

Graduate AI

Lecture 22: Social Choice I

Teachers:

Zico Kolter

Ariel Procaccia (this time)

SOCIAL CHOICE THEORY

- A mathematical theory that deals with aggregation of individual preferences
- Origins in ancient Greece
- Formal foundations: 18th Century (Condorcet and Borda)
- 19th Century: Charles Dodgson
- 20th Century: Nobel prizes to Arrow and Sen



THE VOTING MODEL

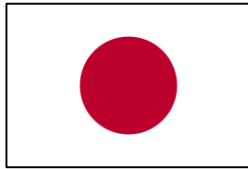
- Set of voters $N = \{1, \dots, n\}$
- Set of alternatives A ;
denote $|A| = m$
- Each voter has a ranking over the alternatives
- Preference profile = collection of all voters' rankings

1	2	3
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>

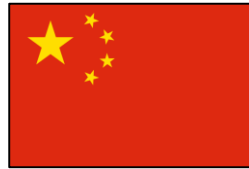
VOTE OVER CUISINES



Indian
(In)



Japanese
(J)



Chinese
(C)



Italian
(It)



Mexican
(M)



VOTING RULES

- **Voting rule** = function from preference profiles to alternatives that specifies the winner of the election
- **Plurality**
 - Each voter awards one point to top alternative
 - Alternative with most points wins
 - Used in almost all political elections



MORE VOTING RULES

- Borda count
 - Each voter awards $m - k$ points to alternative ranked k 'th
 - Alternative with most points wins
 - Proposed in the 18th Century by the chevalier de Borda
 - Used for national elections in Slovenia
 - Similar to rule used in the Eurovision song contest



Lordi

Eurovision 2006 winners

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MORE VOTING RULES

- x beats y in a **pairwise election** if the majority of voters prefer x to y
- **Plurality with runoff**
 - First round: two alternatives with highest plurality scores survive
 - Second round: pairwise election between these two alternatives



MORE VOTING RULES

- Single Transferable vote (STV)
 - $m - 1$ rounds
 - In each round, alternative with least plurality votes is eliminated
 - Alternative left standing is the winner
 - Used in:
 - Ireland, Malta, Australia, and New Zealand
 - US: Maine (governor, US congress), cities like San Francisco and Cambridge



STV: EXAMPLE

2 voters	2 voters	1 voter
<i>a</i>	<i>b</i>	<i>c</i>
<i>b</i>	<i>a</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>b</i>
<i>d</i>	<i>c</i>	<i>a</i>

2 voters	2 voters	1 voter
<i>a</i>	<i>b</i>	<i>c</i>
<i>b</i>	<i>a</i>	<i>b</i>
<i>c</i>	<i>c</i>	<i>a</i>

2 voters	2 voters	1 voter
<i>a</i>	<i>b</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>a</i>

2 voters	2 voters	1 voter
<i>b</i>	<i>b</i>	<i>b</i>



SOCIAL CHOICE AXIOMS

- How do we choose among the different voting rules? Via desirable properties!
- **Majority consistency** = if a majority of voters rank alternative x first, then x should be the winner
- **Poll 1:** Which rule is **not** majority consistent?
 1. Plurality
 2. Plurality with runoff
 3. Borda count
 4. STV



MARQUIS DE CONDORCET

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison



CONDORCET WINNER

- Recall: x beats y in a **pairwise election** if a majority of voters rank x above y
- **Condorcet winner** beats every other alternative in pairwise election
- **Condorcet paradox** = cycle in majority preferences

1	2	3
a	c	b
b	a	c
c	b	a



CONDORCET CONSISTENCY

- Condorcet consistency = select a Condorcet winner if one exists
- Poll 2: Which rule is Condorcet consistent?
 1. Plurality
 2. Borda count
 3. Both
 4. Neither



CONDORCET CONSISTENT RULES

- Copeland

- Alternative's score is #alternatives it beats in pairwise elections
- Why does Copeland satisfy the Condorcet criterion?

- Maximin

- Score of x is $\min_y |\{i \in N: x \succ_i y\}|$
- Why does Maximin satisfy the Condorcet criterion?

