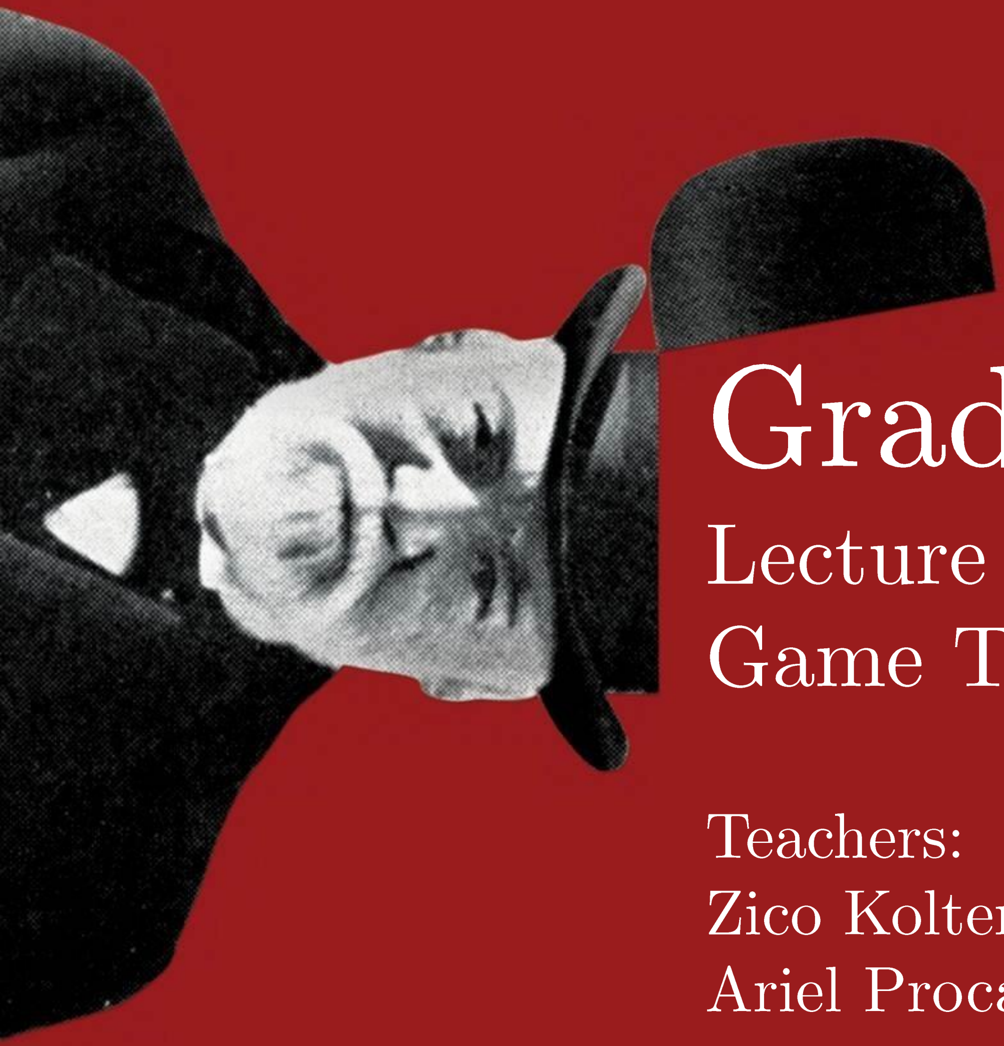
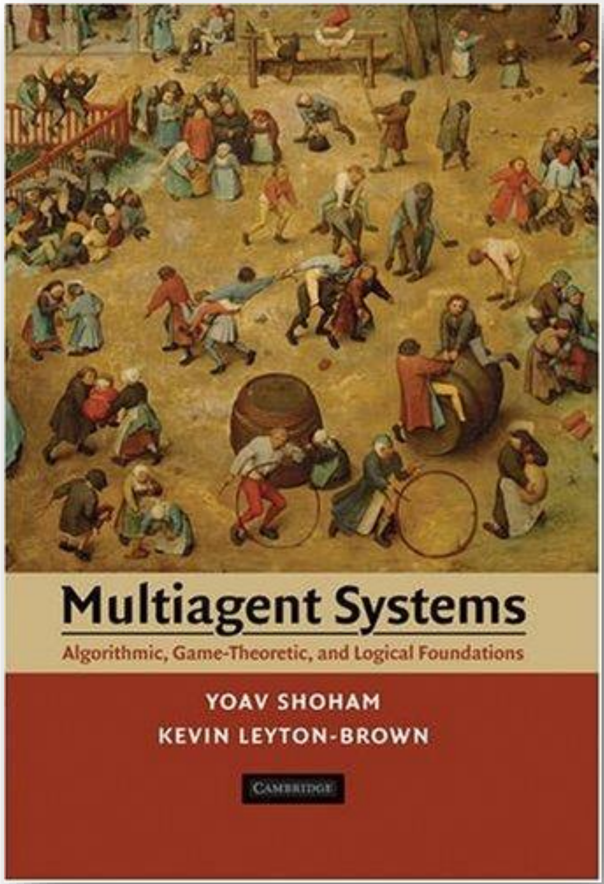


