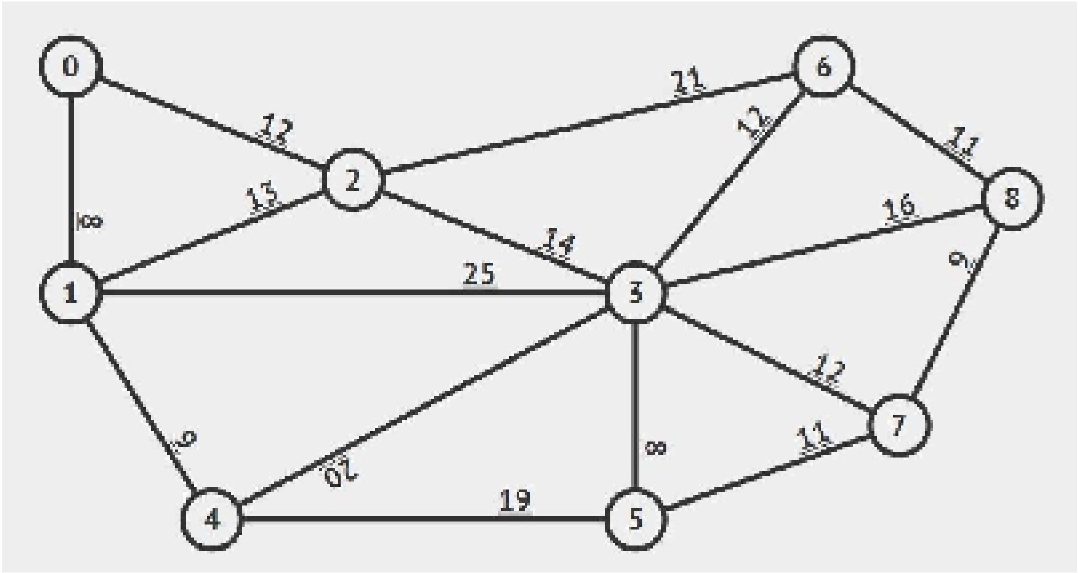
15750 Algos in the RW

1. Greedy Algos

   Minimum-Cost Spanning Tree (MST)

