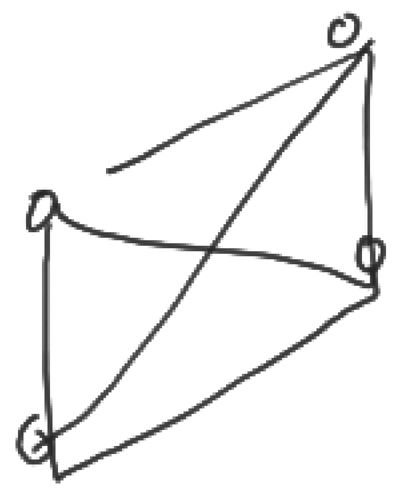
750 L1

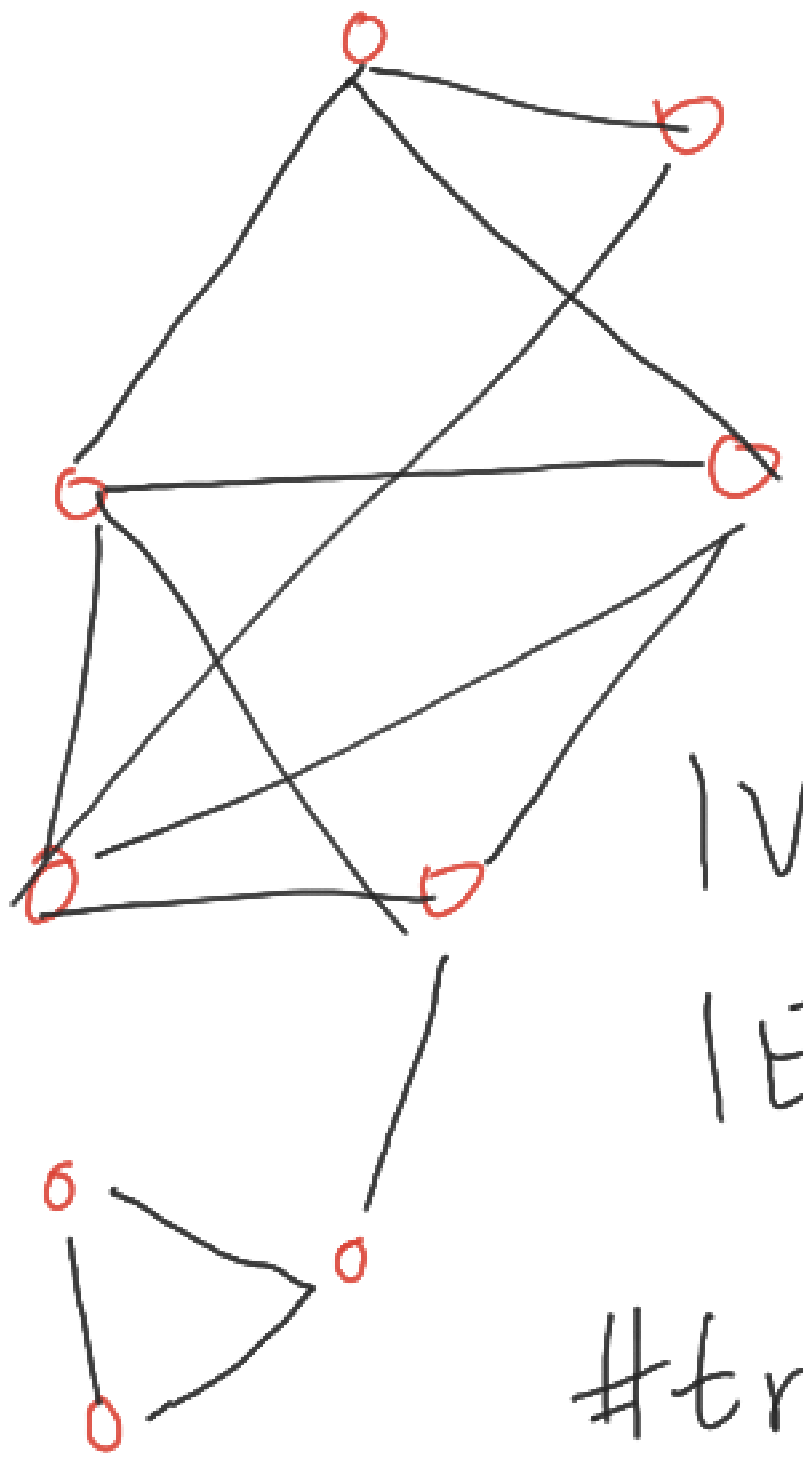
- correct
- precise
- efficient
- robust
- scalable
- understandable / simple
- $\boxed{\text{fair}}$