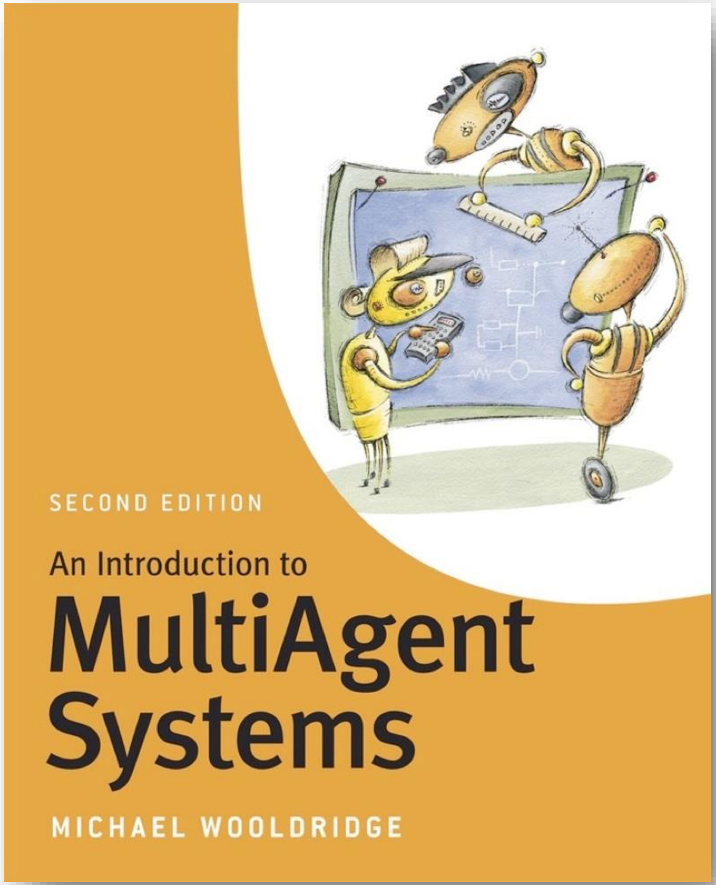
Graduate AI

Lecture 18: Game Theory I

Teachers:
Zico Kolter
Ariel Procaccia (this time)

Multiagent systems





Multiagent systems

Chapters of the Shoham and Leyton-Brown book:

1. Distributed constraint satisfaction
2. Distributed optimization
3. Games in normal form
4. Computing solution concepts of normal-form games
5. Games with sequential actions
6. Beyond the normal and extensive forms
7. Learning and teaching
8. Communication
9. Social choice
10. Mechanism design
11. Auctions
12. Coalitional game theory
13. Logics of knowledge and belief
14. Probability, dynamics, and intention

Legend:

■ "Game theory"

■ Not "game theory"