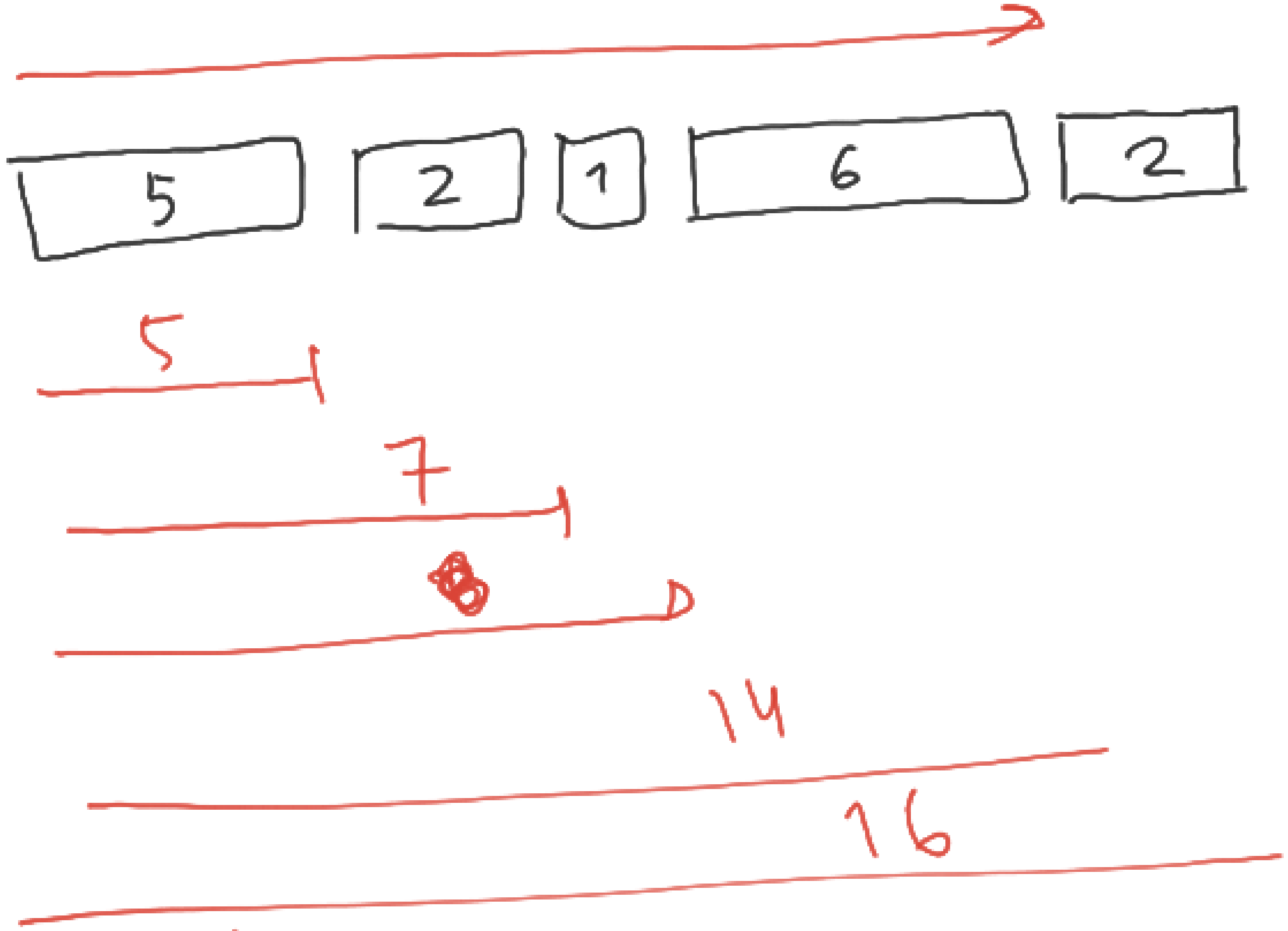
2. Amortized Analysis

$G = (V, E)$



Minimizing Response Times

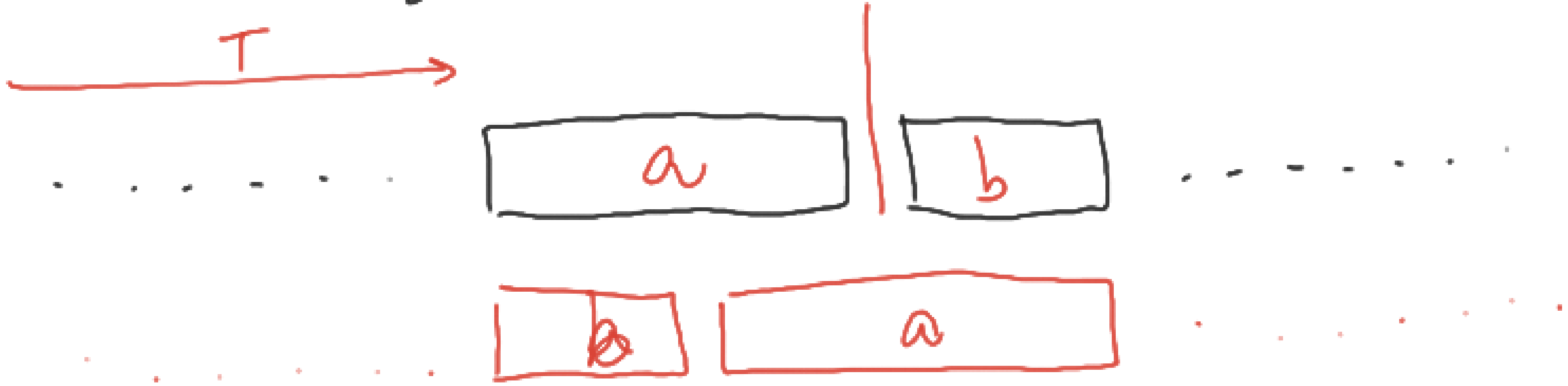
$$\min \frac{1}{n}\sum_j R_j = \frac{50}{5} = 10$$