Graph

Vertices

Edges



$|V| = n$

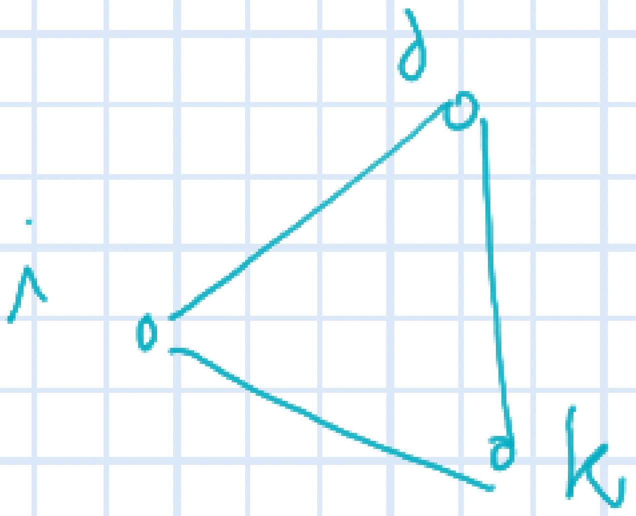
$|E| = m$

$$\frac{\#\text{triangles}}{\binom{n}{3}} =$$

Algo 1:

for all $\binom{n}{3}$ triples of vertices:

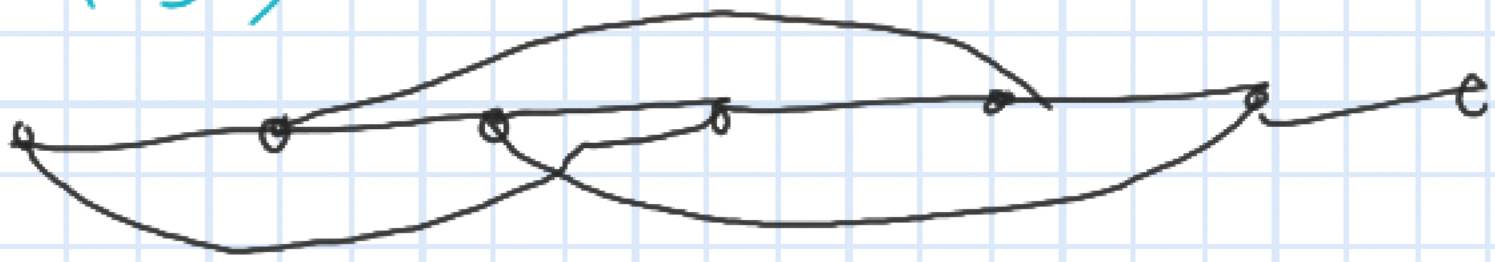
check if this triple forms $\triangle$.



$i\ j\ k$

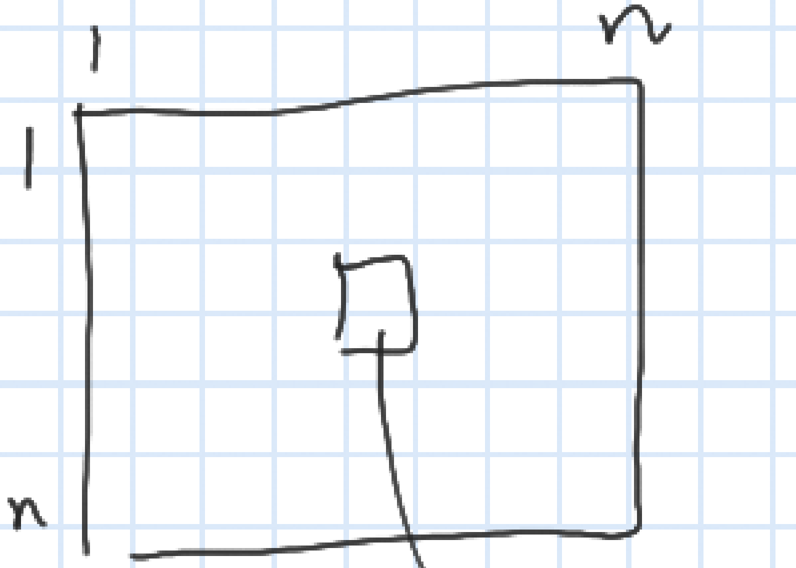
$A_{ij} = 1 \wedge A_{jk} = 1 \wedge A_{ik} = 1$