Multiagent systems

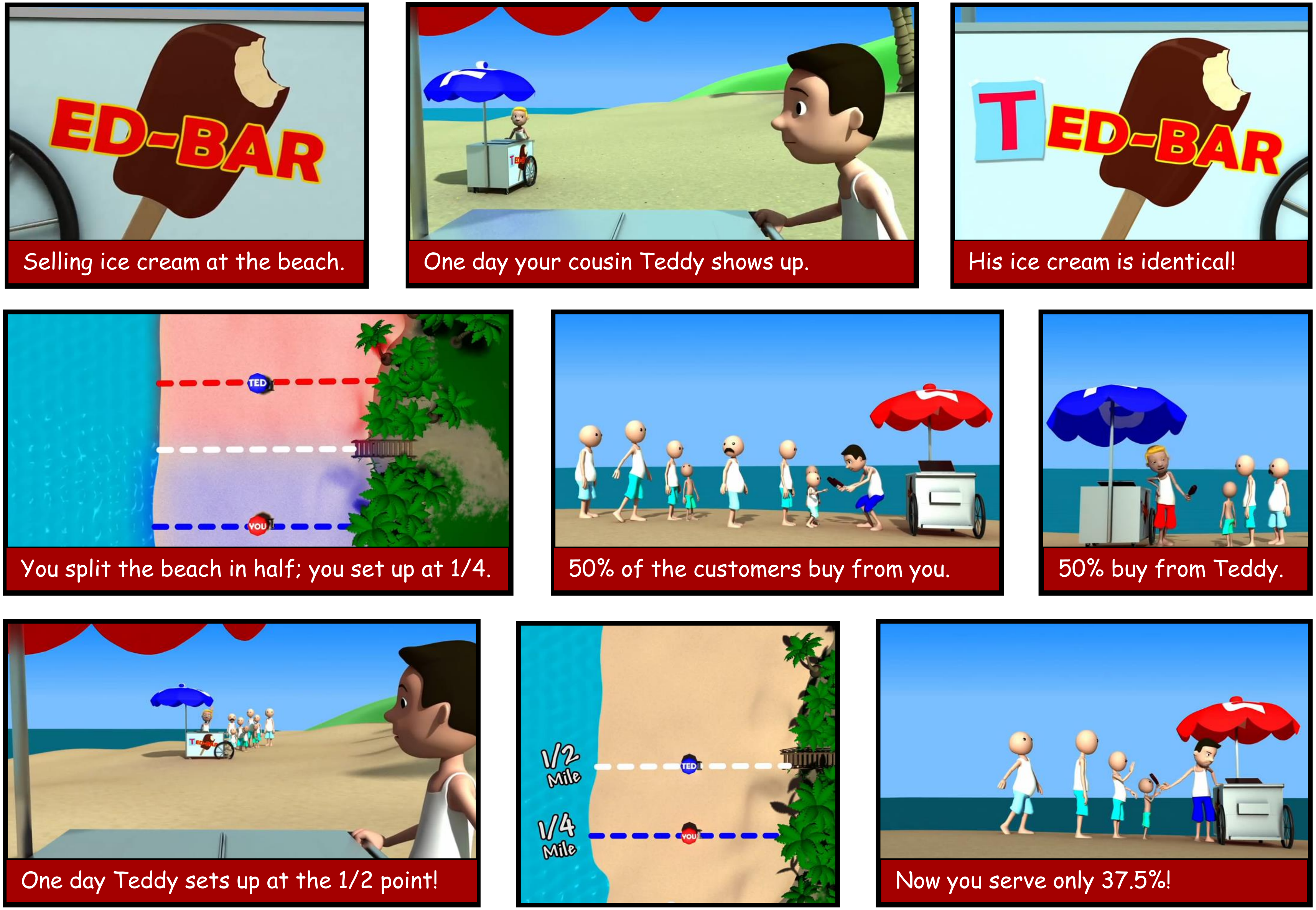
Mike Wooldridge's 2016 publications:

2016

- [j126] Shaheen Fatima, Michael Wooldridge: **Majority bargaining for resource division.** Autonomous Agents and Multi-Agent Systems 30(2): 331-363 (2016)
- [j125] Shaheen Fatima, Tomasz P. Michalak, Michael Wooldridge: **Power and welfare in bargaining for coalition structure formation.** Autonomous Agents and Multi-Agent Systems 30(5): 899-930 (2016)
- [j124] Tomasz P. Michalak, Talal Rahwan, Edith Elkind, Michael Wooldridge, Nicholas R. Jennings: **A hybrid exact algorithm for complete set partitioning.** Artif. Intell. 230: 14-50 (2016)
- [j123] Paul Harrenstein, Paolo Turrini, Michael Wooldridge: **Hard and Soft Preparation Sets in Boolean Games.** Studia Logica 104(4): 813-847 (2016)
- [j122] Julian C. Bradfield, Julian Gutierrez, Michael Wooldridge: **Partial-order Boolean games: informational independence in a logic-based model of strategic interaction.** Synthese 193(3): 781-811 (2016)
- [c208] Mateusz Krzysztof Tarkowski, Piotr L. Szczepanski, Talal Rahwan, Tomasz P. Michalak, Michael Wooldridge: **Closeness Centrality for Networks with Overlapping Community Structure.** AAAI 2016: 622-629
- [c207] Michael Wooldridge, Julian Gutierrez, Paul Harrenstein, Enrico Marchioni, Giuseppe Perelli, Alexis Toumi: **Rational Verification: From Model Checking to Equilibrium Checking.** AAAI 2016: 4184-4191
- [c206] Oskar Skibski, Szymon Matejczyk, Tomasz P. Michalak, Michael Wooldridge, Makoto Yokoo: **k-Coalitional Cooperative Games.** AAMAS 2016: 177-185
- [c205] Julian Gutierrez, Paul Harrenstein, Giuseppe Perelli, Michael Wooldridge: **Expressiveness and Nash Equilibrium in Iterated Boolean Games.** AAMAS 2016: 707-715
- [c204] Piotr L. Szczepanski, Tomasz P. Michalak, Talal Rahwan, Michael Wooldridge: **An Extension of the Owen-Value Interaction Index and Its Application to Inter-Links Prediction.** ECAI 2016: 90-98
- [c203] Oskar Skibski, Henryk Michalewski, Andrzej Nagórko, Tomasz P. Michalak, Andrew James Dowell, Talal Rahwan, Michael Wooldridge: **Non-Utilitarian Coalition Structure Generation.** ECAI 2016: 1738-1739
- [c202] Michael Wooldridge: **From model checking to equilibrium checking.** GI-Jahrestagung 2016: 35
- [c201] Haris Aziz, Paul Harrenstein, Jérôme Lang, Michael Wooldridge: **Boolean Hedonic Games.** KR 2016: 166-175
- [c200] Julian Gutierrez, Giuseppe Perelli, Michael Wooldridge: **Imperfect Information in Reactive Modules Games.** KR 2016: 390-400
- [i11] Marcin Waniek, Tomasz P. Michalak, Talal Rahwan, Michael Wooldridge: **Hiding Individuals and Communities in a Social Network.** CoRR abs/1608.00375 (2016)

Normal-Form Game

- A game in normal form consists of:
    - Set of players $N = \{1, \dots, n\}$
    - Strategy set S
    - For each $i \in N$, utility function $u_i: S^n \to \mathbb{R}$: if each $j \in N$ plays the strategy $s_j \in S$, the utility of player i is $u_i(s_1, \dots, s_n)$
- Next example created by taking screenshots of http://youtu.be/jILgxeNBK_8


ED-BAR
Selling ice cream at the beach.
One day your cousin Teddy shows up.
TED-BAR
His ice cream is identical!
TED
YOU
You split the beach in half; you set up at 1/4.
50% of the customers buy from you.
50% buy from Teddy.
One day Teddy sets up at the 1/2 point!
1/2 Mile
TED
1/4 Mile
YOU
Now you serve only 37.5%!

The Ice Cream Wars

- $N = \{1,2\}$
- $S = [0,1]$
- $u_i(s_i, s_j) = \begin{cases} \frac{s_i+s_j}{2}, & s_i < s_j \\ 1 - \frac{s_i+s_j}{2}, & s_i > s_j \\ \frac{1}{2}, & s_i = s_j \end{cases}$
- To be continued...

