$

A3C

s_1, s_2, s_3, s_4

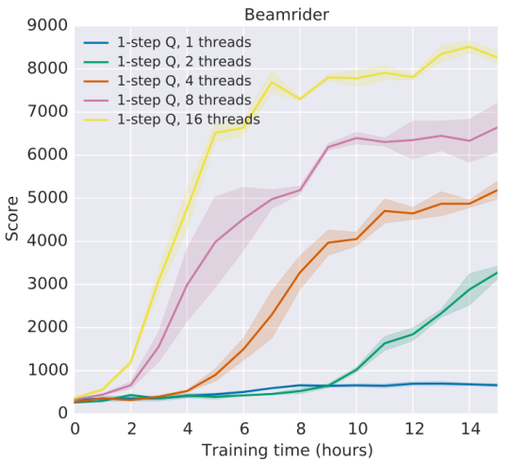
r_1, r_2, r_3

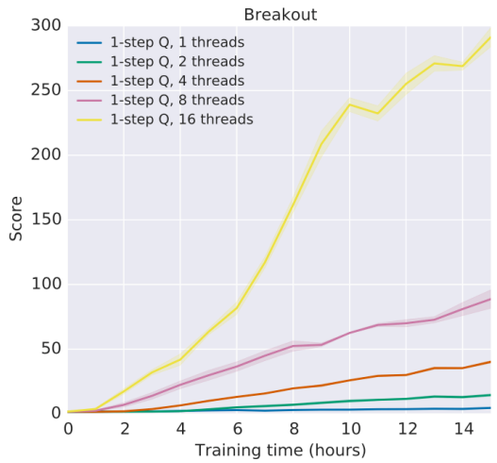
What is the approximation used for the advantage?

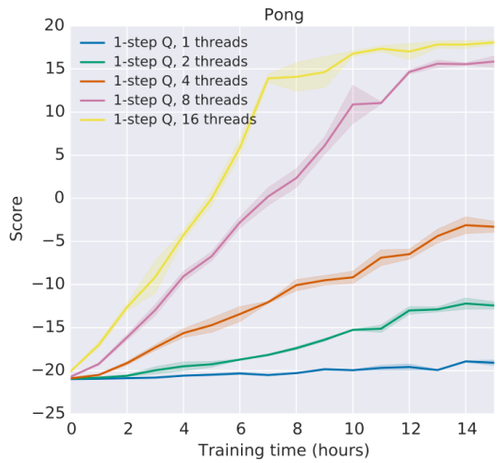
$R_3 = r_3 + \gamma V(s_4, \theta'_v)$ $\qquad$ $A_3 = R_3 - V(s_3; \theta'_v)$

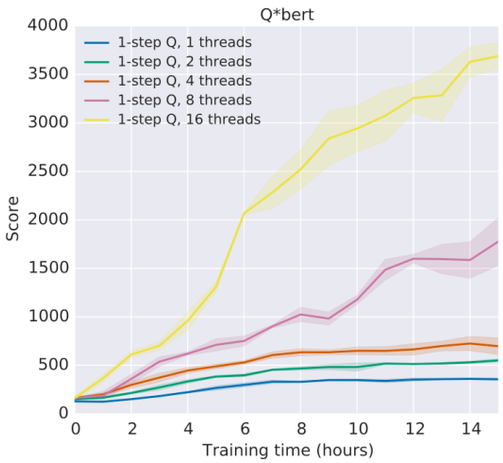
$R_2 = r_2 + \gamma r_3 + \gamma^2 V(s_4, \theta'_v)$ $\qquad$ $A_2 = R_2 - V(s_2; \theta'_v)$

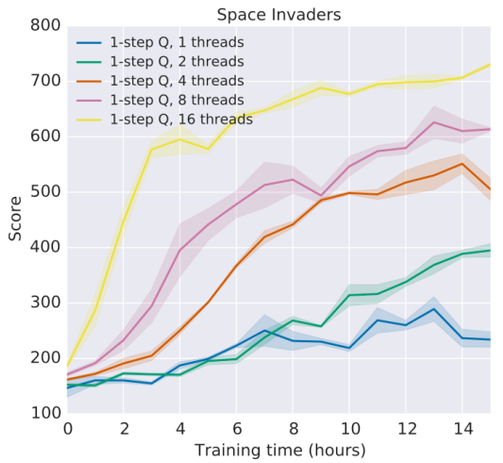
Advantages of Asynchronous (multi-threaded) RL

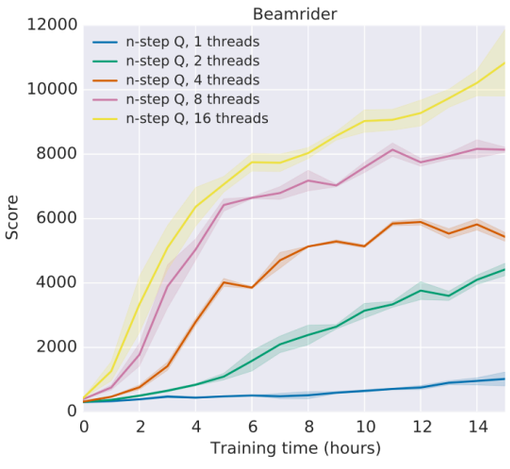


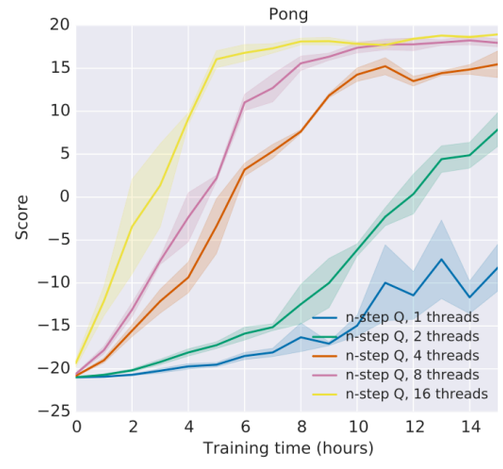


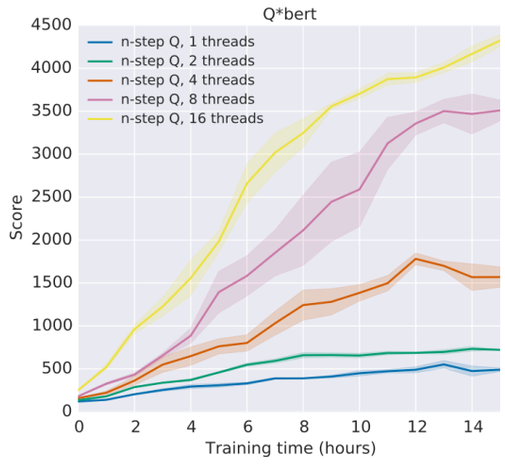


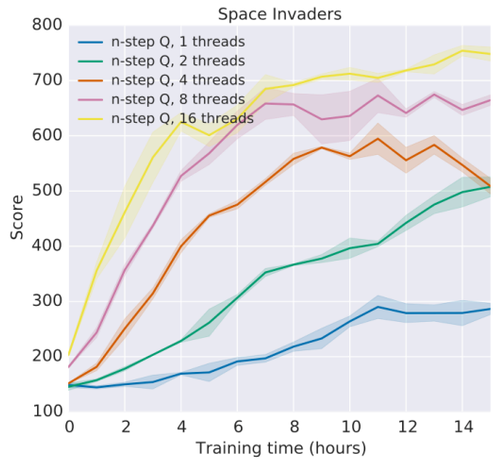


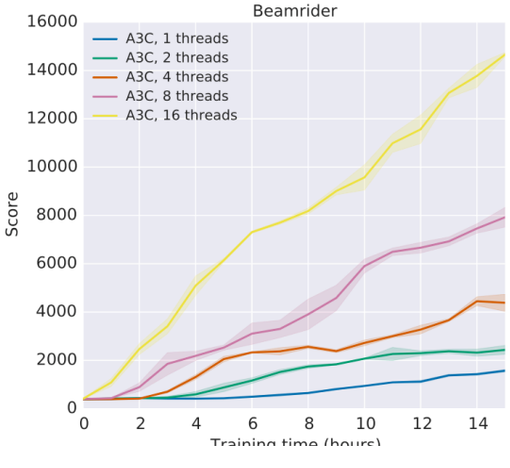


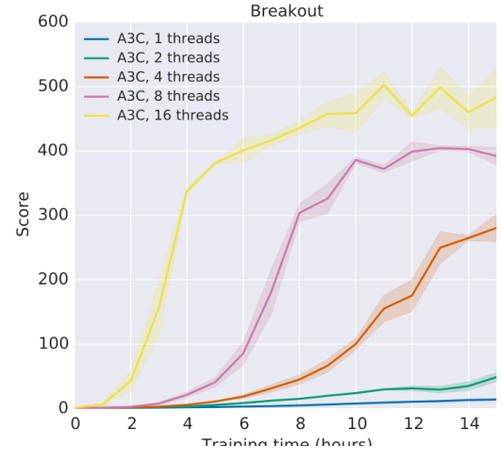


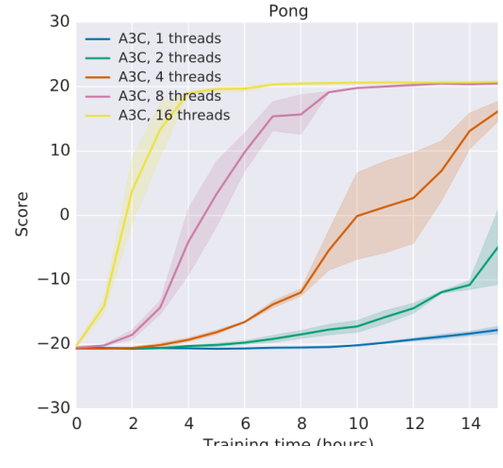


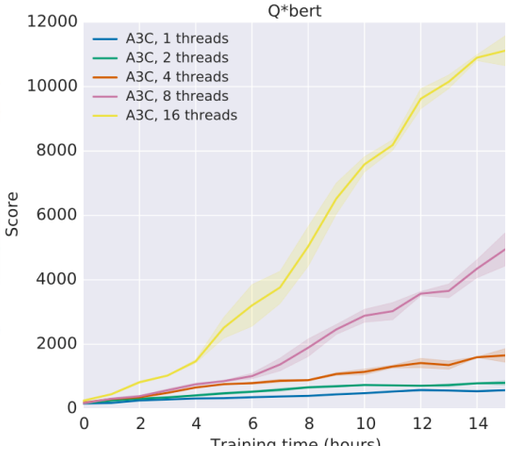


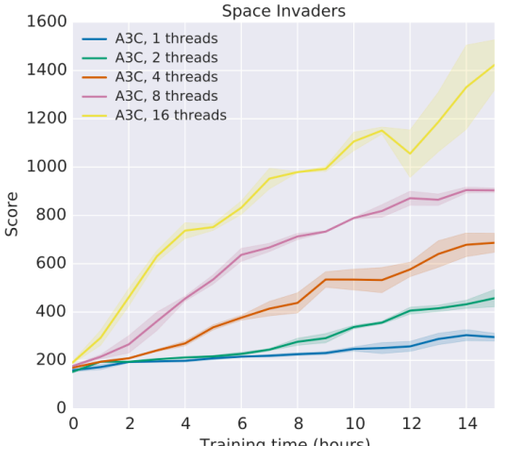












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Evolutionary Methods

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Part of the slides borrowed by Xi Chen, Pieter Abbeel, John Schulman

Policy Optimization

$$\max_{\theta} J(\theta) = \max_{\theta} \mathbb{E}\left[R(\tau) \,|\, \pi_\theta, \mu_0(s_0)\right]$$