$\binom{n}{3} \times \Theta(1) = \Theta(n^3)$



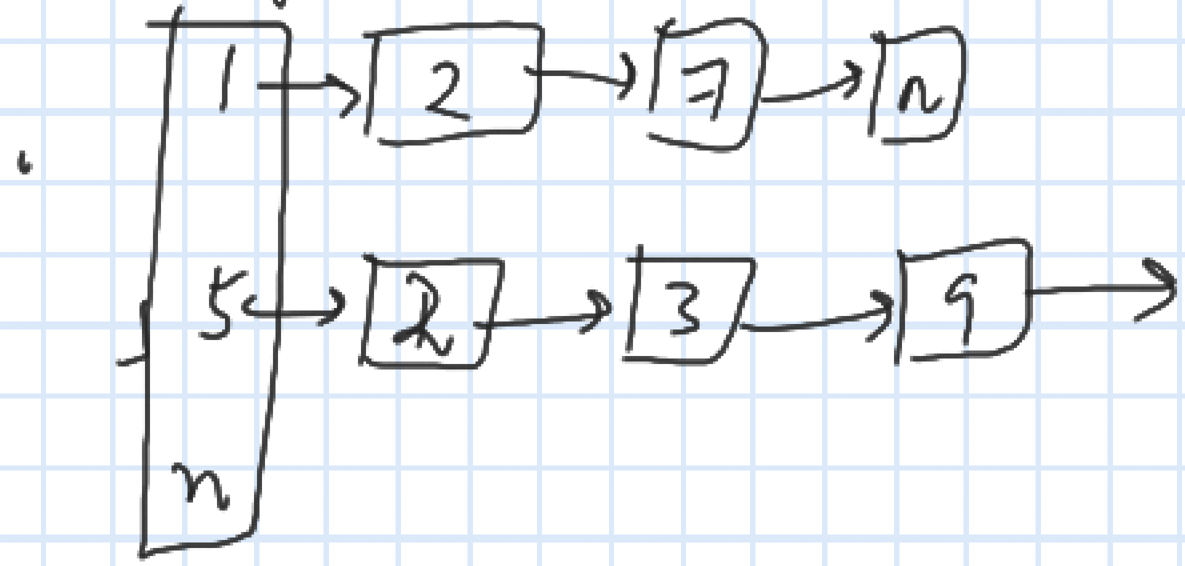
Input Representation

- $\boxed{\text{Adjacency Matrix}}$

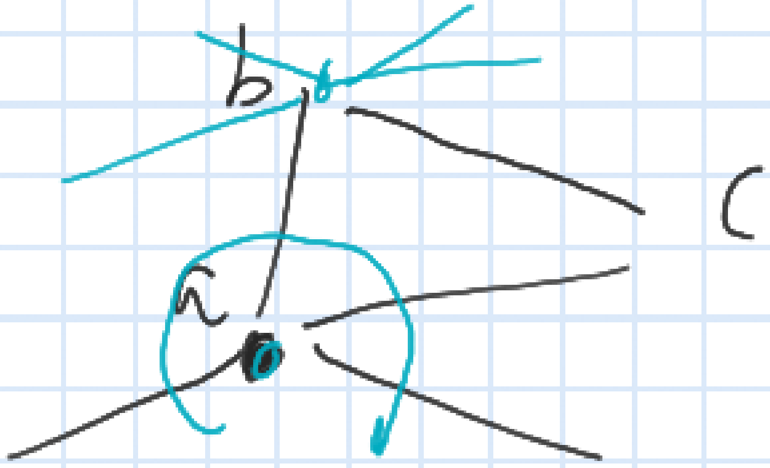


$A_{ij} = 1 \iff \{i, j\} \in E$

- Adjacency list



Algo 2



$\forall$ vertices $i \in V$

for all pairs of edges $\{ij\}$ and $\{ik\}$ adjacent to i

Check if $jk \in E$?