The prisoner's dilemma

- Two men are charged with a crime
- They are told that:
  - If one rats out and the other does not, the rat will be freed, other jailed for nine years
  - If both rat out, both will be jailed for six years
- They also know that if neither rats out, both will be jailed for one year

The prisoner's dilemma

	Cooperate	Defect
Cooperate	-1,-1	-9,0
Defect	0,-9	-6,-6

What would you do?

Understanding the dilemma

- Defection is a dominant strategy
- But the players can do much better by cooperating
- Related to the tragedy of the commons

In real life

- Presidential elections
  - Cooperate = positive ads
  - Defect = negative ads
- Nuclear arms race
  - Cooperate = destroy arsenal
  - Defect = build arsenal
- Climate change
  - Cooperate = curb CO_2 emissions
  - Defect = do not curb

On TV

http://youtu.be/S0qjK3TWZE8

The professor's dilemma

Professor \ Class	Listen	Sleep
Make effort	10^6,10^6	-10,0
Slack off	0,-10	0,0

Dominant strategies?

Nash equilibrium

- Each player's strategy is a best response to strategies of others
- Formally, a Nash equilibrium is a vector of strategies $s = (s_1 \dots, s_n) \in S^n$ such that $\forall i \in N, \forall s_i' \in S, u_i(s) \geq u_i(s_i', s_{-i})$

Nash equilibrium

- Poll 1: How many Nash equilibria does the Professor's Dilemma have?

1. 0
2. 1
3. 2
4. 3

	Listen	Sleep
Make effort	$10^6, 10^6$	-10,0
Slack off	0,-10	0,0

Nash equilibrium

http://youtu.be/CemLiSI5ox8

Russel Crowe was wrong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Computation, Economics, and Game Theory

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Russell Crowe was wrong

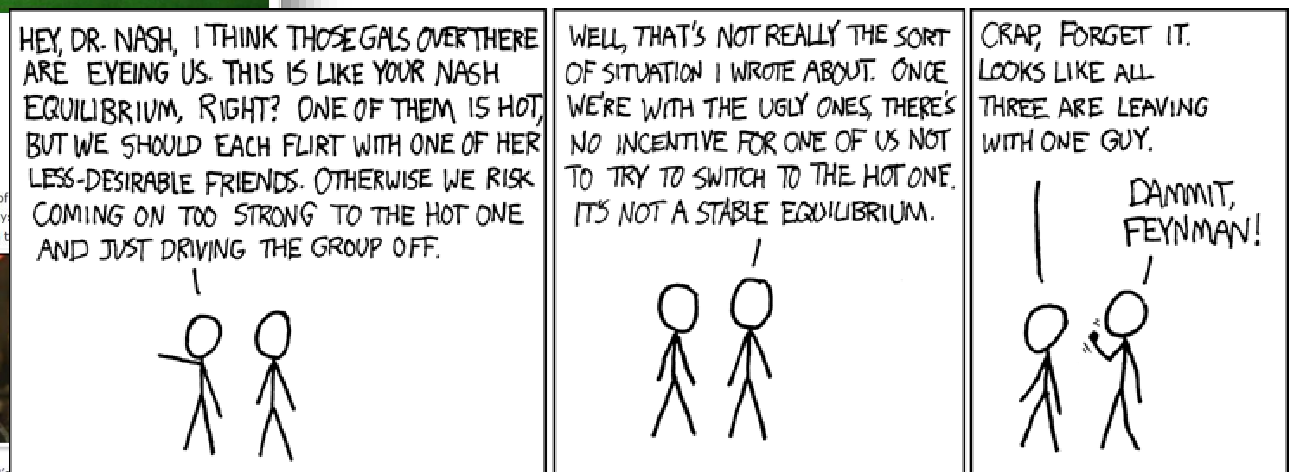
October 30, 2012 by Ariel Procaccia | Edit

Yesterday I taught the first of five algorithmic economics lectures in my undergraduate AI course. This lecture just introduced the basic concepts of game theory, focusing on Nash equilibria. I was contemplating various way making the lecture more lively, and it occurred to me that I could stand on t shoulders of giants. Indeed, didn't Russell Crowe already explain Nash's ideas in A Beautiful Mind, complete with a 1940's-style male chauvinistic example?