τ :a trajectory

Policy Optimization

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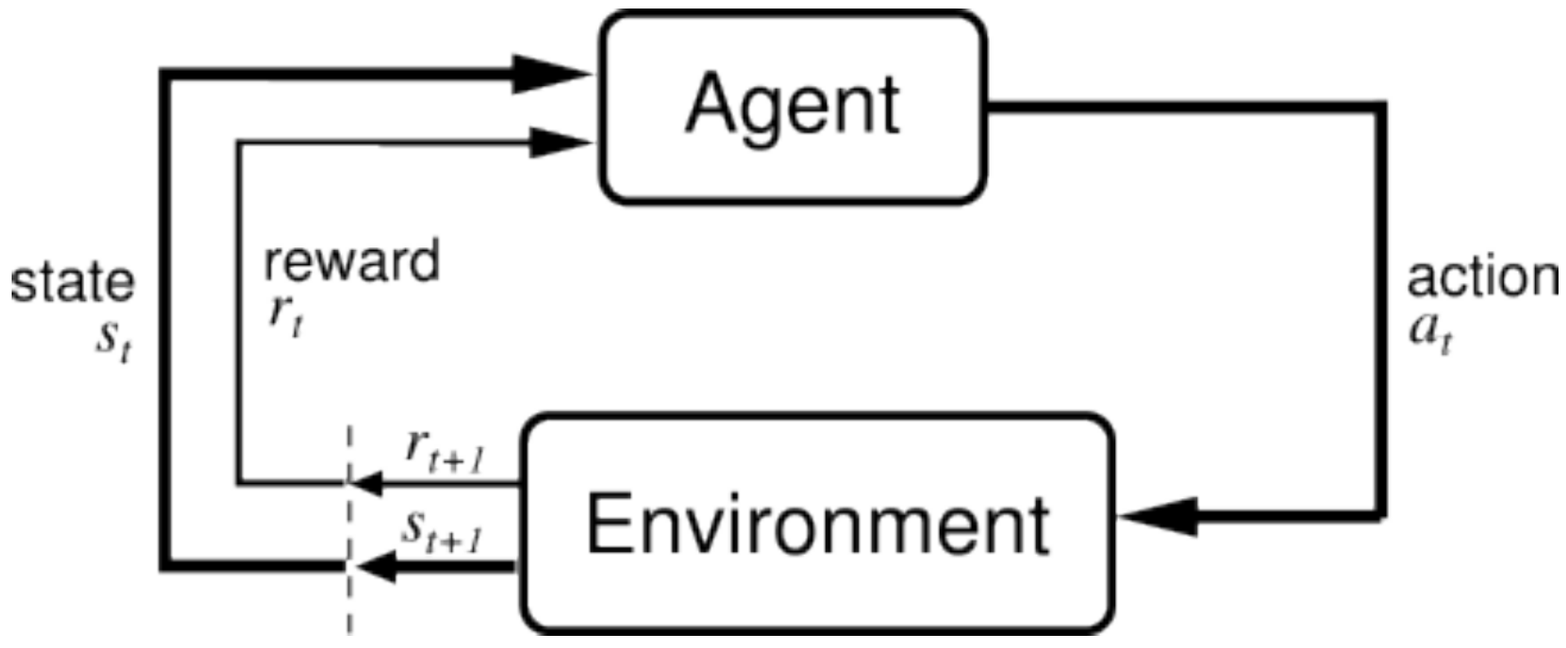
Policy Optimization and RL

$$\max_{\theta} J(\theta) = \max_{\theta} \mathbb{E}\left[R(\tau) \,|\, \pi_\theta, \mu_0(s_0)\right] = \max_{\theta} \mathbb{E}\left[\sum_{t=0}^{T} R(s_t) \,|\, \pi_\theta, \mu_0(s)\right]$$



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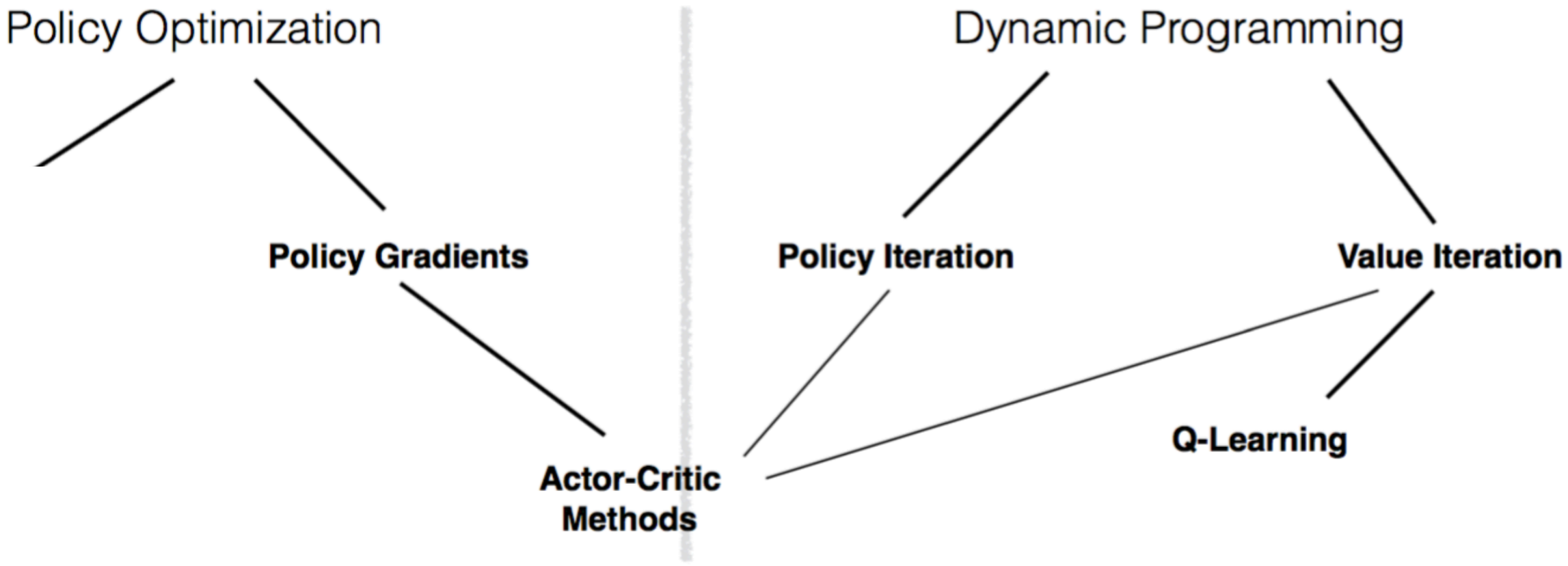

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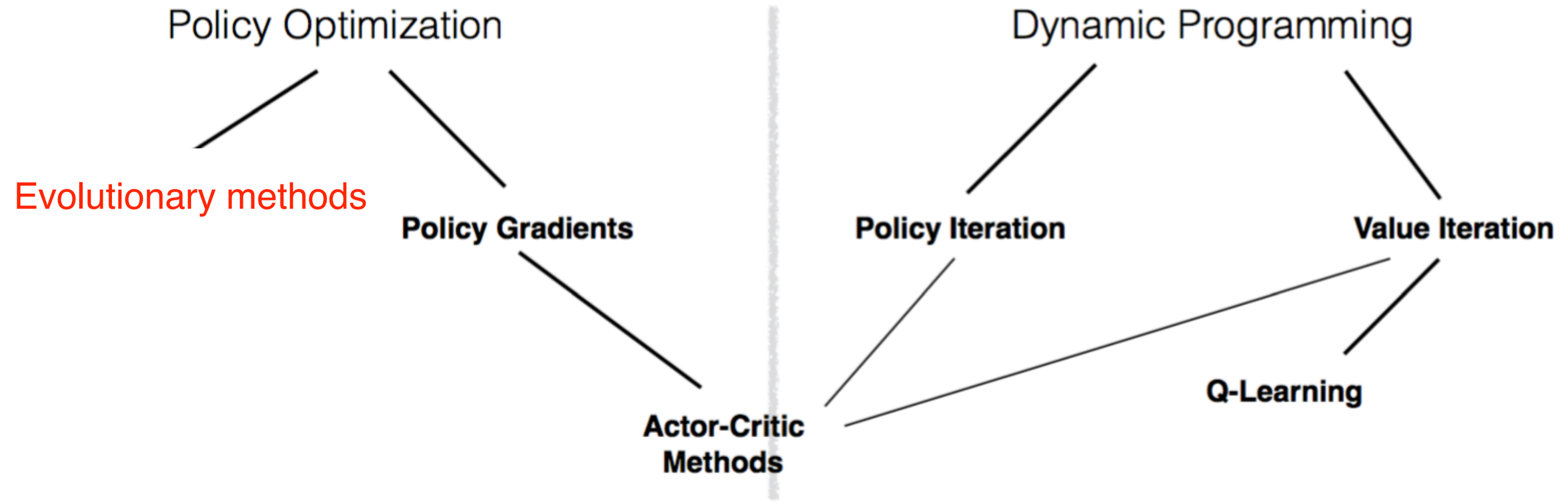


Black-box Policy Optimization

$$\max_{\theta} J(\theta) = \max_{\theta} \mathbb{E}\left[R(\tau) \,|\, \pi_\theta, \mu_0(s_0)\right]$$

No information regarding the structure of the reward

Evolutionary methods

$$\max_{\theta} J(\theta) = \max_{\theta} \mathbb{E}\left[R(\tau) \,|\, \pi_\theta, \mu_0(s_0)\right]$$

General algorithm:

Initialize a population of parameter vectors (*genotypes*)

1. Make random perturbations (*mutations*) to each parameter vector
2. Evaluate the perturbed parameter vector (*fitness*)
3. Keep the perturbed vector if the result improves (*selection*)
4. GOTO 1

Biologically plausible…

Cross-entropy method

Let's consider our parameters to be sampled from a multivariate isotropic Gaussian
We will evolve this Gaussian towards sampled that have highest fitness

CEM:

Initialize $\mu \in \mathbb{R}^d, \sigma \in \mathbb{R}^d_{>0}$

for iteration = 1, 2, …

Sample n parameters $\theta_i \sim N(\mu, \mathrm{diag}(\sigma^2))$

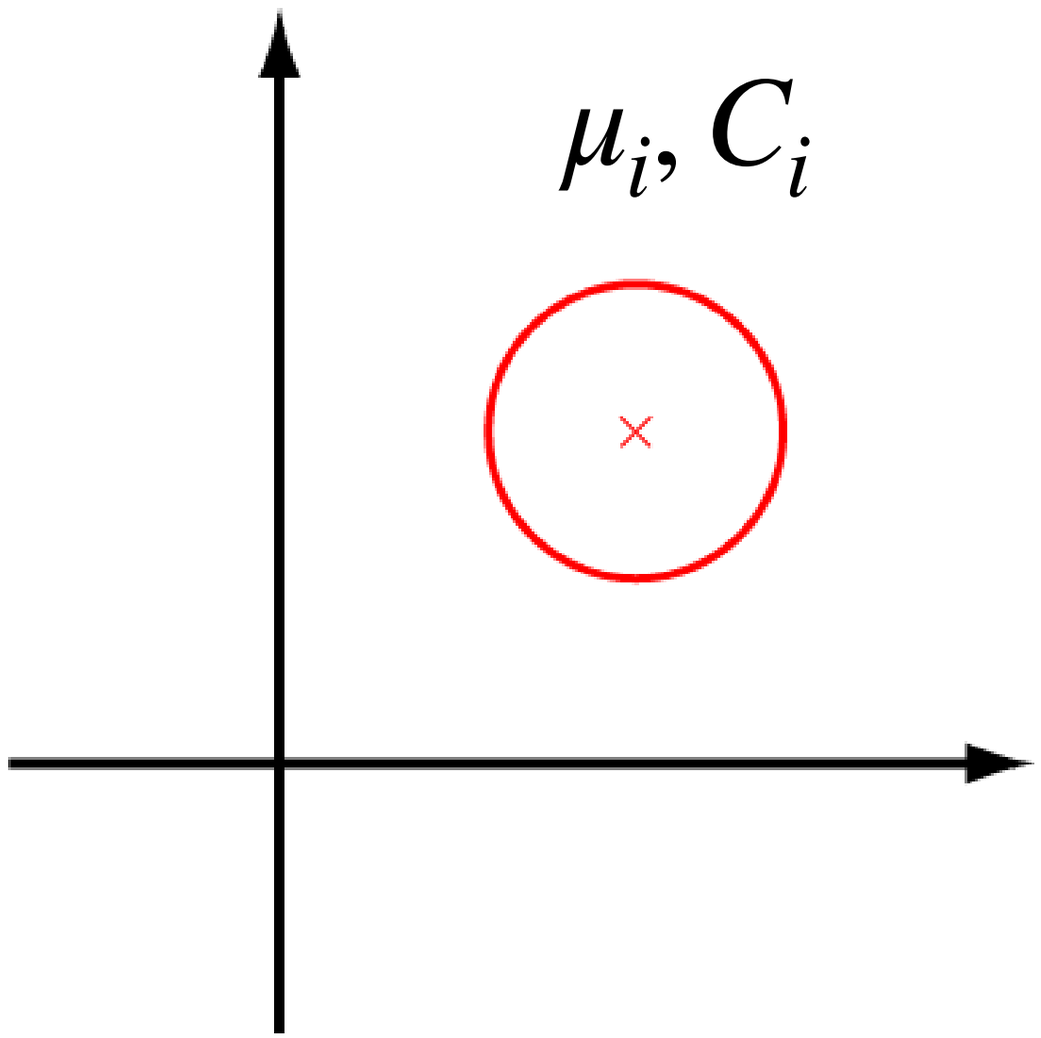
For each θ_i, perform one rollout to get return $R(\tau_i)$