Runtime: $O\Big(\sum_i \underbrace{(\text{degree of } i)}_{d_i}^2\Big) \le n \cdot (\text{max degree})^2 \le n^3$

$\le O\big(d_{max} \cdot \underbrace{\sum_i d_i}\big) = O(d_{max} \cdot m)$

$\binom{d_i}{2} \le O(d_i^2)$

$\le O(nm) \le O(n^3)$

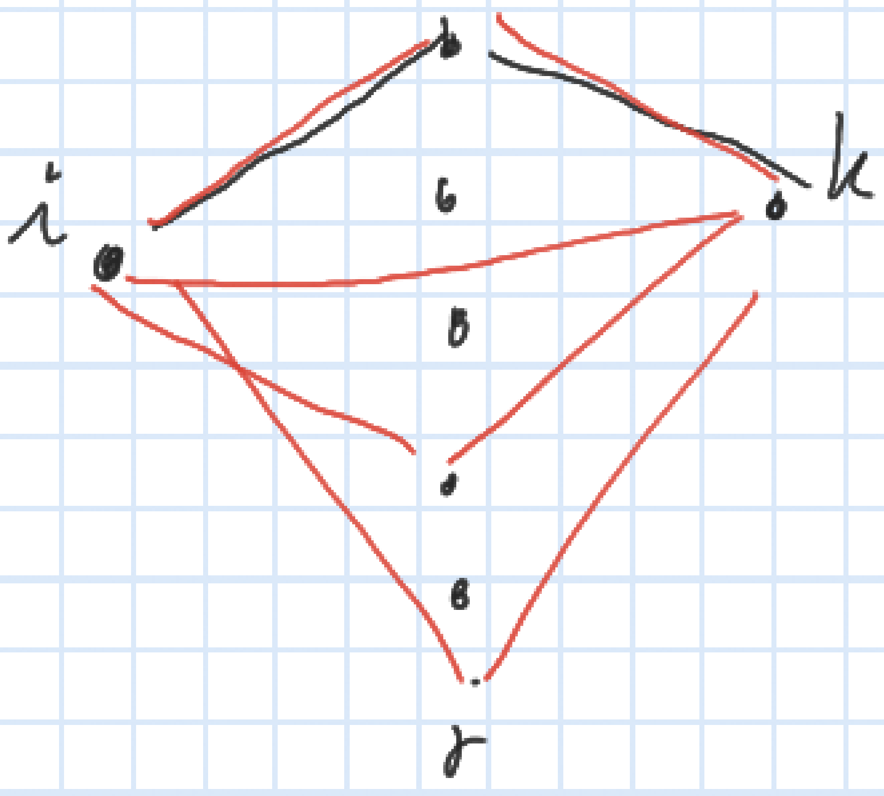
$O(m^{1.5})$

(Conn. graphs: $m \ge n-1$ $\quad m \le \binom{n}{2}$)

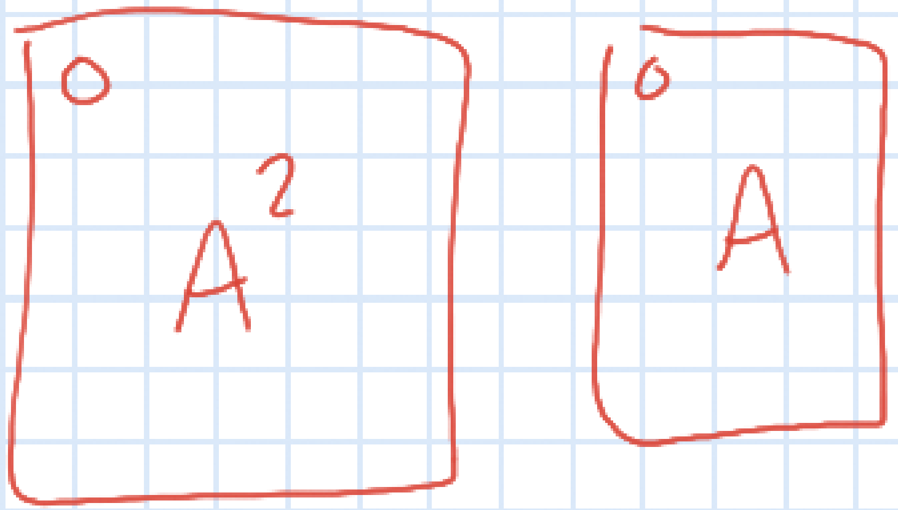
Algo 3

$A \in \{0,1\}^{n \times n}$ $\quad$ $A_{ij} = 1 \iff ij \in E$

$$\boxed{A^2_{ik}} = \underbrace{\sum_j A_{ij} \cdot A_{jk}}_{\text{\# 2-hop paths from } i \to k}$$