$1 + 3 + 5 + 10 + 16$

Shortest Job First

$j_1, j_2 \ldots j_n$



$+ (T + a) + (T + a + b) \ldots$

$+ (T + b) + (T + b + a) + \ldots$

Kruskal

Sort edges in increasing order of cost

$e_1, e_2 \ldots\ldots e_m$

$T \leftarrow \phi$

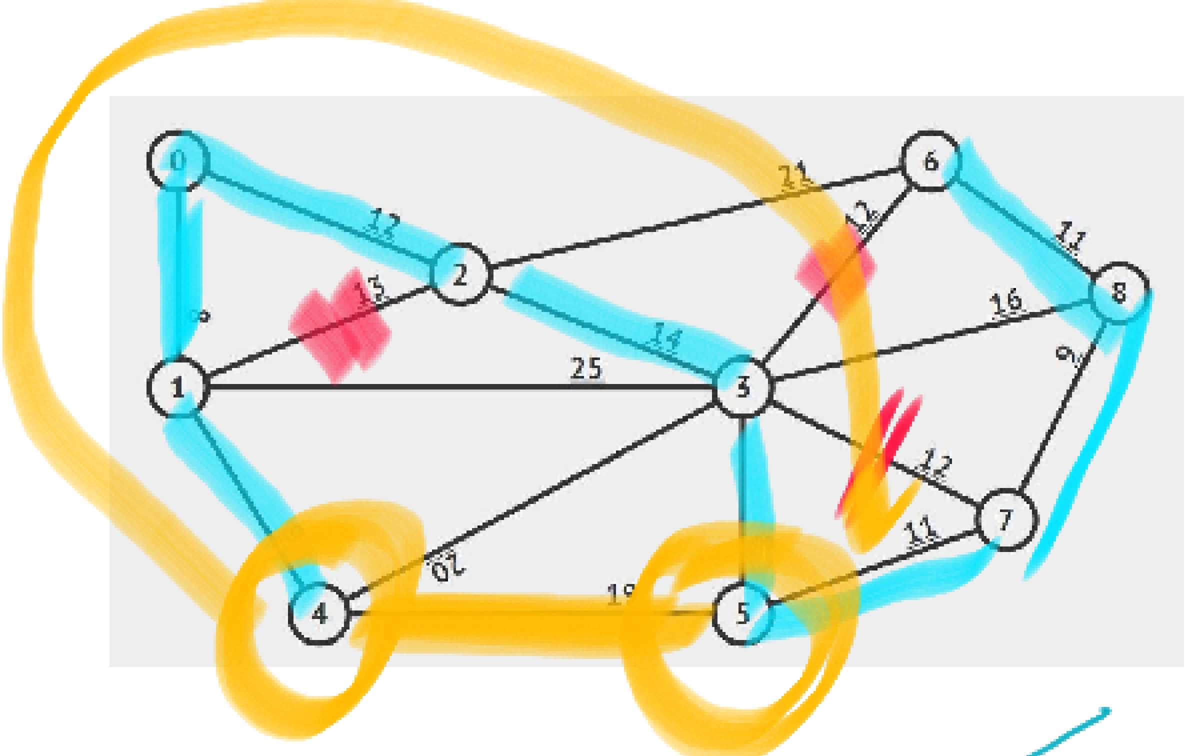
m [For $i = 1$ to m

if adding e_i to T doesn't create cycle

$T \leftarrow T \cup \{e_i\}$]