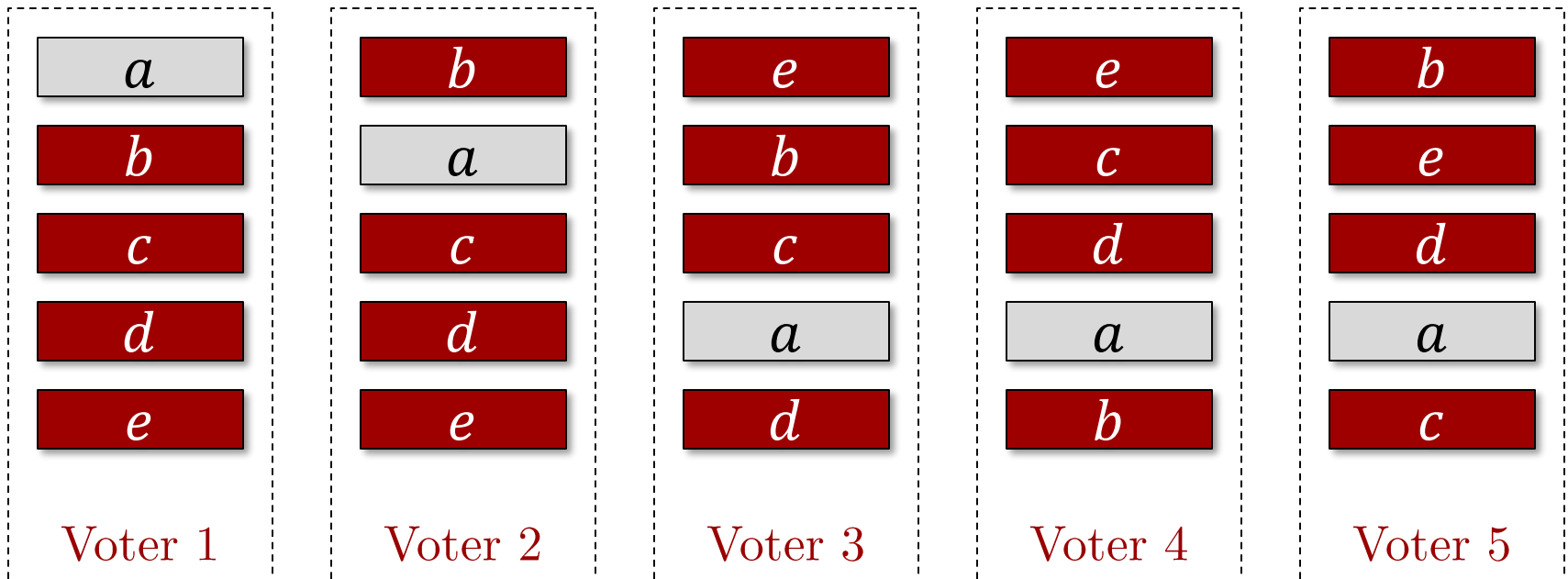


DODGSON'S RULE

- Distance function between profiles: #swaps between adjacent alternatives
- **Dodgson score** of x = the min distance from a profile where x is a Condorcet winner
- Dodgson's rule: select alternative that minimizes Dodgson score
- The problem of computing the Dodgson score is NP-complete!