$\cancel{ijk \in \Delta} \iff \cancel{A^2_{ik} = 1}$ ~~and~~ $\cancel{A_{ik} = 1}$



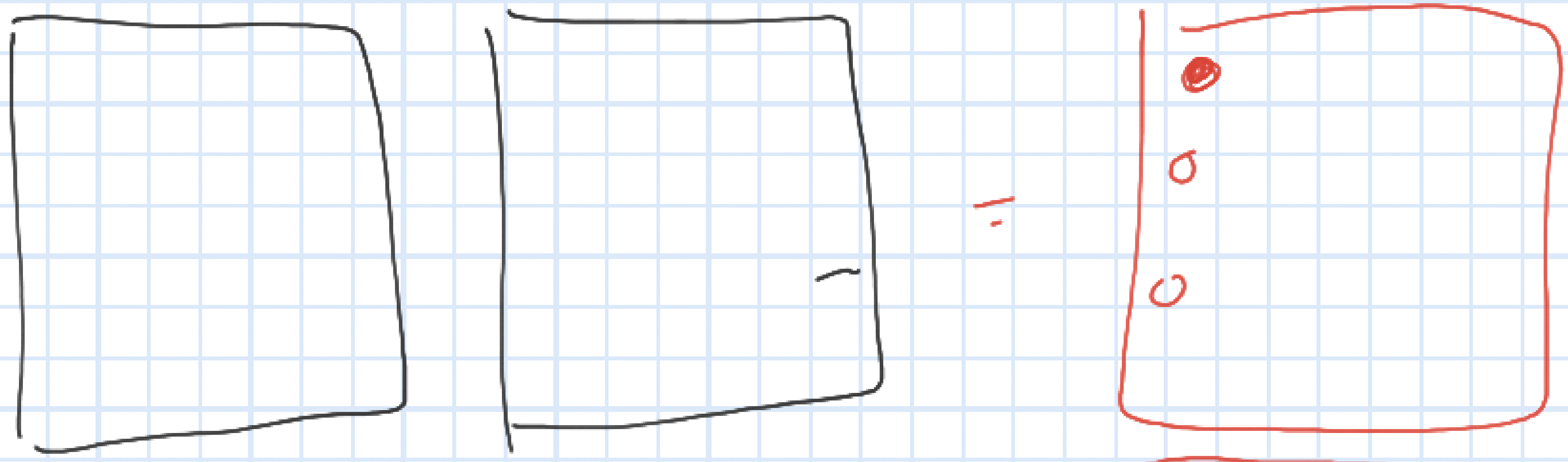
$A^2_{ik} \cdot A_{ik}$ = # of Δs containing edge ik

$\underline{\text{Run time}}$: $\underbrace{\text{time to square}(A)}_{n\times n} + \underbrace{O(\cancel{n^2})}$ $O(m)$

Time to square $(n\times n)$ $\leq$ time to mult 2 $n\times n$ matrices

$A\times B$

Volker Strassen



AB

$\boxed{\Theta(n^3) \text{ time}}$

Strassen

$$\underbrace{\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} B_{11} & \\ B_{21} & B_{22} \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{21} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21} \end{bmatrix}}_{AB}$$

$$S_1 = (a_{11} + a_{21})(b_{11} + b_{12})$$
$$S_2 = (a_{12} + a_{22})(b_{21} + b_{22})$$
$$S_3 = (a_{11} - a_{22})(b_{11} + b_{22})$$
$$S_4 = a_{11}(b_{12} - b_{22})$$
$$S_5 = (a_{21} + a_{22})b_{11}$$
$$S_6 = (a_{11} + a_{12})b_{22}$$
$$S_7 = a_{22}(b_{21} - b_{11})$$