return T

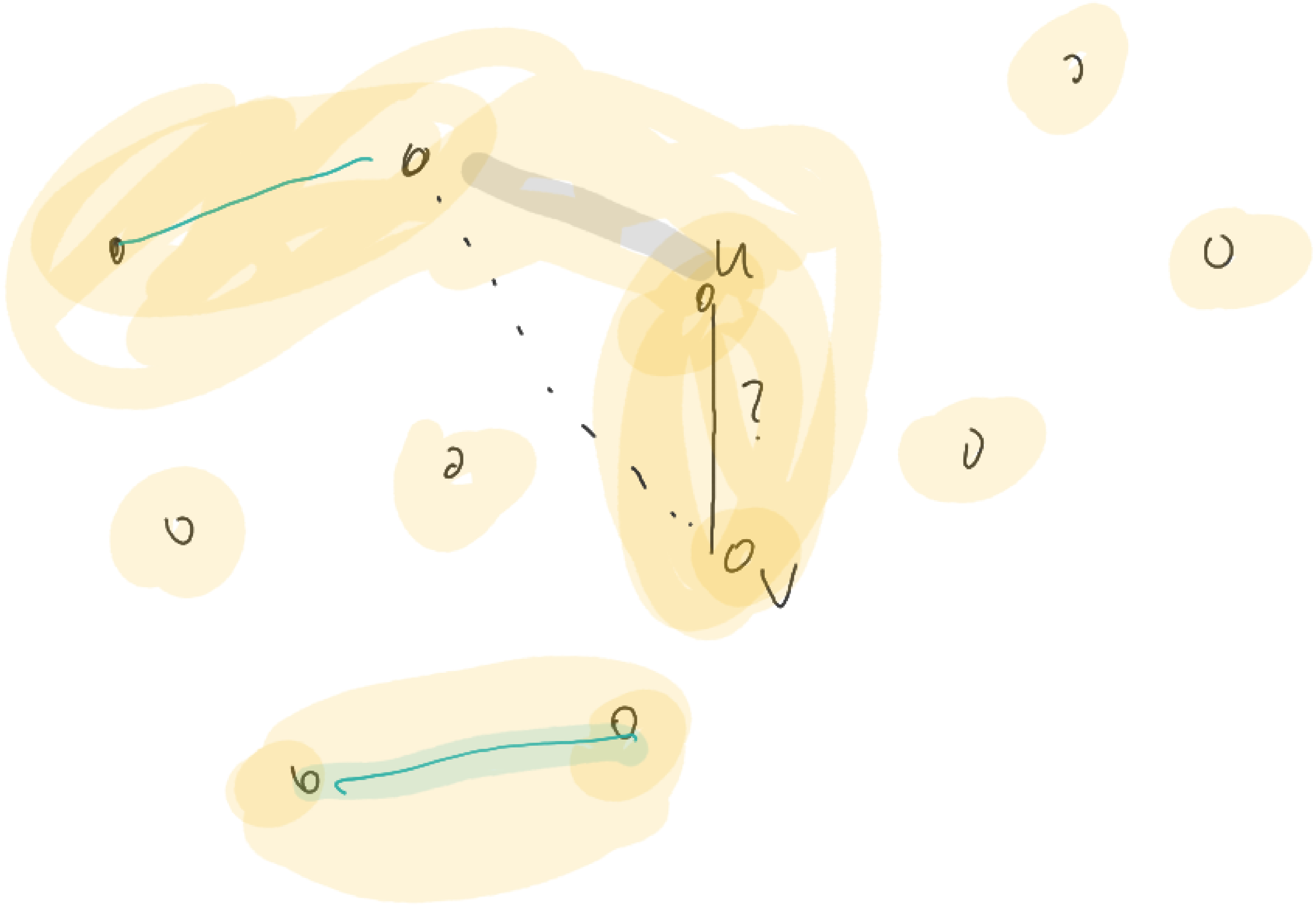
O(n)



① Correct? ✓

② Runtime? ?

