Ph.D. Econometrics I
Heinz School, Carnegie Mellon University
90-906, Spring 2005

Midterm

Instructions You may use any books, notes, calculators, and other aids you like. You may not converse, nor may you cooperate.

Please complete all questions.

Please show all relevant work.

Please interpret your results in plain English.

Use the backs of pages if you need more space.

The parts of question 1 are worth 10 points each. The parts of question 2 are worth 15 points each. Question 3 is worth 15 points.

1. Consider the following very simple wage regression:

$$W_i = \beta_1 + \beta_2 HS_i + \epsilon_i \tag{1}$$

In these equations, HS is a dummy variable taking the value 1 if a person has graduated from high school and a value of 0 otherwise, W is wage.

As we discussed in class, it seems reasonable to believe that $\hat{\beta}_{2,OLS} = \overline{W}_{HS=1} - \overline{W}_{HS=0}$, where $\overline{W}_{HS=1}$ means average wage among high school grads (ie among observations where $HS = 1$).

In the following parts, we will try to prove this fact. If you fail to prove any of the individual parts, please continue on anyway, assuming that you have proved the previous parts.

(a) Use the first-order conditions for the OLS minimization problem in the context of the model of equation 1 to prove that $\hat{\beta}_{2,OLS} = \overline{W}_{HS=1} - \hat{\beta}_{1,OLS}$

(b) Prove that $\hat{\beta}_{1,OLS} = \overline{W} - \hat{\beta}_{2,OLS}\overline{HS}$

(c) Prove that $\hat{\beta}_{2,OLS} = \frac{\overline{W}_{HS=1} - \overline{W}}{1 - \overline{HS}}$

(d) Prove that $\hat{\beta}_{2,OLS} = \overline{W}_{HS=1} - \overline{W}_{HS=0}$

2. Consider the following estimator for β in the classical linear regression model:

   $\tilde{\beta} = \left((X+Z)'X\right)^{-1}(X+Z)'Y$

   Assume Z is an $N \times K$ fixed matrix, and that $(X+Z)'X$ is positive definite.

   (a) Is $\tilde{\beta}$ biased?

(b) What is the variance of $\tilde{\beta}$?

(c) Is this variance greater than less than or equal to the variance of $\hat{\beta}_{OLS}$?

3. Consider the following regression model, which we will estimate by OLS:

$$Y = X\beta + \epsilon \tag{2}$$

Now, suppose we multiply X_3 by 100 (say by measuring it in cents rather than in dollars). What effect will this have on the OLS estimates. Please answer and prove your answer.