Select the top k% of θ, and fit a new diagonal Gaussian to those samples. Update μ, σ

endfor

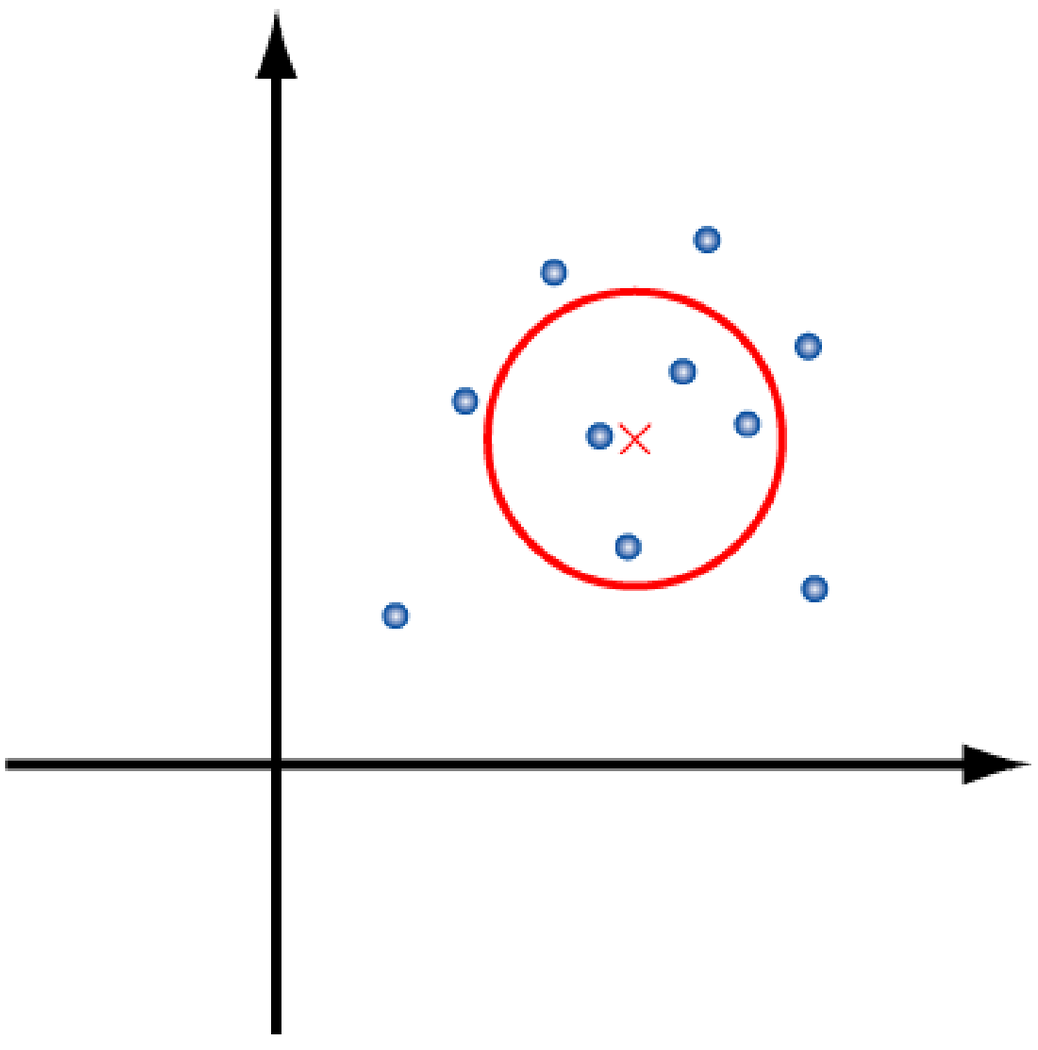
Covariance Matrix Adaptation

Let's consider our parameters to be sampled from a multivariate Gaussian
We will evolve this Gaussian towards sampled that have highest fitness



- Sample
- Select elites
- Update mean
- Update covariance
- iterate

Covariance Matrix Adaptation



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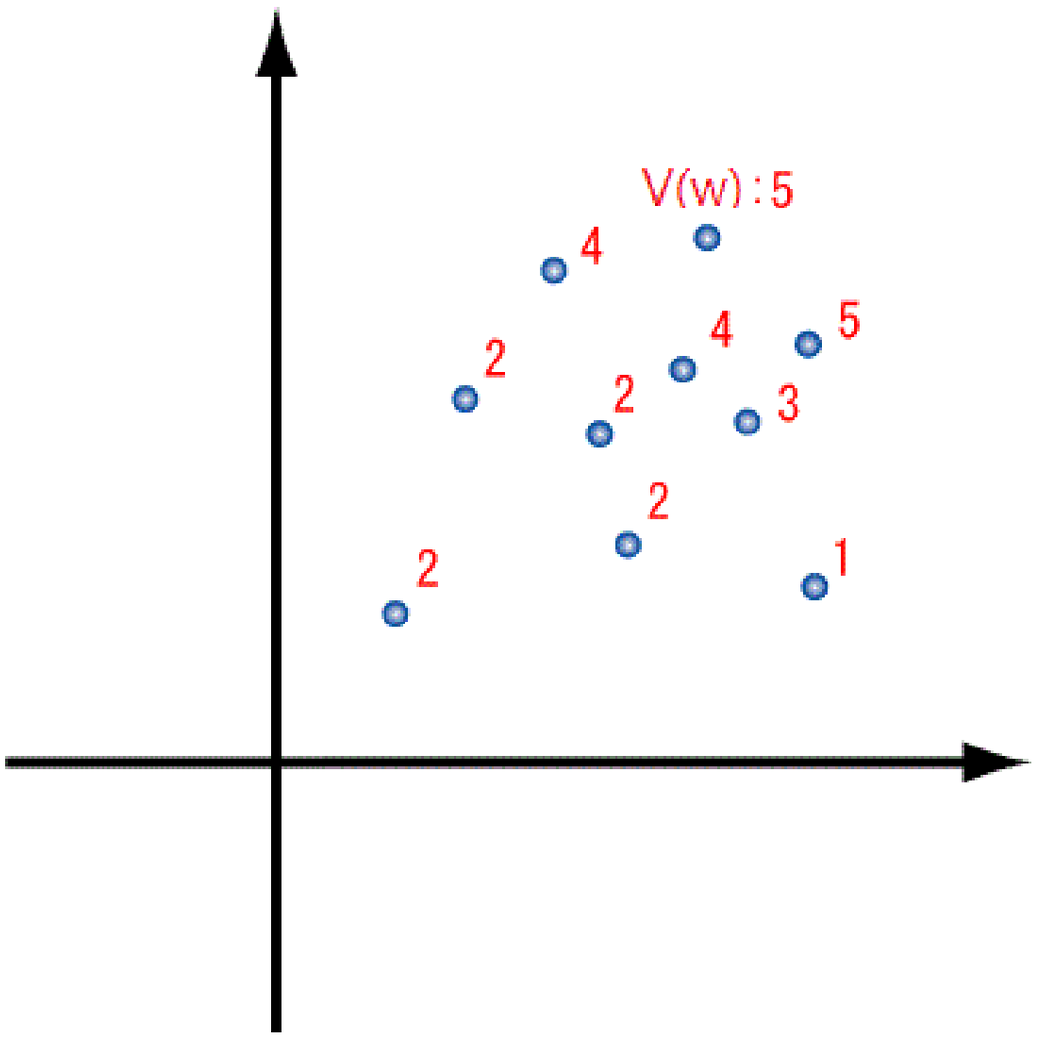
Covariance Matrix Adaptation



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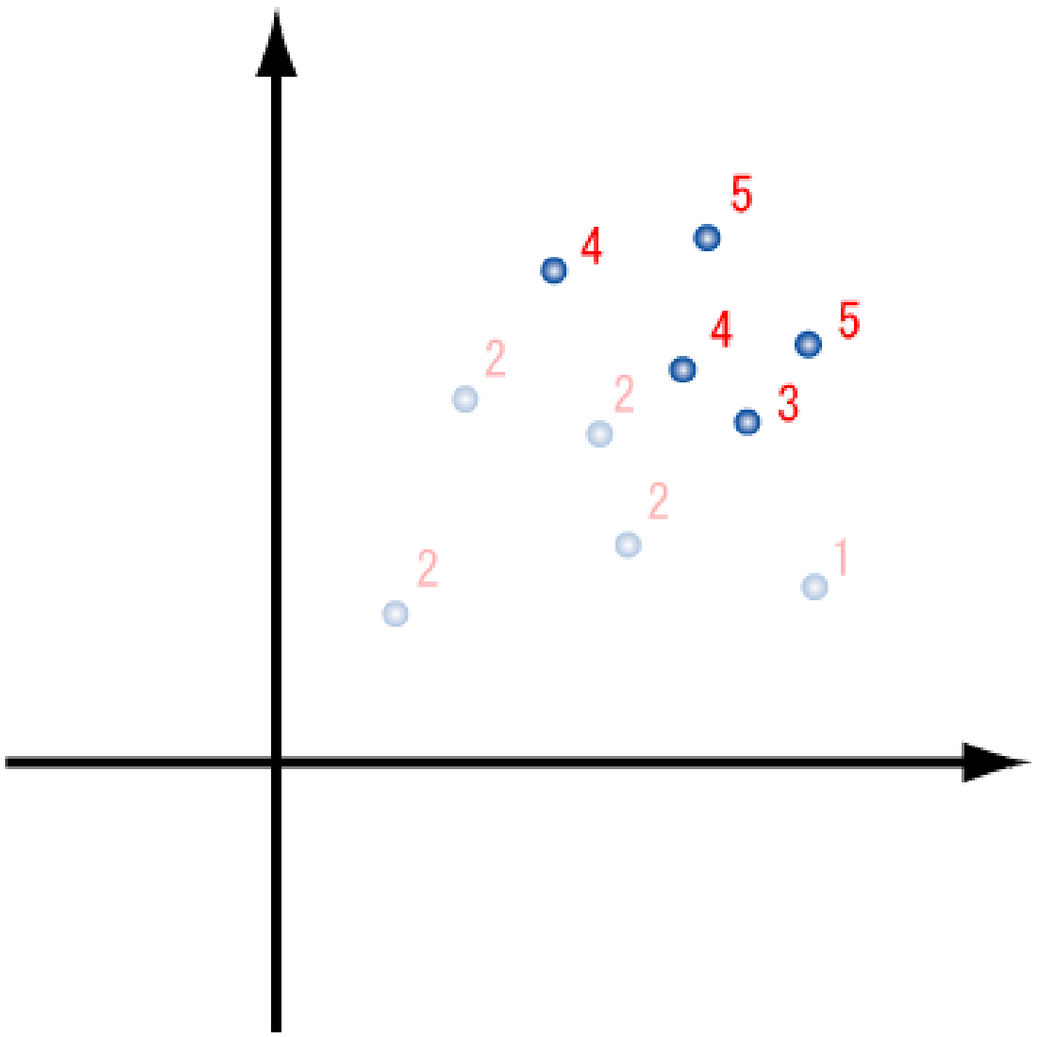
Covariance Matrix Adaptation