O(mn)


u
?
v

Kruskal II

Sort m #s $O(m \log m)$
+ $O(m+n)$ operations

- $T \leftarrow \emptyset$
- ~~for~~ all $v \in V$: makeset(v) — n makeset
- Sort edges $e_1, e_2 \dots e_m$ — sort m #s

for $i = 1$ to m
    $e_i = (u, v)$
    if (find(u) ≠ find(v)) — 2m find
        - $T \leftarrow T \cup \{e_i\}$
        - Union (u, v) — ~~m~~ n union

return T

Data Structure

Set Union-Find

1. Createset (i)
2. Find(i) — ↑ returns "name" of set containing i
3. Union(i, j) — merges sets containing i & j

Thm: Kruskal's Algo finds MST in $O(m \log m)$

$+ O(m+n)$ Union-find OPs

Claim: $\exists$ implementation of UF where k ops cost $O(k \log m)$ (starting from an empty state)

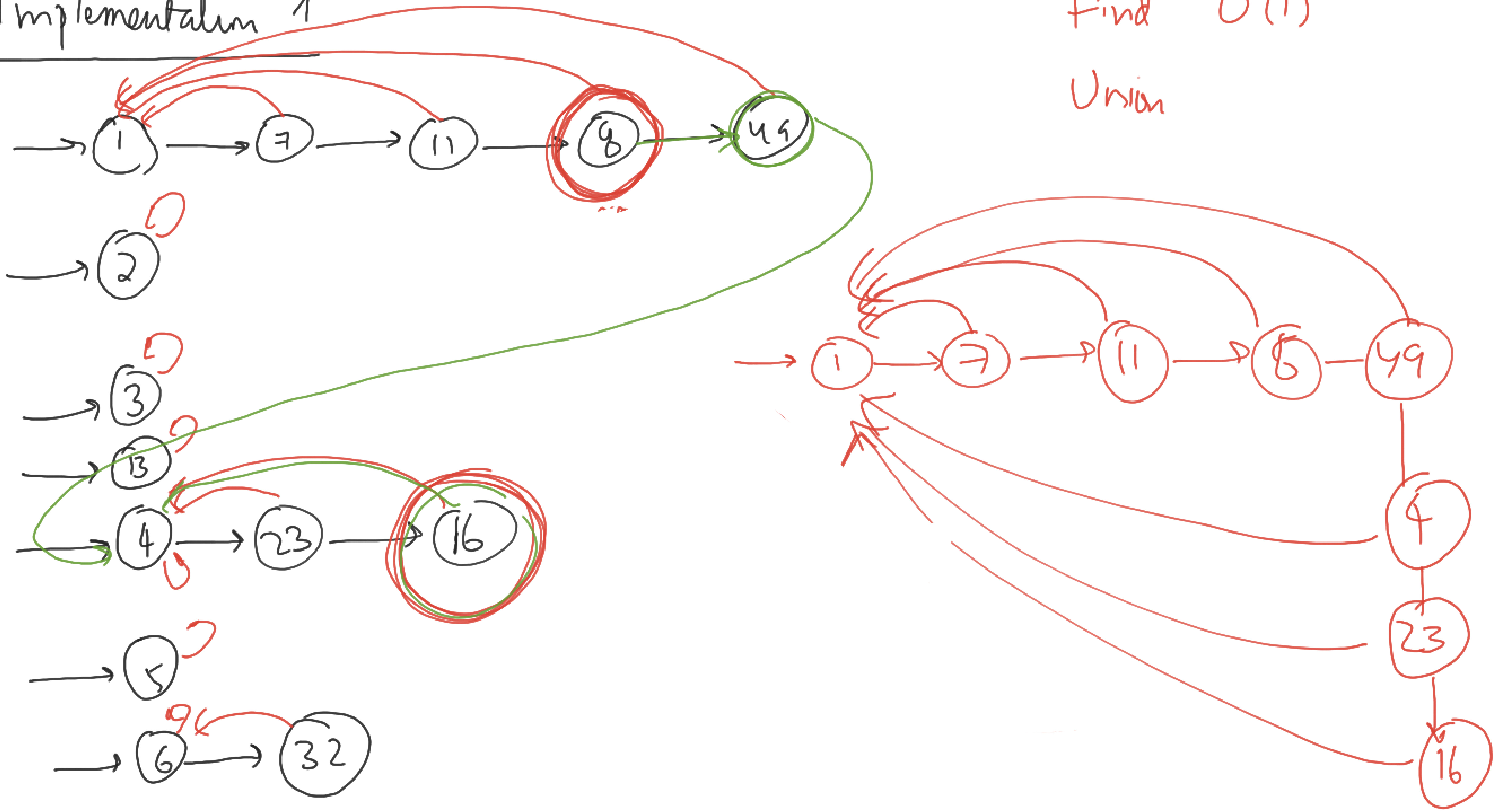
Claim: - - - - - - k ops cost $O(k \cdot \log^* m)$