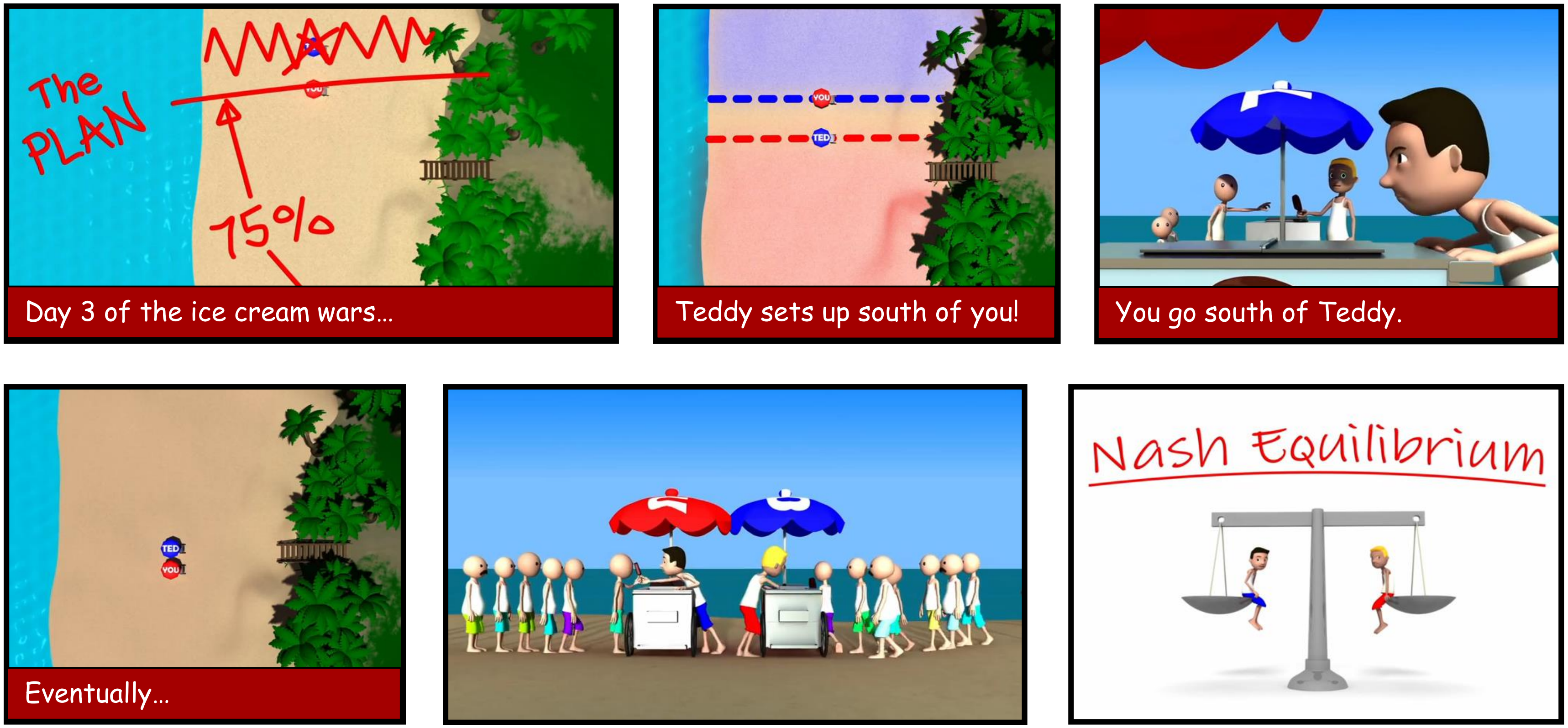
The first and last time I watched the movie was when it was released in 2001. Back then I was an undergrad freshman, working for 20+ hours a week on the programming exercises of Hebrew U's Intro to CS course, which was taught by some guy called Noam Nisan. I didn't know anything about game theory, and Crowe's explanation made a lot of sense at the time.