algebra shows that the product AB is

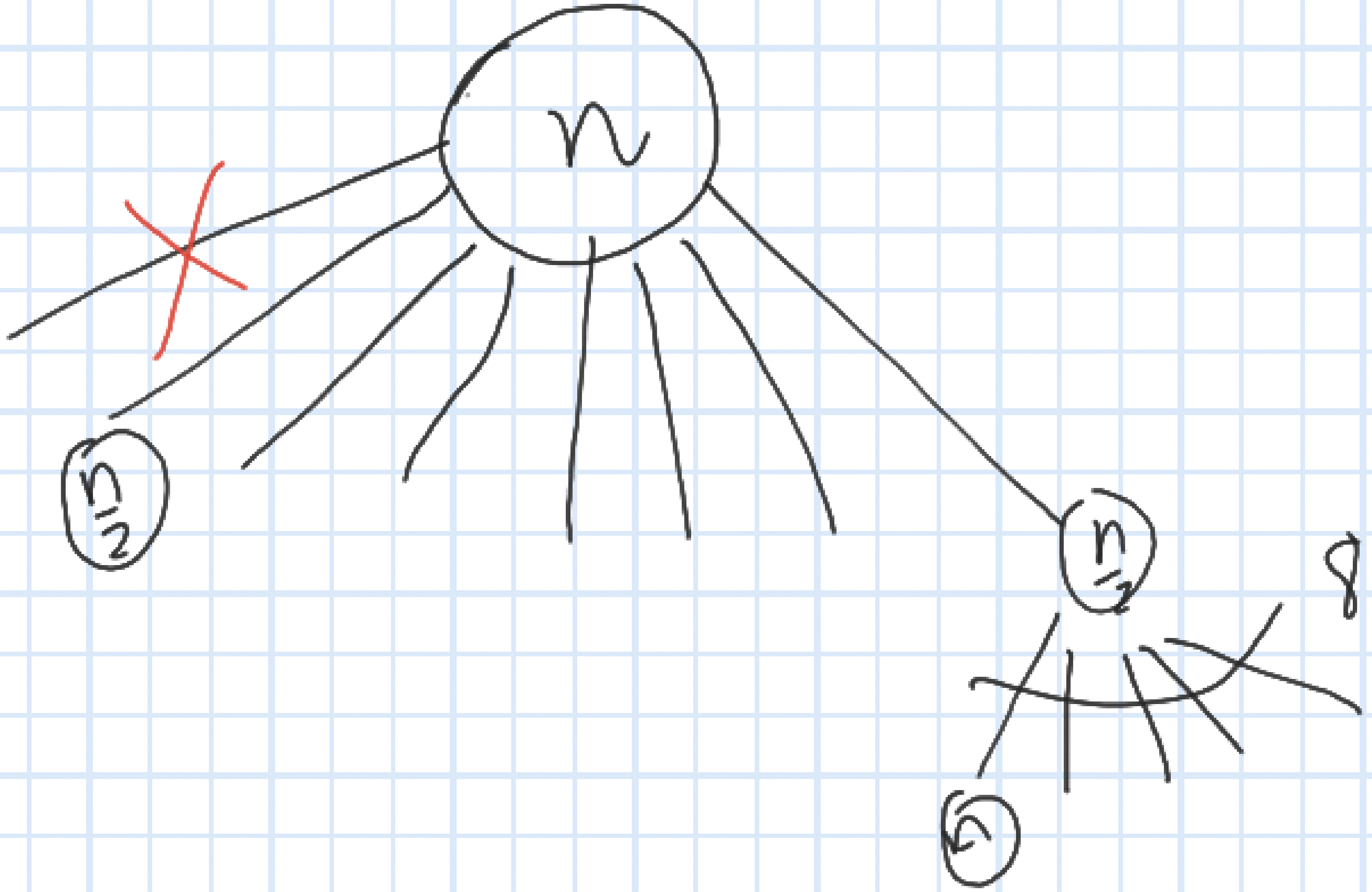
$$\begin{bmatrix} S_2 + S_3 - S_6 - S_7 & S_4 + S_6 \\ S_5 + S_7 & S_1 - S_3 - S_5 + S_5 \end{bmatrix}$$

$$\underbrace{T(n)}_{\text{amount of time to multiply two } n \times n \text{ matrices}} = 8(T(n/2)) + O(n^2).$$