DODGSON UNLEASHED

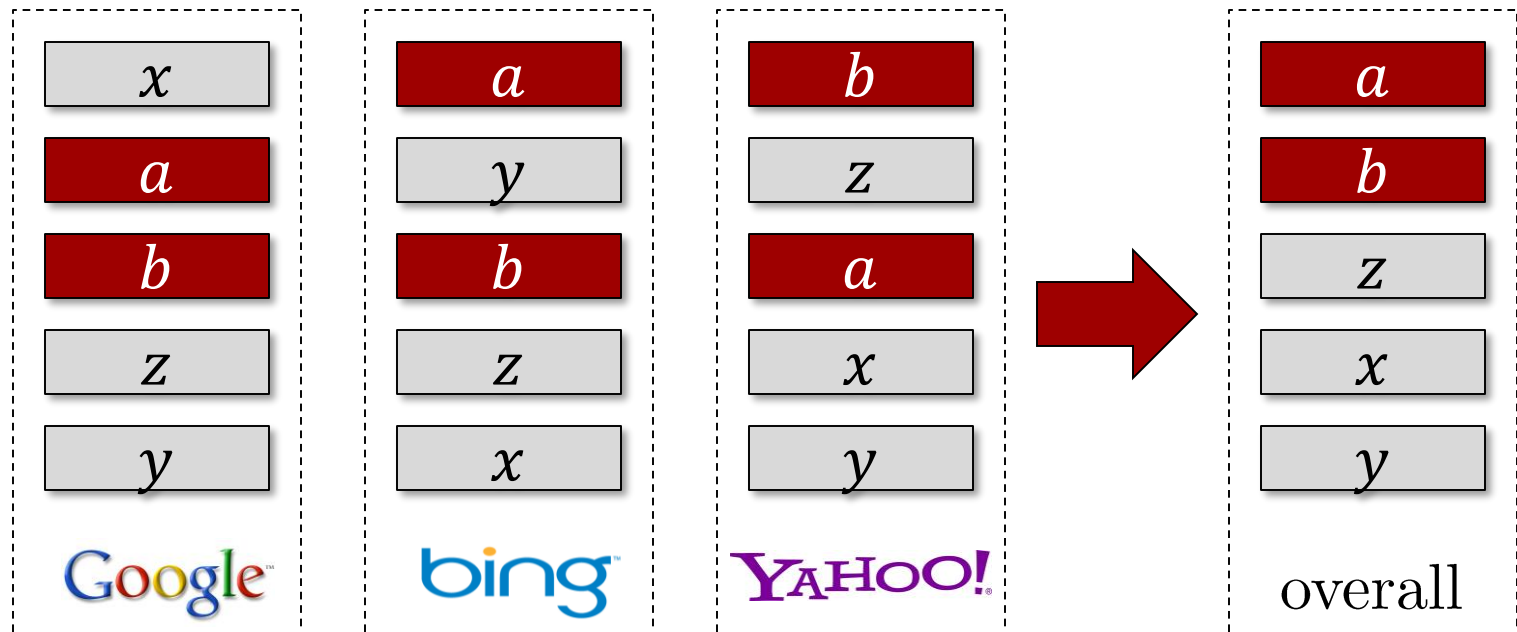


APPLICATION: WEB SEARCH

- **Generalized Condorcet:** if there is a partition X, Y of A such that a majority prefers every $x \in X$ to every $y \in Y$, then X is ranked above Y
- Assumption: spam website identified by a majority of search engines
- When aggregating results from different search engines, spam websites will be ranked last [Dwork et al. 2001]



APPLICATION: WEB SEARCH



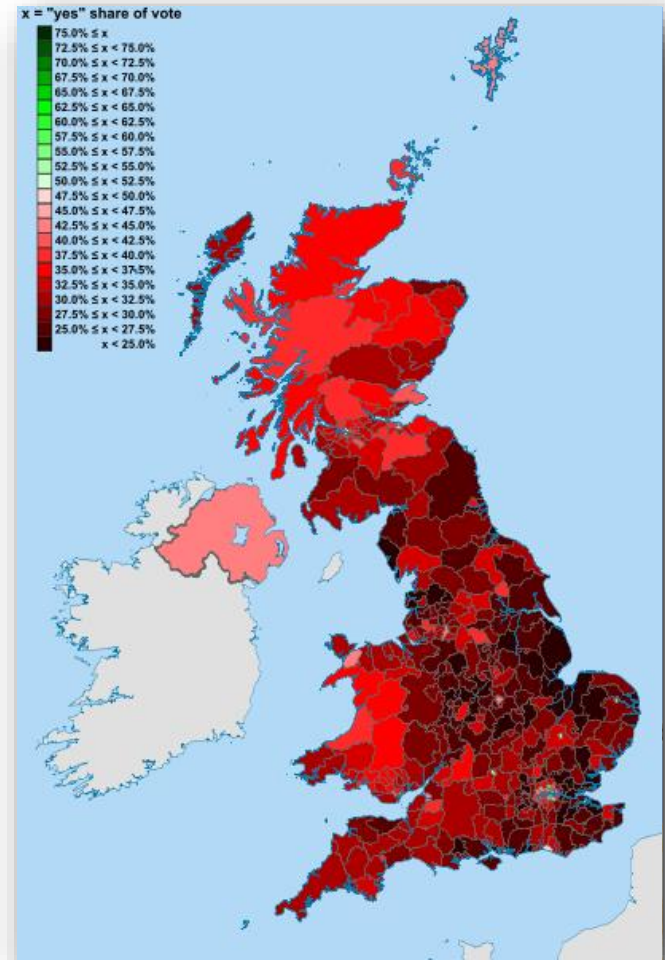
AWESOME EXAMPLE

- Plurality: a
- Borda: b
- Condorcet winner: c
- STV: d
- Plurality with runoff: e

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

IS SOCIAL CHOICE PRACTICAL?

- UK referendum: Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better...
- ... but STV seen as beneficial to the hated Nick Clegg
- Hard to change political elections!



COMPUTATIONAL SOCIAL CHOICE

- However:
 - in online voting...
 - in human computation...
 - in multiagent systems...