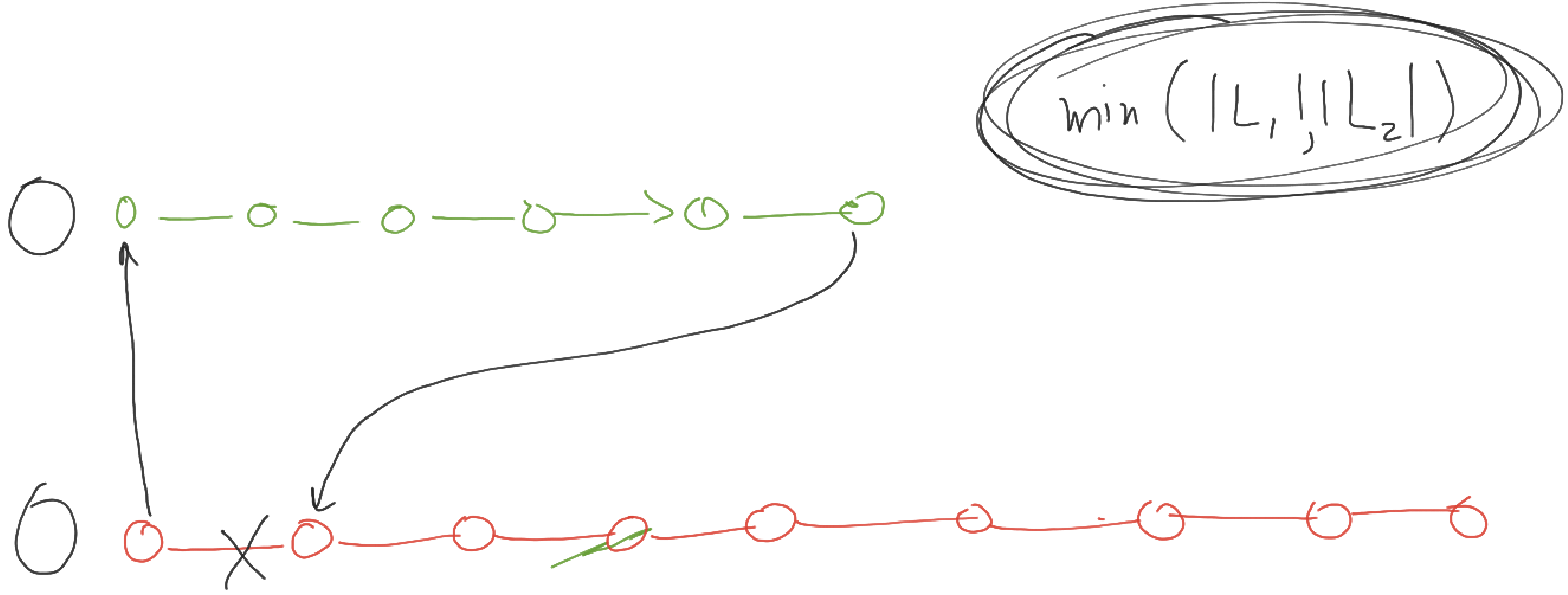
$\log^*(2^{65526}) \leq 6$

Implementation 1

Makeset O(1)
Find O(1)

