15780: GRADUATE AI (SPRING 2017)

Practice Midterm Exam

February 23, 2017

Topic	Total Score	Score
Heuristic Search	25	
VC Dimension	25	
Integer Programming	25	
Convex Optimization	25	
Total	100	

1 Heuristic Search [25 points]

Consider the problem of informed search with a heuristic. For each state x, let $h^*(x)$ be the length of the cheapest path from x to a goal.

Prove or disprove the following statements:

1.1 [15 points] If $h(x) = 2h^*(x)$ for all states x, then A^* tree search with the heuristic h is optimal.

1.2 [10 points] If h is a consistent heuristic, A^* graph search with the heuristic $h'(x) = h(x)/2$ is optimal.

2 Learning Theory [25 points]

Determine the VC dimension of the following function classes.

2.1 [15 points] Define F to be the set of strings of length 3 composed of the symbols 0, 1, and $*$. Each $f \in F$ acts as a pattern matcher; i.e., when applied to a binary string s, it either accepts or rejects s. For example, when we apply the schema $f = 1 * *$ to the string $s = 101$, it accepts, and when we apply f to $s' = 010$, it rejects. What is the VC dimension of F?

2.2 [10 points] The union of n intervals on the real line.

3 Integer Programming [25 points]

Consider an undirected graph $G = (V, E)$. A *minimum dominating set* is a smallest subset S of V such that every node not in S is adjacent to at least one node in S. A *minimum independent dominating set* is a smallest subset S of V such that (1) every node not in S is adjacent to at least one node in S and (2) no pair of nodes in S are adjacent. In your answer, you can use $N(i)$ to denote the set of neighbors of node i (i.e., $N(i)$ is a set of nodes adjacent to i) for each node $i \in V$. Note that $i \notin N(i)$. You also can use $(i, j) \in E$ to denote the edge between node $i \in V$ and node $j \in V$.

3.1 [15 points] Formulate an integer linear program to find a minimum dominating set.

3.2 [10 points] Formulate an integer linear program to find a minimum independent dominating set.

4 Convex Optimization [25 points]

Consider a linear program of the standard form: minimize $\mathbf{c}^T\mathbf{x}$ such that $\mathbf{A}\mathbf{x} \leq \mathbf{b}$. Here $\mathbf{x} \in \mathbb{R}^n$ is the vector of variables, and $\mathbf{c} \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{m \times n}$, and $\mathbf{b} \in \mathbb{R}^m$ are constants.

Prove from the definitions that this is a convex program.