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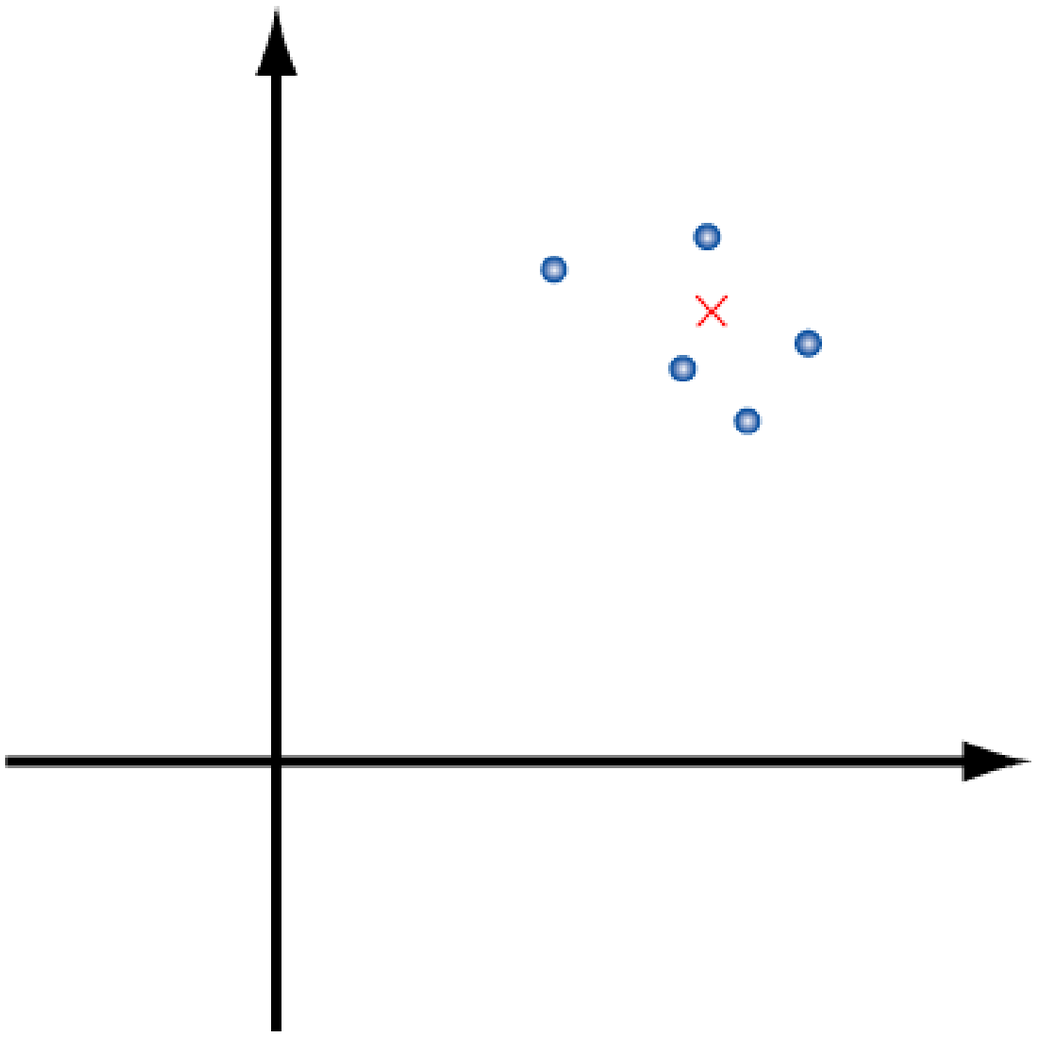
Covariance Matrix Adaptation



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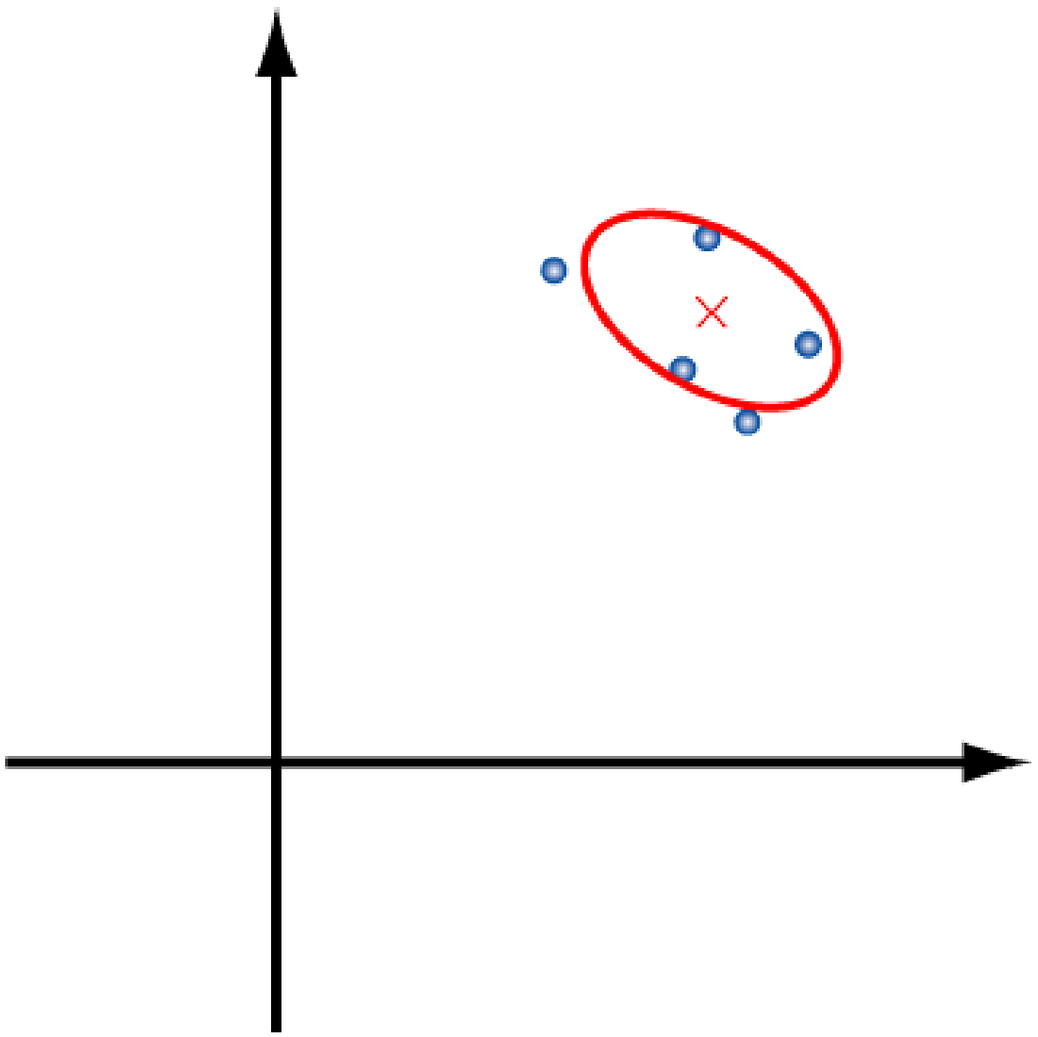

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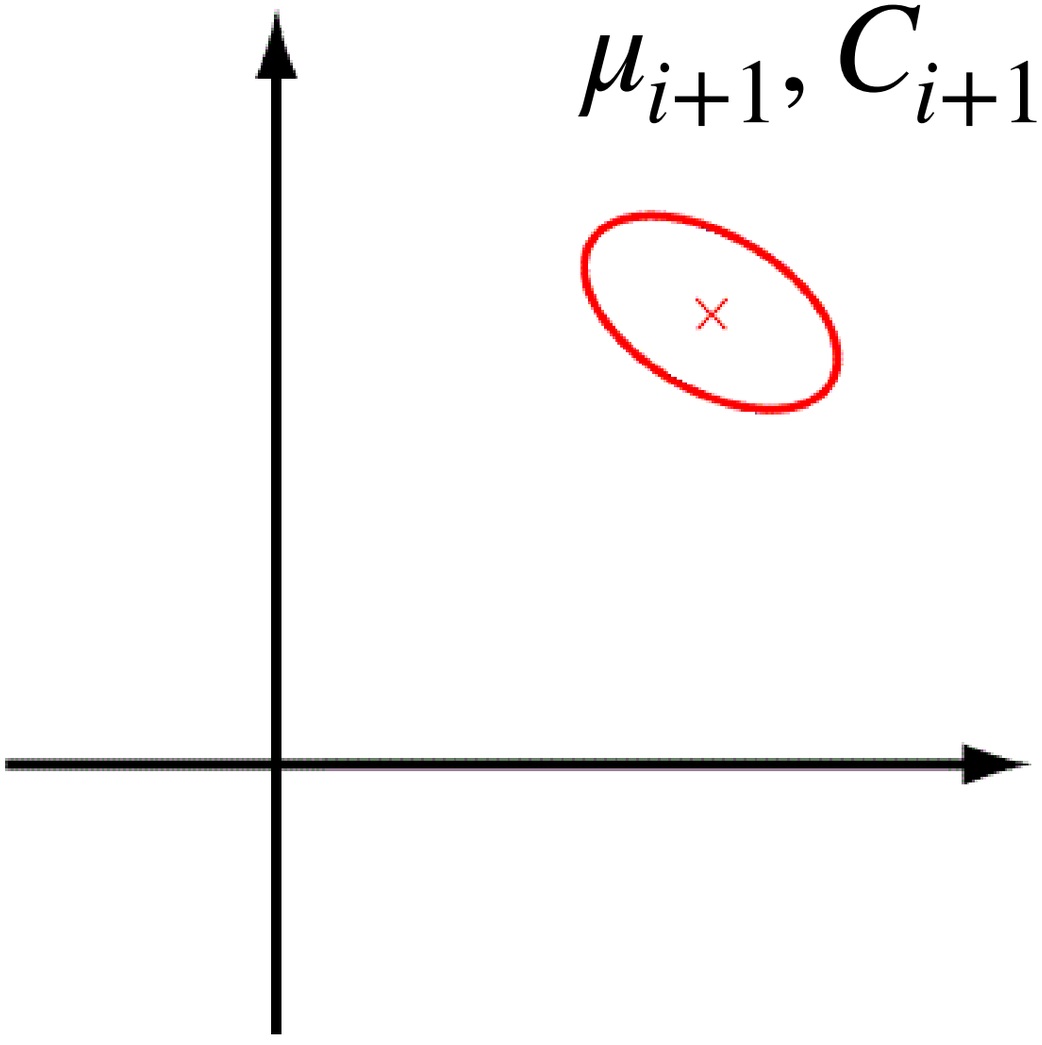
Covariance Matrix Adaptation



- Sample
- Select elites
- Update mean
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- iterate

CMA-ES, CEM

Work embarrassingly well in low-dimensions

Method	Mean Score	Reference
Nonreinforcement learning		
Hand-coded	631,167	Dellacherie (Fahey, 2003)
Genetic algorithm	586,103	(Böhm et al., 2004)
Reinforcement learning		
Relational reinforcement learning+kernel-based regression	≈50	Ramon and Driessens (2004)
Policy iteration	3183	Bertsekas and Tsitsiklis (1996)
Least squares policy iteration	<3000	Lagoudakis, Parr, and Littman (2002)
Linear programming + Bootstrap	4274	Farias and van Roy (2006)
Natural policy gradient	≈6800	Kakade (2001)
CE+RL	21,252	
CE+RL, constant noise	72,705	
CE+RL, decreasing noise	348,895	

István Szita and András Lörincz. "Learning Tetris using the noisy cross-entropy method". In: *Neural computation* 18.12 (2006), pp. 2936–2941

$\mu \in \mathbb{R}^{22}$

Approximate Dynamic Programming Finally Performs Well in the Game of Tetris

[NIPS 2013]

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Question

- Evolutionary methods work well on relatively low-dim problems
- Can they be used to optimize deep network policies?