Union

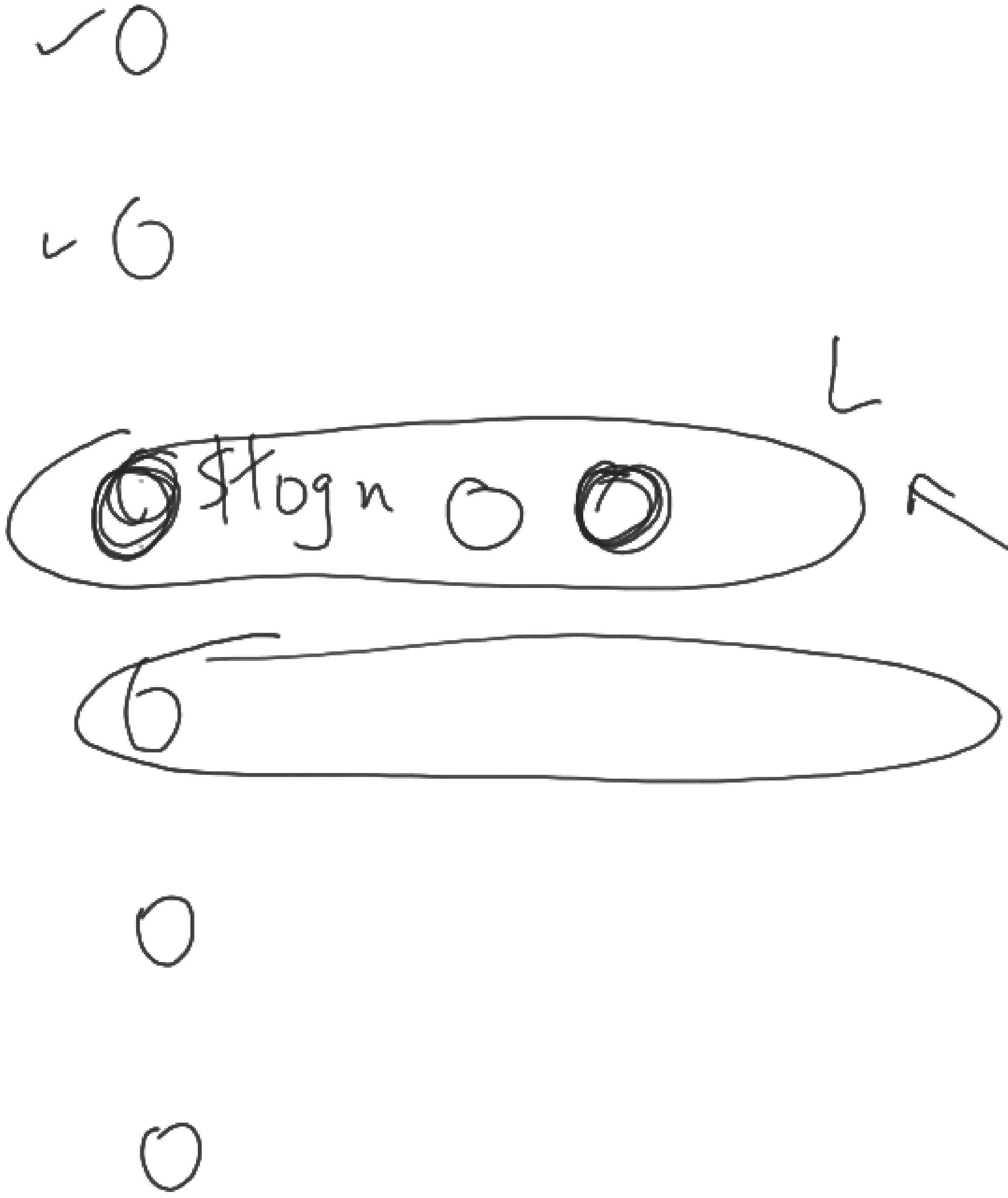



min(|L1|,|L2|)

n makesets ~~unions~~ finds $O(1)$

(starting from the empty state)

