Recitation 7: Memory Access Patterns

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15213 Section A

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- Office hours:
  - NSH 2504 (lab) / 2507 (conference room)
  - Thursday 5-6

- Lab 4
  - due Thursday, 24 Oct @ 11:59pm
  - Submission is online
    - http://www2.cs.cmu.edu/afs/cs/academic/class/15213-f02/www/L4.html

Today's Plan

- Loop Unrolling
- Blocking

Loop Unrolling

```
void combine5(vec_ptr v, int *dest)
{
  int length = vec_length(v);
  int limit = length-2;
  int *data = get_vec_start(v);
  int sum = 0;
  int i;
  /* Combine 3 elements at a time */
  for (i = 0; i < limit; i+=3) {
    sum += data[i] + data[i+1]
           + data[i+2];
  }
  /* Finish any remaining elements */
  for (; i < length; i++) {
    sum += data[i];
  }
  *dest = sum;
}
```

- Optimization
  - Combine multiple iterations into single loop body
  - Amortizes loop overhead across multiple iterations
  - Finish extras at end

Practice Problem

- Problem 5.12 and 5.13

Solution 5.12

```
void inner5(vec_ptr u, vec_ptr v, data_t *dest)
{
  int i;
  int length = vec_length(u);
  int limit = length-3;
  data_t *udata = get_vec_start(u);
  data_t *vdata = get_vec_start(v);
  data_t sum = (data_t) 0;

  /* Do four elements at a time */
  for (i = 0; i < limit; i += 4) {
    sum += udata[i] * vdata[i] + udata[i+1] * vdata[i+1]
     + udata[i+2] * vdata[i+2] + udata[i+3] * vdata[i+3];
  }

  /* Finish off any remaining elements */
  for (; i < length; i++)
    sum += udata[i] * vdata[i];

  *dest = sum;
}
```

Solution 5.12

A. We must perform two loads per element to read values for **`udata`** and **`vdata`**. There is only one unit to perform these loads, and it requires one cycle.

B. The performance for floating point is still limited by the 3 cycle latency of the floating-point adder.

Solution 5.13

```
void inner6(vec_ptr u, vec_ptr v, data_t *dest)
{
  int i;
  int length = vec_length(u);
  int limit = length-3;
  data_t *udata = get_vec_start(u);
  data_t *vdata = get_vec_start(v);
  data_t sum0 = (data_t) 0;
  data_t sum1 = (data_t) 0;

  /* Do four elements at a time */
  for (i = 0; i < limit; i+=4) {
    sum0 += udata[i] * vdata[i];
    sum1 += udata[i+1] * vdata[i+1];
    sum0 += udata[i+2] * vdata[i+2];
    sum1 += udata[i+3] * vdata[i+3];
  }
  /* Finish off any remaining elements */
  for (; i < length; i++)
    sum0 = sum0 + udata[i] * vdata[i];

  *dest = sum0 + sum1;
}
```

Solution 5.13

- For each element, we must perform two loads with a unit that can only load one value per clock cycle.
- We must also perform one floating-point multiplication with a unit that can only perform one multiplication every two clock cycles.
- Both of these factors limit the CPE to 2.

Summary of Matrix Multiplication

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k]
           * b[k][j];
    c[i][j] = sum;
  }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
       c[i][j] += r
         * b[k][j];
  }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

```
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
       c[i][j] +=
         a[i][k] * r;
  }
}
```

Improving Temporal Locality by Blocking

- Example: Blocked matrix multiplication
  - “block” (in this context) does not mean “cache block”.
  - Instead, it mean a sub-block within the matrix.
  - Example: N = 8; sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} X \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (i.e., A_{xy}) can be treated just like scalars.

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Blocked Matrix Multiply (bijk)

```
for (jj=0; jj<n; jj+=bsize) {
  for (i=0; i<n; i++)
    for (j=jj; j < min(jj+bsize,n); j++)
      c[i][j] = 0.0;
  for (kk=0; kk<n; kk+=bsize) {
    for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
    }
  }
}
```

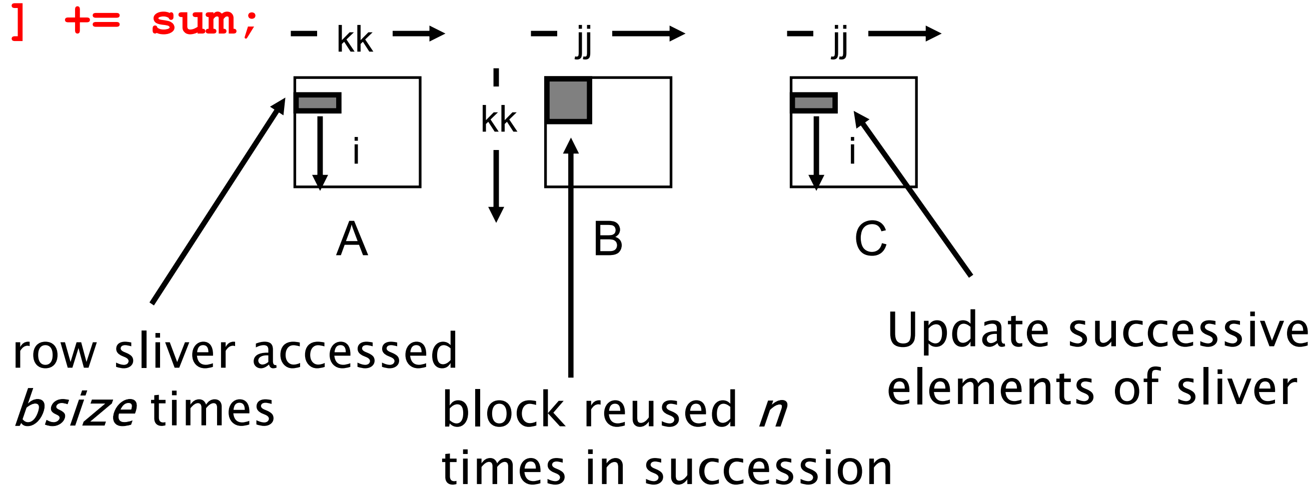
- Provides temporal locality as block is reused multiple times
- Constant cache performance