I easily found the relevant scene on youtube. In the scene, Nash's friends are trying to figure out how to seduce a beautiful blonde and her less beautiful friends. Then Nash/Crowe has an epiphany. The hubbub of the seedy Princeton bar is drowned by inspirational music, as Nash announces:

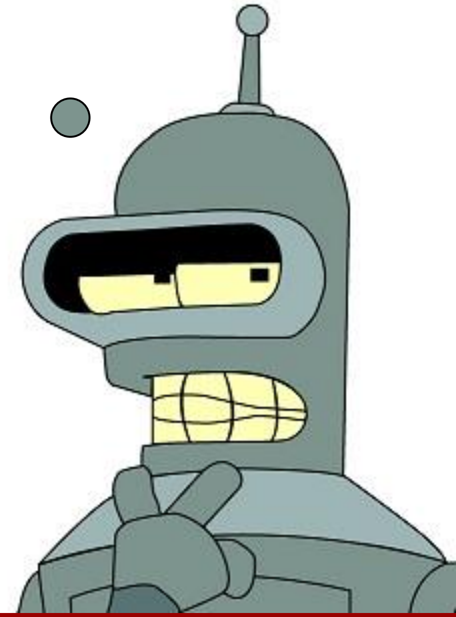
January 2012
December 2011
November 2011
October 2011
September 2011
August 2011
July 2011
June 2011



End of the Ice Cream Wars



This is why competitors open their stores next to one another!



Rock-paper-scissors

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Nash equilibrium?

Mixed strategies

- A mixed strategy is a probability distribution over (pure) strategies
- The mixed strategy of player $i \in N$ is x_i, where

$$x_i(s_i) = \Pr[i \text{ plays } s_i]$$

- The utility of player $i \in N$ is

$$u_i(x_1, \dots, x_n) = \sum_{(s_1,\dots,s_n) \in S^n} u_i(s_1, \dots, s_n) \cdot \prod_{j=1}^{n} x_j(s_j)$$

Exercise: Mixed NE

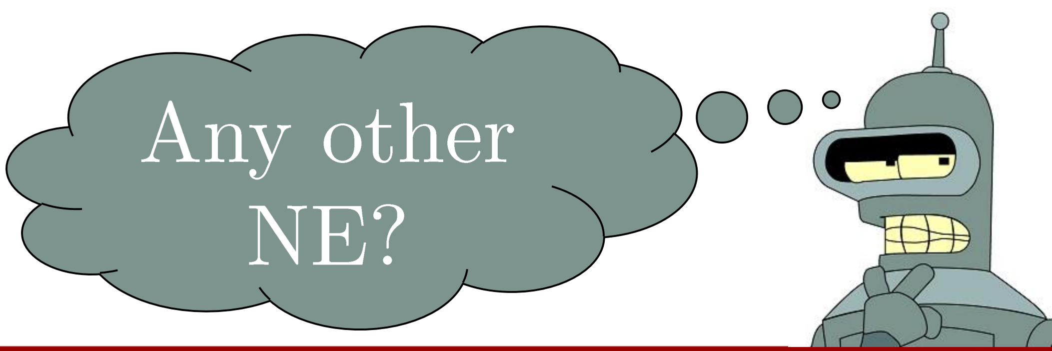
- Exercise: player 1 plays $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$, player 2 plays $\left(0, \frac{1}{2}, \frac{1}{2}\right)$. What is u_1?
- Exercise: Both players play $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. What is u_1?