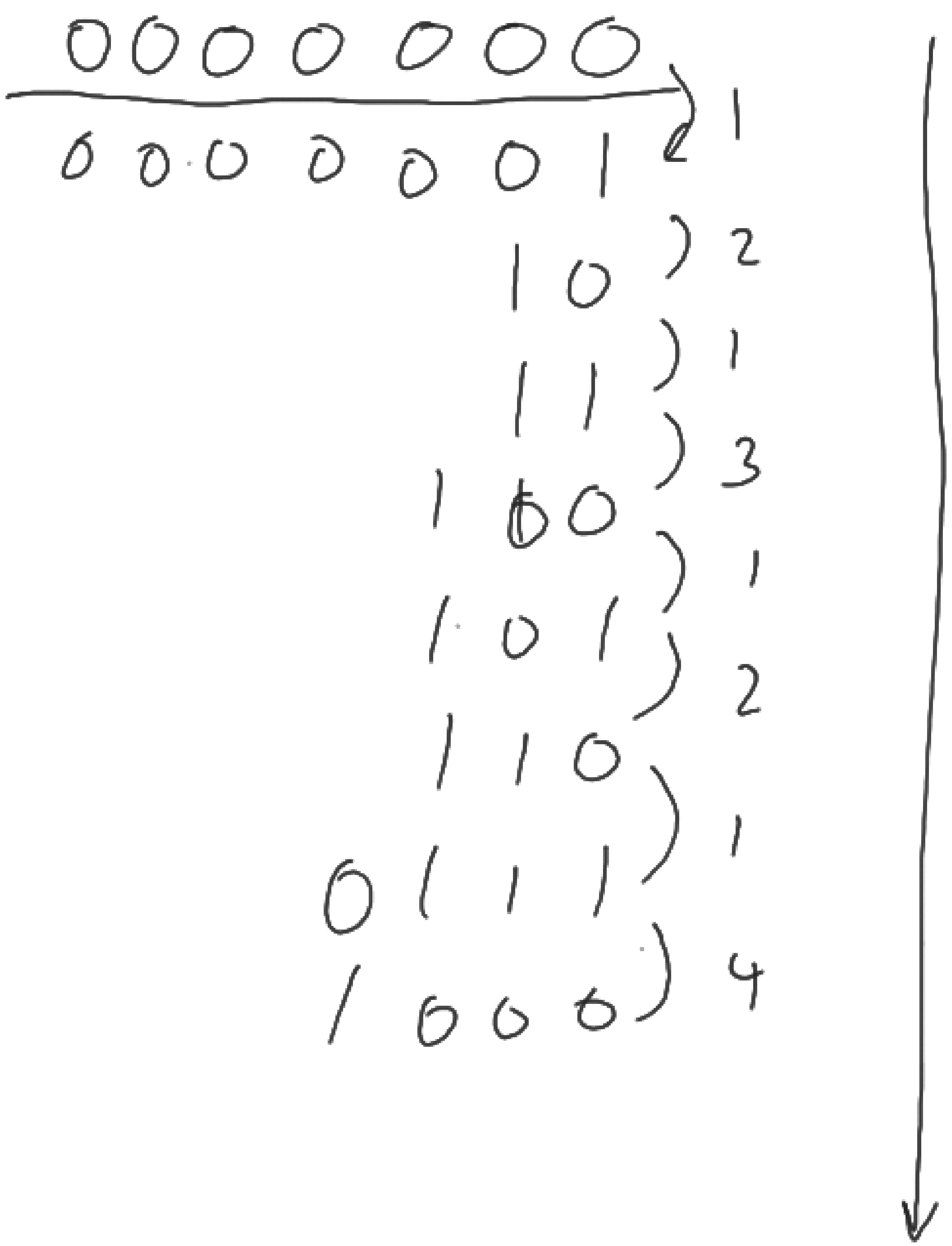
k unions will take time $O(n \log n)$



each person $\$\log_2 n$ starts with

list length L

$\Rightarrow$ each person in this list has $\geq \log_2(n/L)$ money



Sps start at all-zeros

K increments

① total cost? $\leq 2K$

② What is max cost of an operation among these K? $\log_2 K + O(1)$