Ph.D. Econometrics I
Heinz School, Carnegie Mellon University
90-906, Spring 2006

Midterm

Instructions You may use any books, notes, calculators, and other aids you like. You may not converse, nor may you cooperate.

Please complete all questions.

Please show all relevant work.

Please interpret your results in plain English.

You have 1.5 hours to complete the test.

1. Consider the following simple regression model (Y and X have mean zero, so we can omit the intercept):

$$Y = x\beta_1 + \epsilon \tag{1}$$

Imagine that the five data points have been sorted on the x variable, so that $x_1 < x_2 < x_3 < x_4 < x_5$.

Consider the following three estimators of β_1:
i) the OLS estimator, $\widehat{\beta}_{1,OLS}$
ii) $\tilde{\beta}_{1A} = \frac{Y_4 - Y_2}{x_4 - x_2}$
iii) $\tilde{\beta}_{1B} = \frac{Y_5 - Y_1}{x_5 - x_1}$

Assume all the assumptions of the CLRM hold.

   (a) Why are ii) and iii) reasonable estimators of β_1?

   (b) Rank the variance of the three estimators, explain.

   (c) Now, consider a dataset with two data points in it. Show that the OLS estimator of β_1 and the $\tilde{\beta}$ estimator are identical. Hint: since there are only two datapoints, if $\overline{x} = 0$ then $x_1 = -x_2$.

   (d) Now, consider a dataset with three data points in it. Show that the OLS estimator of β_1 is the weighted average of the two "tilde" estimators: $\frac{y_1 - y_3}{x_1 - x_3}$ and $\frac{y_2 - y_3}{x_2 - x_3}$

   (e) Interpret these two facts.

2. Consider the following model:

$$Y = (X\beta)\epsilon \tag{2}$$

Assume that $E(\epsilon) = 1$, $V(\epsilon) = \sigma^2$, and that x is fixed and $X'X$ is invertible.

   (a) What is $E(Y)$?

   (b) Define $\nu = Y - X\beta$. What is $E(\nu)$?

   (c) For the equation $Y = X\beta + \nu$, which of the assumptions of the CLRM are true?

   (d) What can we say about the usefulness of running OLS on $Y = X\beta + \nu$ as a way of estimating β?

3. Consider the following model:

$$Y = \beta_1 + \beta_2 x_2 + \epsilon \quad (3)$$
$$x_2 = \beta_3 + \beta_4 x_3 + \nu \quad (4)$$

Suppose that ϵ and ν are independent, that each of ϵ and ν have expectation 0 and variance σ^2, that x_3 is fixed, and that all relevant matrixes are invertible.

Now, suppose we regress Y on x_3 and a constant, as if we believed this to be the true model:

$$Y = \gamma_1 + \gamma_2 x_3 + \delta \quad (5)$$

What can we say about $\widehat{\gamma}_{1,OLS}$ and $\widehat{\gamma}_{2,OLS}$? Are they unbiased for anything? Best for anything?