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Exercise: Mixed NE

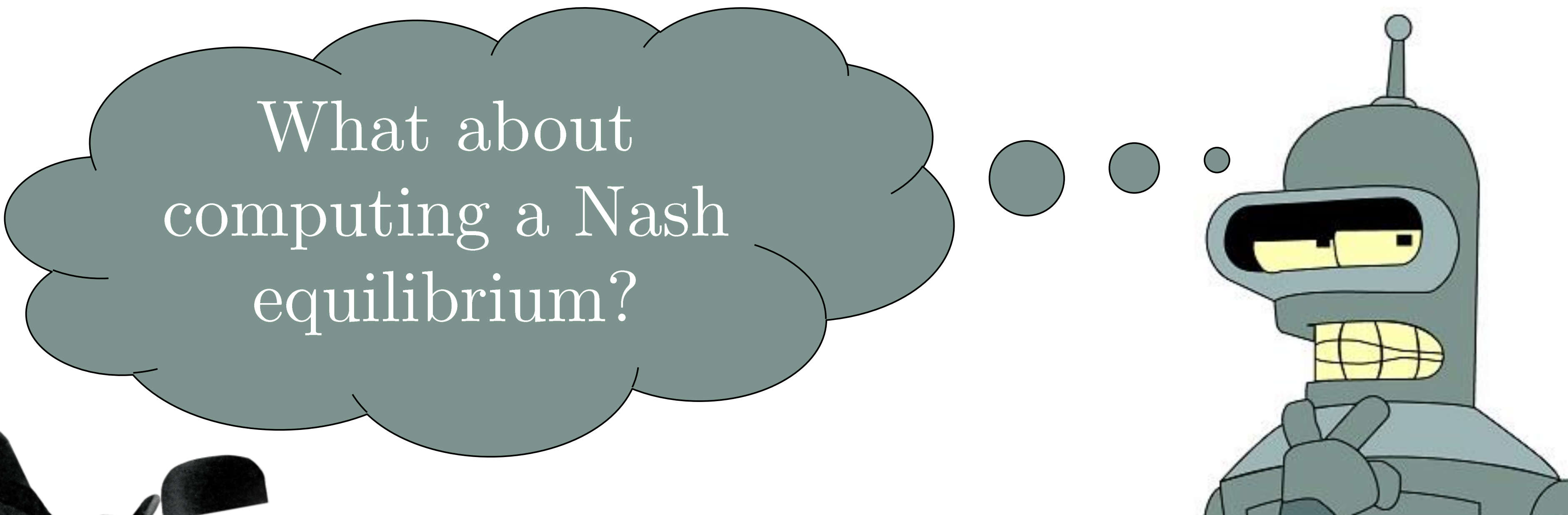
- Poll 2: Which is a NE?
  1. $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(\frac{1}{2},\frac{1}{2},0\right)\right)$
  2. $\left(\left(\frac{1}{2},\frac{1}{2},0\right),\left(\frac{1}{2},0,\frac{1}{2}\right)\right)$
  3. $\left(\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right)$
  4. $\left(\left(\frac{1}{3},\frac{2}{3},0\right),\left(\frac{2}{3},0,\frac{1}{3}\right)\right)$

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0



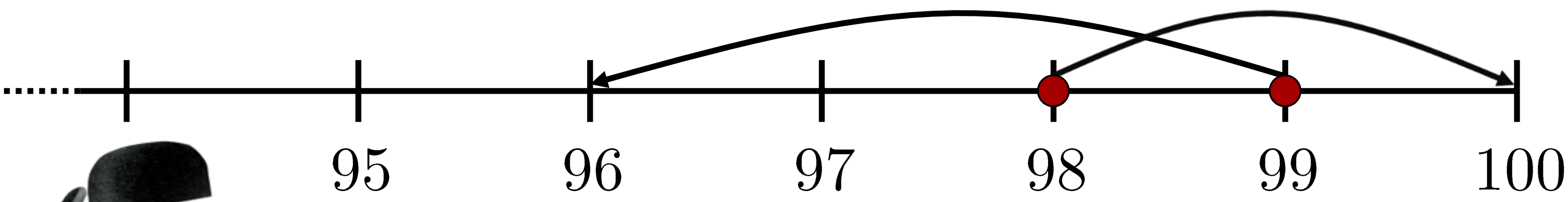
Nash's Theorem

- Theorem [Nash, 1950]: In any (finite) game there exists at least one (possibly mixed) Nash equilibrium



Does NE make sense?

- Two players, strategies are $\{2, \dots, 100\}$
- If both choose the same number, that is what they get
- If one chooses s, the other t, and $s < t$, the former player gets $s + 2$, and the latter gets $s - 2$
- Poll 3: What would you choose?



SUMMARY

- Terminology:
  - Normal-form game
  - Nash equilibrium
  - Mixed strategies
- Nobel-prize-winning ideas:
  - Nash equilibrium ☺