PG VS ES

We are sampling in both cases…

Policy Gradients Review

$$\max_{\theta} . \; J(\theta) = \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[R(\tau) \right]$$

$$\begin{aligned}
\nabla_\theta J(\theta) &= \nabla_\theta \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[R(\tau) \right] \\
&= \nabla_\theta \sum_\tau P_\theta(\tau) R(\tau) \\
&= \sum_\tau \nabla_\theta P_\theta(\tau) R(\tau) \\
&= \sum_\tau P_\theta(\tau) \frac{\nabla_\mu P_\theta(\tau)}{P_\theta(\tau)} R(\tau) \\
&= \sum_\tau P_\theta(\tau) \nabla_\theta \log P_\theta(\tau) R(\tau) \\
&= \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[\nabla_\theta \log P_\theta(\tau) R(\tau) \right]
\end{aligned}$$

Sample estimate:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)}) R(\tau^{(i)})$$

ES

Considers distribution over policy parameters

$$\max_{\mu} . \; U(\mu) = \mathbb{E}_{\theta \sim P_\mu(\theta)} \left[F(\theta) \right]$$

$$\begin{aligned}
\nabla_\mu U(\mu) &= \nabla_\mu \mathbb{E}_{\theta \sim P_\mu(\theta)} \left[F(\theta) \right] \\
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&= \int \nabla_\mu P_\mu(\theta) F(\theta) d\theta \\
&= \int P_\mu(\theta) \frac{\nabla_\mu P_\mu(\theta)}{P_\mu(\theta)} F(\theta) d\theta \\
&= \int P_\mu(\theta) \nabla_\mu \log P_\mu(\theta) F(\theta) d\theta \\
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From trajectories to actions

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)}) R(\tau^{(i)}) \quad \Rightarrow \quad \nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(a_t^{(i)} \mid s_t^{(i)}) R(s_t^{(i)}, a_t^{(i)})$$

$$\nabla_\theta \log P(\tau^{(i)}; \theta) = \nabla_\theta \log \left[\prod_{t=0}^{T} \underbrace{P(s_{t+1}^{(i)} \mid s_t^{(i)}, a_t^{(i)})}_{\text{dynamics}} \cdot \underbrace{\pi_\theta(a_t^{(i)} \mid s_t^{(i)})}_{\text{policy}} \right]$$

$$= \nabla_\theta \left[\sum_{t=0}^{T} \underbrace{\log P(s_{t+1}^{(i)} \mid s_t^{(i)}, a_t^{(i)})}_{\text{dynamics}} + \underbrace{\log \pi_\theta(a_t^{(i)} \mid s_t^{(i)})}_{\text{policy}} \right]$$

$$= \nabla_\theta \left[\sum_{t=0}^{T} \underbrace{\log \pi_\theta(a_t^{(i)} \mid s_t^{(i)})}_{\text{policy}} \right]$$

$$= \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^{(i)} \mid s_t^{(i)})$$

PG

Considers distribution over actions

$$\max_{\theta} . \; J(\theta) = \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[R(\tau) \right]$$

$$\begin{aligned}
\nabla_\theta J(\theta) &= \nabla_\theta \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[R(\tau) \right] \\
&= \nabla_\theta \sum_\tau P_\theta(\tau) R(\tau) \\
&= \sum_\tau \nabla_\theta P_\theta(\tau) R(\tau) \\
&= \sum_\tau P_\theta(\tau) \frac{\nabla_\mu P_\theta(\tau)}{P_\theta(\tau)} R(\tau) \\
&= \sum_\tau P_\theta(\tau) \nabla_\theta \log P_\theta(\tau) R(\tau) \\
&= \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[\nabla_\theta \log P_\theta(\tau) R(\tau) \right]
\end{aligned}$$

Sample estimate:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \,|\, s_t^{(i)}) R(s_t^{(i)}, a_t^{(i)})$$

ES

Considers distribution over policy parameters

$$\max_{\mu} . \; U(\mu) = \mathbb{E}_{\theta \sim P_\mu(\theta)} \left[F(\theta) \right]$$

$$\begin{aligned}
\nabla_\mu U(\mu) &= \nabla_\mu \mathbb{E}_{\theta \sim P_\mu(\theta)} \left[F(\theta) \right] \\
&= \nabla_\mu \int P_\mu(\theta) F(\theta) d\theta \\
&= \int \nabla_\mu P_\mu(\theta) F(\theta) d\theta \\
&= \int P_\mu(\theta) \frac{\nabla_\mu P_\mu(\theta)}{P_\mu(\theta)} F(\theta) d\theta \\
&= \int P_\mu(\theta) \nabla_\mu \log P_\mu(\theta) F(\theta) d\theta \\
&= \mathbb{E}_{\theta \sim P_\mu(\theta)} \left[\nabla_\mu \log P_\mu(\theta) F(\theta) \right]
\end{aligned}$$

Sample estimate:

$$\nabla_\mu U(\mu) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\mu \log P_\mu(\theta^{(i)}) F(\theta^{(i)})$$

A concrete example

- Suppose $\theta \sim P_\mu(\theta)$ is a Gaussian distribution with mean μ, and covariance matrix $\sigma^2 I$

$$\log P_\mu(\theta) = -\frac{||\theta - \mu||^2}{2\sigma^2} + \text{const}$$

$$\nabla_\mu \log P_\mu(\theta) = \frac{\theta - \mu}{\sigma^2}$$

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$$\nabla_\mu \log P_\mu(\theta) = \frac{\theta - \mu}{\sigma^2}$$

- If we draw two parameter samples θ_1, θ_2, and obtain two trajectories τ_1, τ_2:

$$\mathbb{E}_{\theta \sim P_\mu(\theta)} \left[\nabla_\mu log P_\mu(\theta) R(\tau) \right] \approx \frac{1}{2} \left[R(\tau_1) \frac{\theta_1 - \mu}{\sigma^2} + R(\tau_2) \frac{\theta_2 - \mu}{\sigma^2} \right]$$

Sampling parameter vectors

- Suppose $\theta \sim P_\mu(\theta)$ is a Gaussian distribution with mean μ, and covariance matrix $\sigma^2 I$

Imagine we have access to random vectors $\epsilon \sim \mathcal{N}(0,I)$

$$\theta_1 = \mu + \sigma * \epsilon_1,\ \epsilon_1 \sim \mathcal{N}(0,I)$$

$$\theta_2 = \mu + \sigma * \epsilon_2,\ \epsilon_2 \sim \mathcal{N}(0,I)$$

The theta samples have the desired mean and variance

A concrete example

- Suppose $\theta \sim P_\mu(\theta)$ is a Gaussian distribution with mean μ, and covariance matrix $\sigma^2 I$

$$\log P_\mu(\theta) = -\frac{||\theta - \mu||^2}{2\sigma^2} + \text{const}$$

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$$\approx \frac{1}{2\sigma}\left[R(\tau_1)\epsilon_1 + R(\tau_2)\epsilon_2\right]$$

$\theta_1 = \mu + \sigma * \epsilon_1,\ \epsilon_1 \sim \mathcal{N}(0, I)$

$\theta_2 = \mu + \sigma * \epsilon_2,\ \epsilon_2 \sim \mathcal{N}(0, I)$

Natural Evolutionary Strategies

Algorithm 1 Evolution Strategies

Input: Learning rate α, noise standard deviation σ, initial policy parameters θ_0
for $t = 0, 1, 2, \ldots$ **do**
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    Compute returns $F_i = F(\theta_t + \sigma\epsilon_i)$ for $i = 1, \ldots, n$
    Set $\theta_{t+1} \leftarrow \theta_t + \alpha \frac{1}{n\sigma} \sum_{i=1}^{n} F_i \epsilon_i$
end for

Connection to Finite Differences

- Antithetic sampling
  - Sample a pair of policies with mirror noise $(\theta_+ = \mu + \sigma\epsilon, \theta_- = \mu - \sigma\epsilon)$

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  - SPSA: Finite Difference with random direction

$$\begin{aligned}\nabla_\mu \mathbb{E}\left[R(\tau)\right] &\approx \frac{1}{2}\left[R(\tau_+)\frac{\theta_+ - \mu}{\sigma^2} + R(\tau_-)\frac{\theta_- - \mu}{\sigma^2}\right] \\ &= \frac{1}{2}\left[R(\tau_+)\frac{\sigma\epsilon}{\sigma^2} + R(\tau_-)\frac{-\sigma\epsilon}{\sigma^2}\right] \\ &= \frac{\epsilon}{2\sigma}\left[R(\tau_+) - R(\tau_-)\right]\end{aligned}$$

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vs

Finite Difference

$$\frac{\partial U}{\partial \theta_j}(\theta) = \frac{U(\theta + \epsilon e_j) - U(\theta - \epsilon e_j)}{2\epsilon}$$

Finite Differences

We can compute the gradient g using standard finite difference methods, as follows:

$$\frac{\partial U}{\partial \theta_j}(\theta) = \frac{U(\theta + \epsilon e_j) - U(\theta - \epsilon e_j)}{2\epsilon}$$

Where:

$$e_j = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow \quad j\text{'th entry}$$

Evolution methods VS Policy Gradients

Sample estimate:

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)}) R(s_t^{(i)}, a_t^{(i)})$$

Sample estimate:

$$\nabla_\mu U(\mu) \approx \frac{1}{N}\sum_{i=1}^{N}\nabla_\mu \log P_\mu(\theta^{(i)}) F(\theta^{(i)})$$

- Open Question: Policy Gradient at action level or parameter level?

We add noise \epsilon in our **actions** (\epsilon-greedy)!

$$\nabla_\theta F_{PG}(\theta) = \mathbb{E}_\epsilon \left\{ R(\mathbf{a}(\epsilon, \theta)) \nabla_\theta \log p(\mathbf{a}(\epsilon, \theta); \theta) \right\}$$

We add noise \ksi in our policy (neural) **parameters**!

$$\nabla_\theta F_{ES}(\theta) = \mathbb{E}_\xi \left\{ R(\mathbf{a}(\xi, \theta)) \nabla_\theta \log p(\tilde{\theta}(\xi, \theta); \theta) \right\}$$

Evolution Strategies as a Scalable Alternative to Reinforcement Learning

Tim Salimans **Jonathan Ho** **Xi Chen** **Szymon Sidor** **Ilya Sutskever**
OpenAI

Neural net architectures that work well with (stochastic) gradient descent optimization do not work will with ES. Contributions:

- Virtual batch norm
- Discretization of continuous actions - better exploration during mutation!
- Parallelization with a need for tiny only cross-worker communication!!