Blocked Matrix Multiply Analysis

- Innermost loop pair multiplies a *1 X bsize* sliver of *A* by a *bsize X bsize* block of *B* and accumulates into *1 X bsize* sliver of *C*
- Loop over *i* steps through *n* row slivers of *A* & *C*, using same *B*

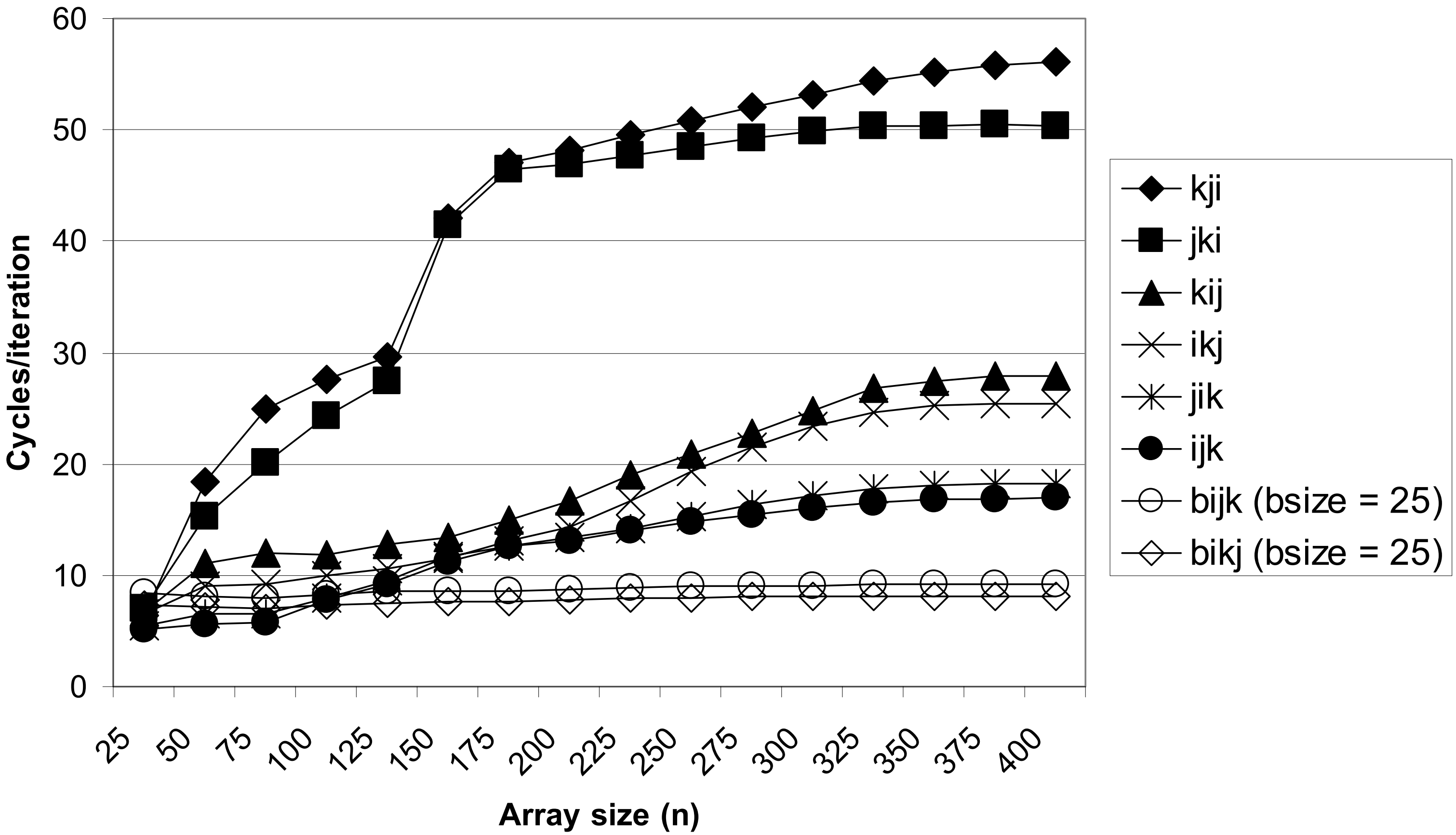
```
for (i=0; i<n; i++) {
    for (j=jj; j < min(jj+bsize,n); j++) {
      sum = 0.0
      for (k=kk; k < min(kk+bsize,n); k++) {
        sum += a[i][k] * b[k][j];
      }
      c[i][j] += sum;
    }
```

Innermost Loop Pair



Pentium Blocked Matrix Multiply Performance

- Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)
  - relatively insensitive to array size.



Summary

- All systems favor "cache friendly code"
  - Getting absolute optimum performance is very platform specific
    - Cache sizes, line sizes, associativities, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)