$$T(n) = O(n^3)$$

$$T'(n) = 7 \cdot T'(n/2) + O(n^2)$$

$$T'(n) = O(n^{\log_2 7}) = O(n^{2.81..})$$



1

8

8^2

$\vdots$

$$S_1 = (a_{11} + a_{21})(b_{11} + b_{12})$$
$$S_2 = (a_{12} + a_{22})(b_{21} + b_{22})$$
$$S_3 = (a_{11} - a_{22})(b_{11} + b_{22})$$
$$S_4 = a_{11}(b_{12} - b_{22})$$
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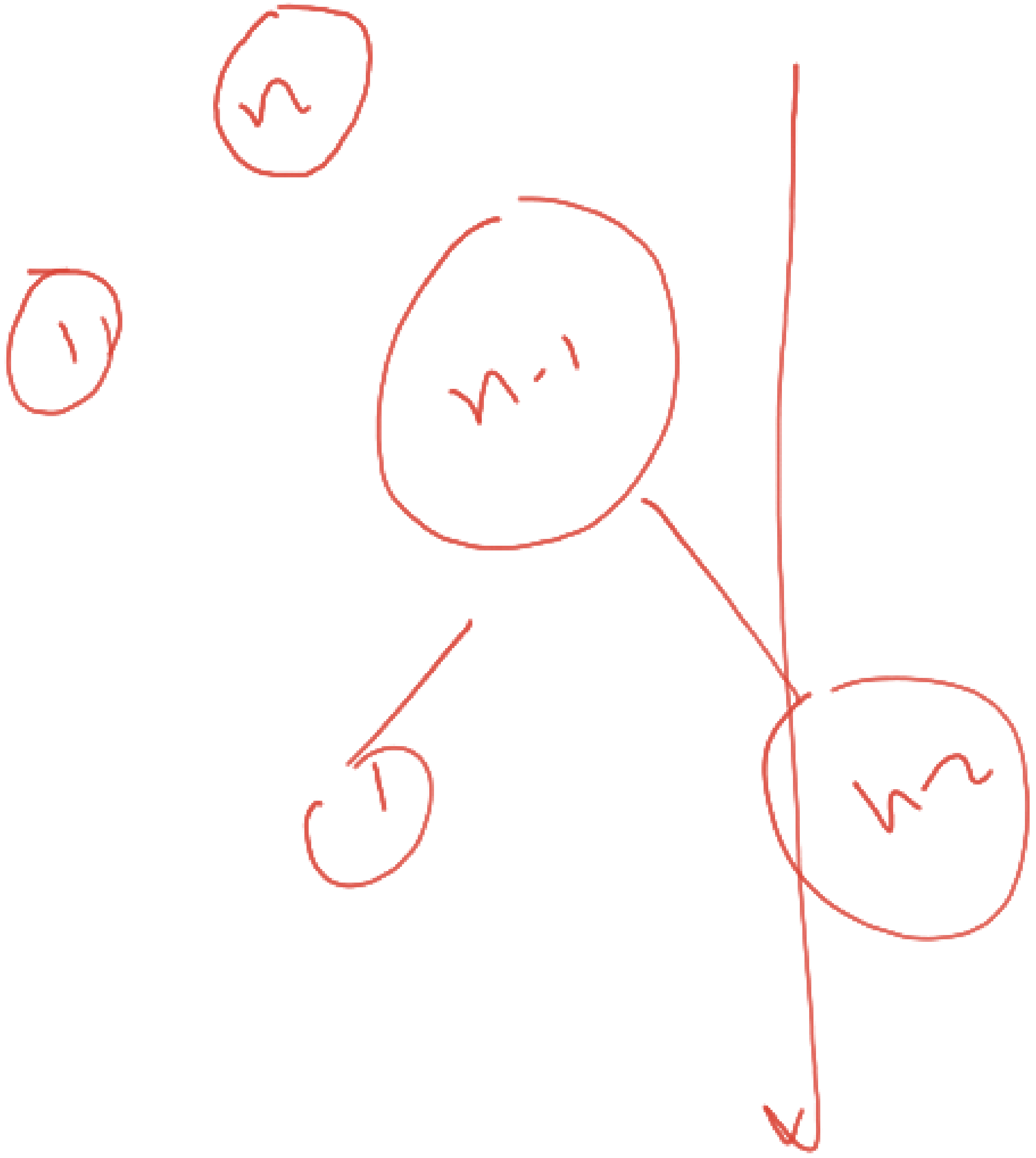
$\text{MatMult}(n) = n^{2.36-}$

$= n^{2.81\ldots}$

$8^{\log_2 n} = n^{\log_2 8} = \boxed{n^3}$

$7^{\log_2 n} = \boxed{n^{\log_2 7}}$

$n^{2.8}$


n
n-1
n-2
1
1