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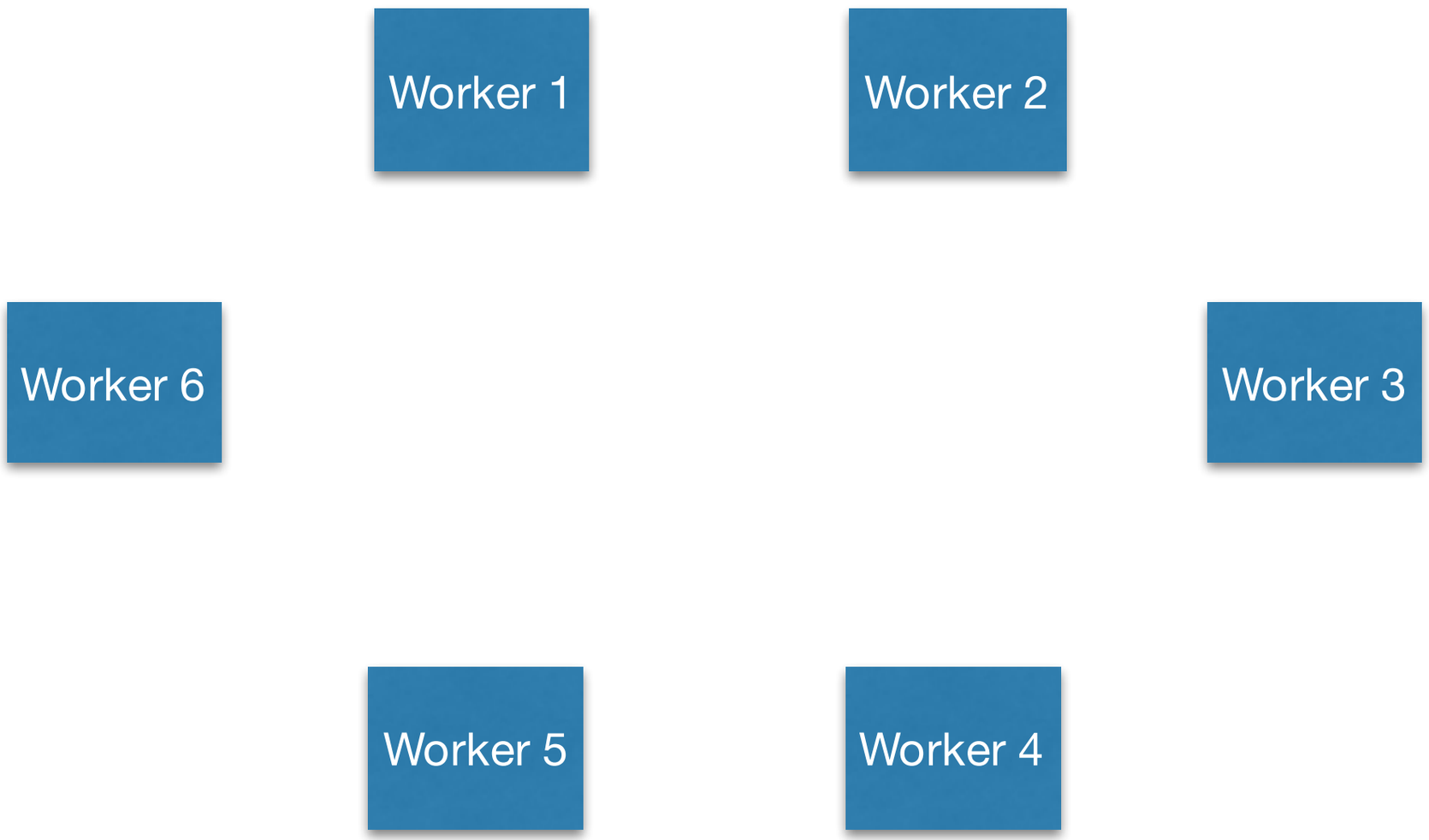
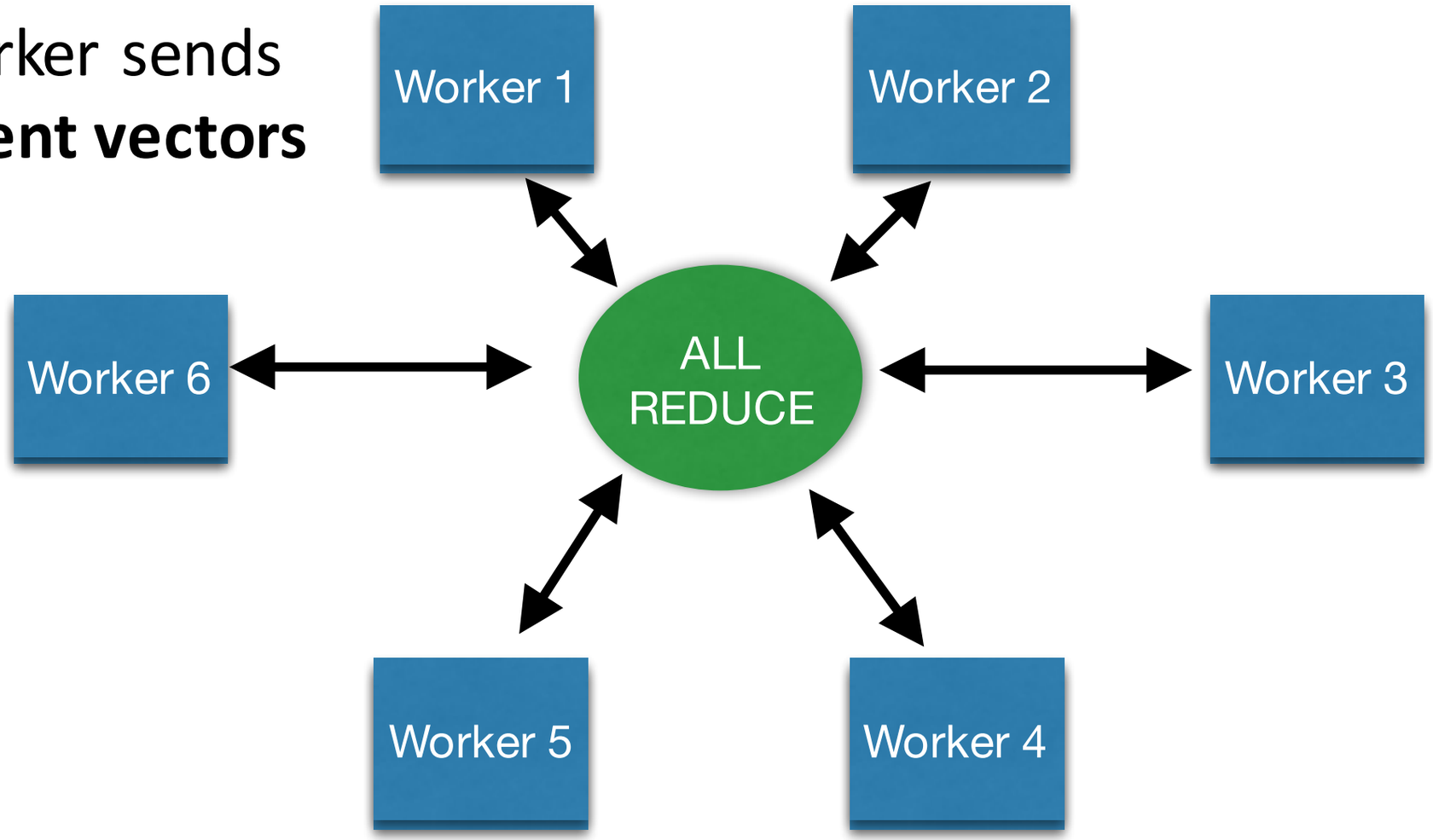
Distributed SGD



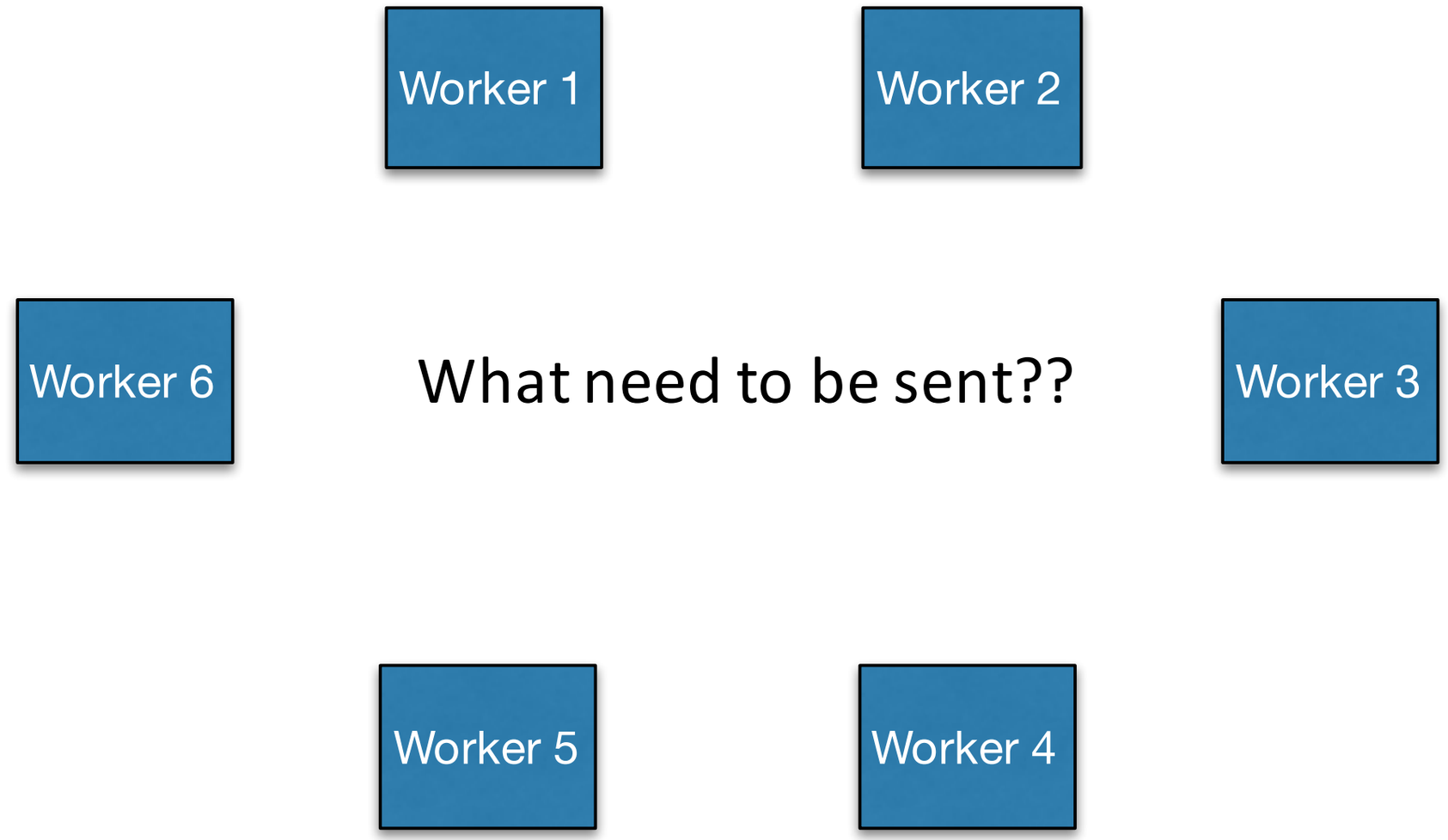
Used in Asynchronous RL!

Distributed SGD

Each worker sends
big gradient vectors



Distributed Evolution



Distributed Evolution

Algorithm 1 Evolution Strategies

1: **Input:** Learning rate α, noise standard deviation σ, initial policy parameters θ_0
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3: Sample $\epsilon_1, \ldots \epsilon_n \sim \mathcal{N}(0, I)$
4: Compute returns $F_i = F(\theta_t + \sigma\epsilon_i)$ for $i = 1, \ldots, n$
5: Set $\theta_{t+1} \leftarrow \theta_t + \alpha \frac{1}{n\sigma} \sum_{i=1}^{n} F_i \epsilon_i$
6: **end for**

Worker 5

Worker 4

Distributed Evolution

Worker 1

Worker 2

Worker 6

θ and R(τ)?

Worker 3

θ is big!

but $\theta = \mu + \sigma\epsilon$

Same for all workers

Only need seed of random number generator!

Distributed Evolution

Algorithm 2 Parallelized Evolution Strategies

Input: Learning rate α, noise standard deviation σ, initial policy parameters θ_0
Initialize: n workers with known random seeds, and initial parameters θ_0
for $t = 0, 1, 2, \ldots$ **do**
  for each worker $i = 1, \ldots, n$ **do**
    Sample $\epsilon_i \sim \mathcal{N}(0, I)$
    Compute returns $F_i = F(\theta_t + \sigma\epsilon_i)$
  end for
  Send all scalar returns F_i from each worker to every other worker
  for each worker $i = 1, \ldots, n$ **do**
    Reconstruct all perturbations ϵ_j for $j = 1, \ldots, n$
    Set $\theta_{t+1} \leftarrow \theta_t + \alpha \frac{1}{n\sigma} \sum_{j=1}^{n} F_j \epsilon_j$
  end for
end for

[Salimans, Ho, Chen, Sutskever, 2017]

Distributed Evolution

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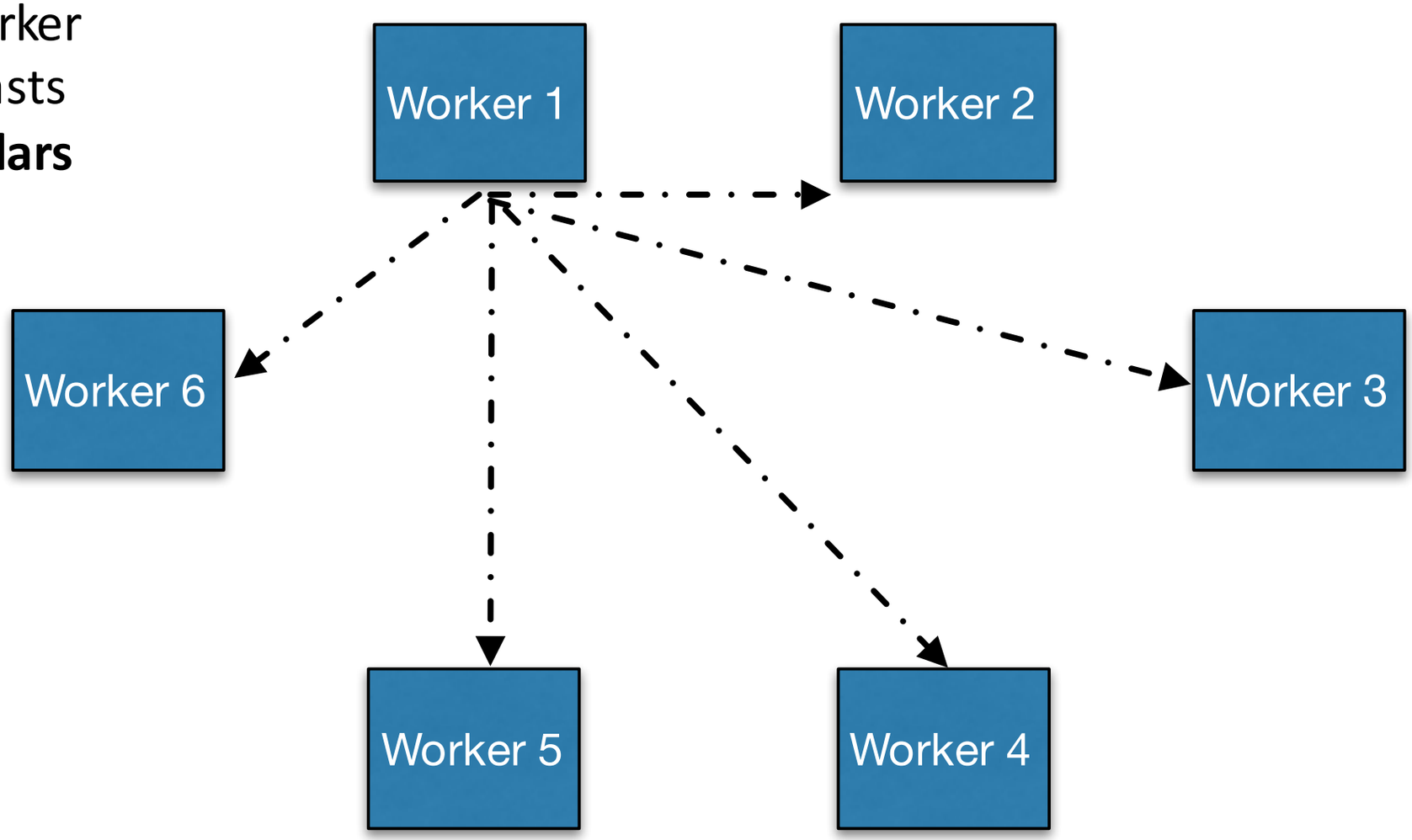
  end for
end for

[Salimans, Ho, Chen, Sutskever, 2017]

Distributed Evolution

Each worker broadcasts **tiny scalars**



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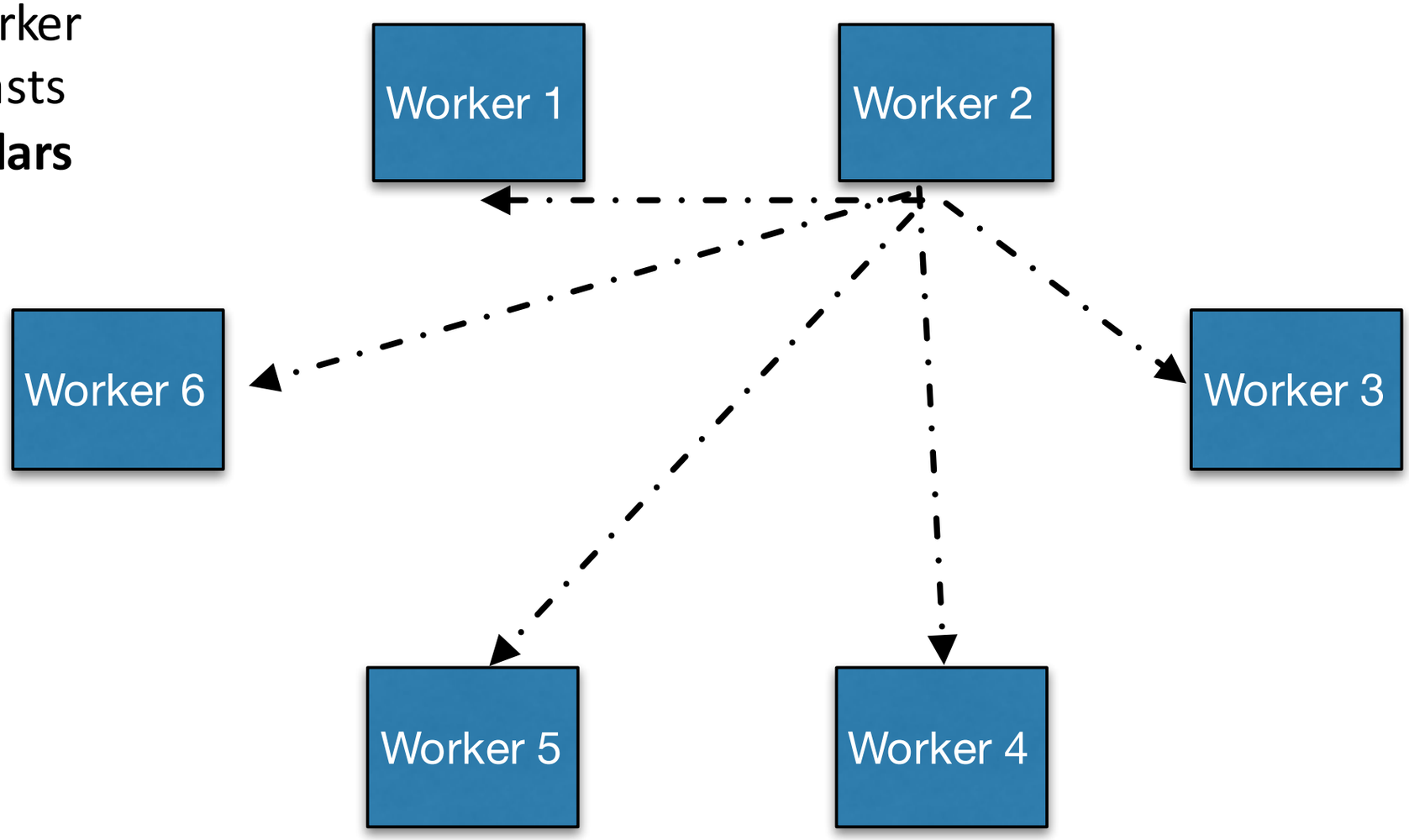

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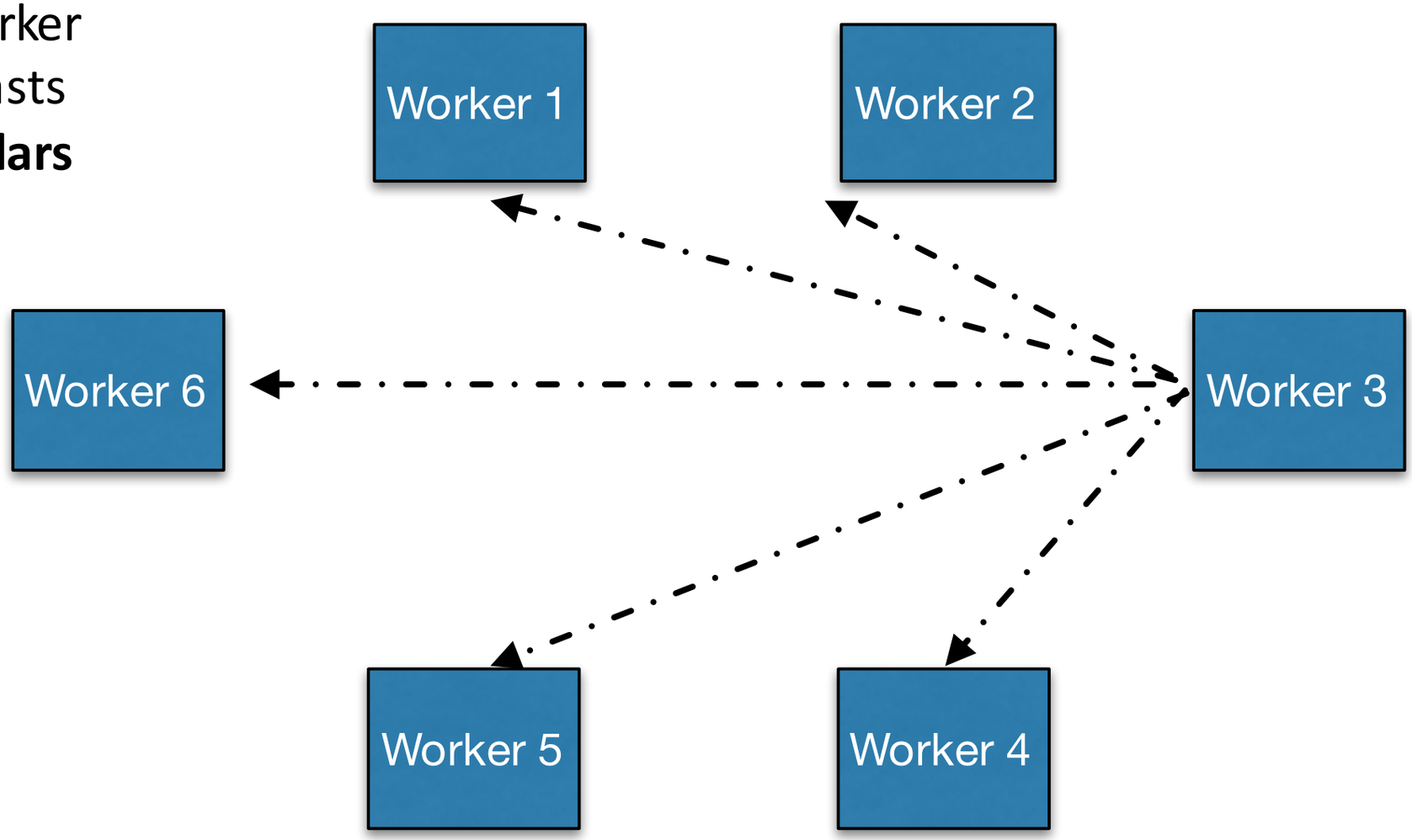


Distributed Evolution Scales Very Well :-)

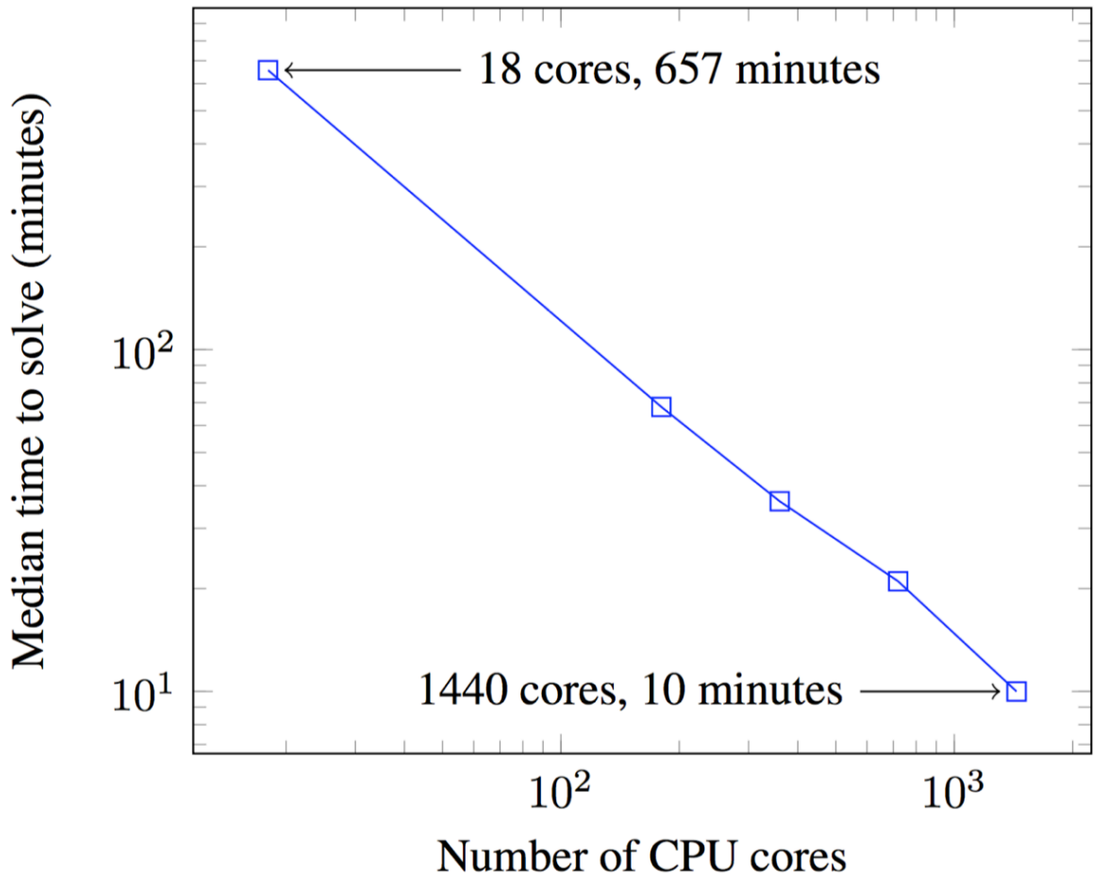
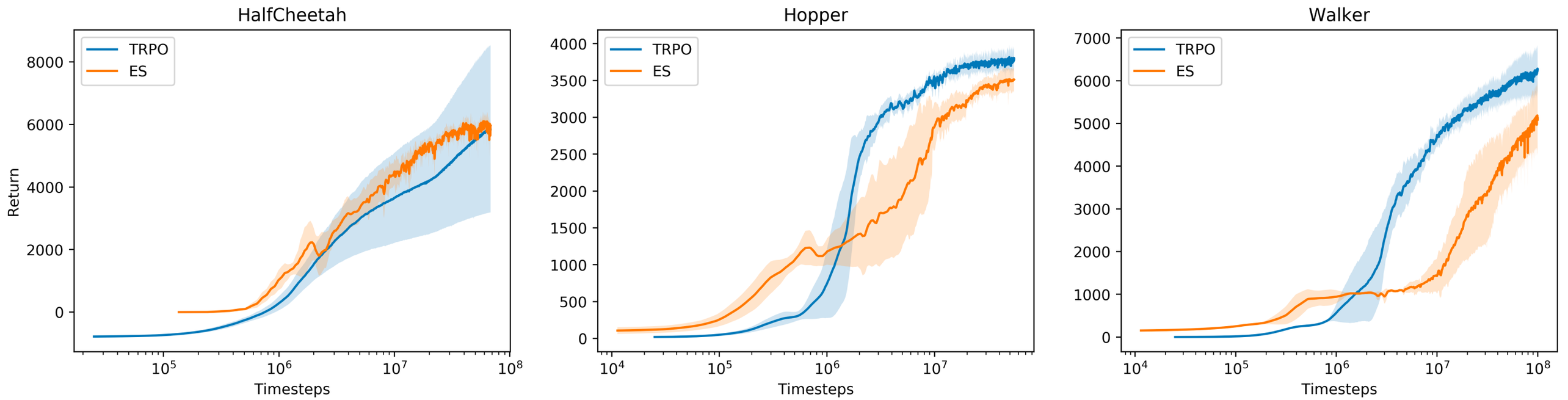


Figure 1. Time to reach a score of 6000 on 3D Humanoid with different number of CPU cores. Experiments are repeated 7 times and median time is reported.

[Salimans, Ho, Chen, Sutskever, 2017]

Distributed Evolution Requires More Samples :-(



[Salimans, Ho, Chen, Sutskever, 2017]

Alternative derivations

Monte Carlo Policy Gradients (REINFORCE), gradient direction:

$$\hat{g} = \hat{\mathbb{E}}_t \left[\nabla_\theta \log \pi_\theta(a_t \mid s_t) \hat{A}_t \right]$$

How would we implement this in TF or Pytorch

$$L^{PG}(\theta) = \hat{\mathbb{E}}_t \left[\log \pi_\theta(a_t \mid s_t) \hat{A}_t \right]$$

Equivalently:

$$L^{IS}_{\theta_{old}}(\theta) = \mathbb{E}_{\tau \sim \theta_{old}} \left[\frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{old}}(a_t \mid s_t)} \hat{A}_t \right]$$

Derivation of likelihood ratio derivative from importance sampling

$$
\begin{aligned}
J(\theta) &= \mathbb{E}_{\tau \sim P_\theta(\tau)}\left[R(\tau)\right] \\
&= \sum_{\tau} P(\tau \mid \theta) R(\tau) \\
&= \sum_{\tau} P(\tau \mid \theta_{old}) \frac{P(\tau \mid \theta)}{P(\tau \mid \theta_{old})} R(\tau) \\
&= \sum_{\tau \sim P(\tau \mid \theta_{old})} \frac{P(\tau \mid \theta)}{P(\tau \mid \theta_{old})} R(\tau) \\
&= \mathbb{E}_{\tau \sim \theta_{old}} \frac{P(\tau \mid \theta)}{P(\tau \mid \theta_{old})} R(\tau)
\end{aligned}
$$

$$
\mathbb{E}_{\tau \sim \theta_{old}} \frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{old}}(a_t \mid s_t)} \hat{A}_t
$$

$$
\nabla J(\theta)\big|_{\theta_{old}} = \mathbb{E}_{\tau \sim \theta_{old}} \frac{\nabla_\theta P(\tau \mid \theta)\big|_{\theta_{old}}}{P(\tau \mid \theta_{old})} R(\tau)
$$

$$
= \mathbb{E}_{\tau \sim \theta_{old}} \nabla_\theta \log P(\tau \mid \theta)\big|_{\theta_{old}} R(\tau)
$$

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Next: Instead of leaving the stepwise to some schedule of the learning rate (heuristically chosen), make it part of the algorithm!

Hard to choose stepsizes

- Policy gradients

$$\hat{g} = \hat{\mathbb{E}}_t \Big[\nabla_\theta \log \pi_\theta(a_t \mid s_t) \hat{A}_t \Big]$$

- Can differentiate the following loss

$$L^{PG}(\theta) = \hat{\mathbb{E}}_t \Big[\log \pi_\theta(a_t \mid s_t) \hat{A}_t \Big].$$

- Input data is nonstationary due to changing policy: observation and reward distributions change
- Bad step is more damaging than in supervised learning, since it affects visitation distribution

- Step too big
  Bad policy->data collected under bad policy-> we cannot recover
  (in SL, data does not depend on neural network weights)
- Step too small
  Not efficient use of experience
  (in SL, data can be trivially re-used)

SGD:

$$\theta_{new} = \theta_{old} + \epsilon \cdot \hat{g}$$

Natural Gradient Descent

$$\frac{-\nabla_\theta \mathcal{L}(\theta)}{\|\nabla_\theta \mathcal{L}(\theta)\|} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \underset{d \text{ s.t. } \|d\| \leq \epsilon}{\arg\min} \mathcal{L}(\theta + d)$$

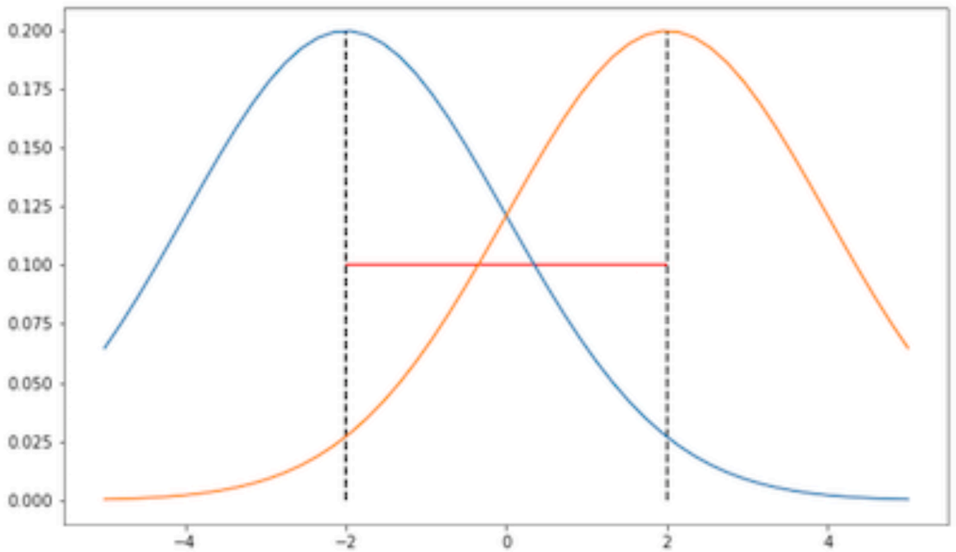
Take a step in direction d where d stays within a a ball of radius epsilon.

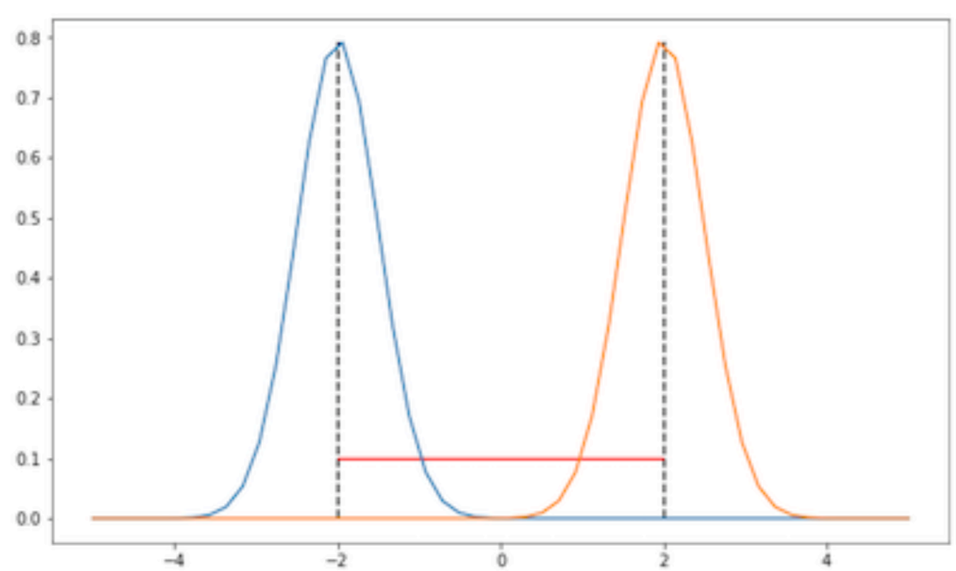
Gradient Descent in Parameter Space

Take a step in direction d where d stays within a a ball of radius epsilon.

$$\frac{-\nabla_\theta \mathcal{L}(\theta)}{\|\nabla_\theta \mathcal{L}(\theta)\|} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \arg\min_{d \text{ s.t. } \|d\| \leq \epsilon} \mathcal{L}(\theta + d)$$

Euclidean distance in parameter space





Imagine we are trying to minimize -loglik, and want to update the parameters of the distribution.

Same distance in parameter space results in very different distances in distribution space

Ideally, **we want to take steps based on distance in distribution space. Way more robust, less tweaking of the step.**

What metric shall we use?

Gradient Descent in Distribution Space

We will use the Kullback-Leibler divergence (KL) to measure distances between distributions before and after the update.

$$\frac{-\nabla_\theta \mathcal{L}(\theta)}{\|\nabla_\theta \mathcal{L}(\theta)\|} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \arg\min_{d \text{ s.t. } \|d\| \leq \epsilon} \mathcal{L}(\theta + d)$$

Instead:

$$d^* = \arg\min_{d \text{ s.t. } \mathrm{KL}[p_\theta \| p_{\theta+d}] = c} \mathcal{L}(\theta + d)$$

$$D_{\mathrm{KL}}(P\|Q) = \sum_i P(i) \log\left(\frac{P(i)}{Q(i)}\right)$$

$$D_{\mathrm{KL}}(P\|Q) = \int_{-\infty}^{\infty} p(x) \log\left(\frac{p(x)}{q(x)}\right) dx$$

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Unconstrained penalized objective:

$$d^* = \underset{d}{\arg\min}\, \mathcal{L}(\theta + d) + \lambda \left(\mathrm{KL}[p_\theta \| p_{\theta+d}] - c\right)$$

First order Taylor expansion for the loss and second order for the KL:

$$\approx \underset{d}{\arg\min}\, \mathcal{L}(\theta) + \nabla_\theta \mathcal{L}(\theta)^{\mathrm{T}} d + \frac{1}{2} \lambda\, d^{\mathrm{T}} \mathrm{F} d - \lambda c$$

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Natural Gradient Descent

Newton's method

$$
\begin{aligned}
0 &= \frac{\partial}{\partial d}\mathcal{L}(\theta) + \nabla_\theta \mathcal{L}(\theta)^{\mathrm{T}} d + \frac{1}{2}\lambda\, d^{\mathrm{T}}\mathrm{F} d - \lambda c \\
&= \nabla_\theta \mathcal{L}(\theta) + \lambda\, \mathrm{F} d \\
\lambda\, \mathrm{F} d &= -\nabla_\theta \mathcal{L}(\theta) \\
d &= -\frac{1}{\lambda}\mathrm{F}^{-1}\nabla_\theta \mathcal{L}(\theta)
\end{aligned}
$$

The natural gradient:

$$
\tilde{\nabla}_\theta \mathcal{L}(\theta) = \mathrm{F}^{-1}\nabla_\theta \mathcal{L}(\theta)
$$