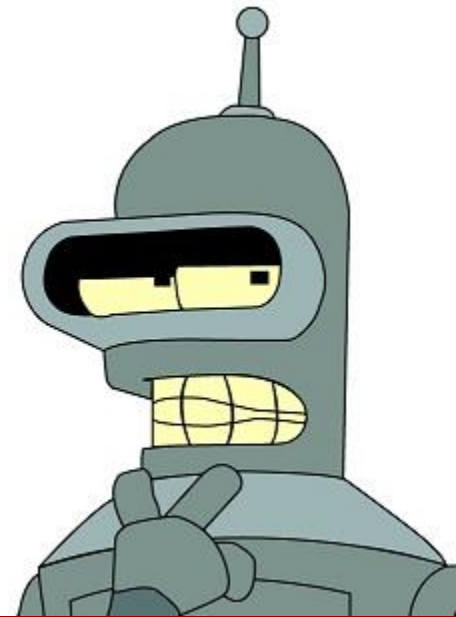
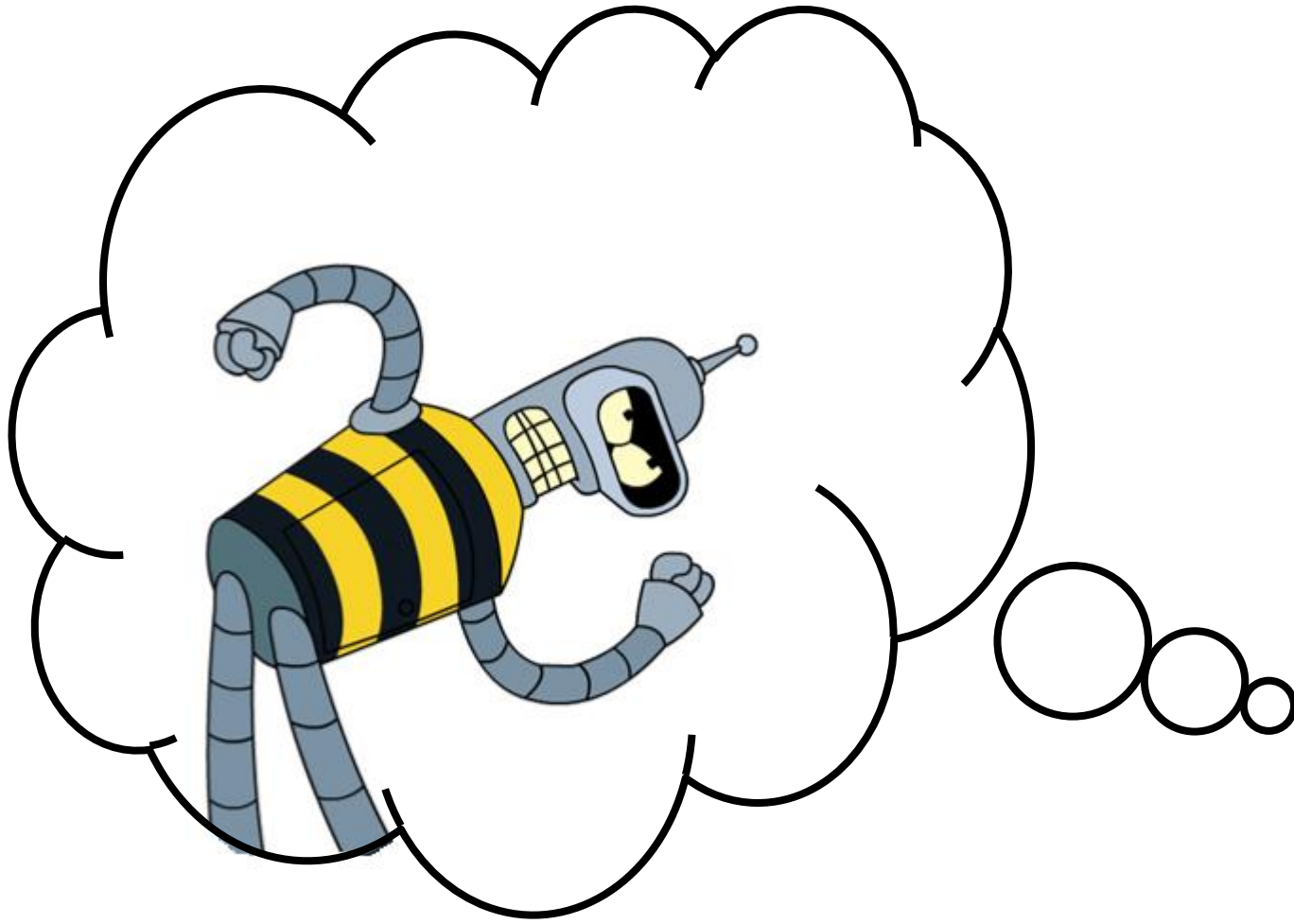
the designer is free to employ any voting rule!



EXAMPLE: ROBOBEEES

- Robobees need to decide on a joint plan (alternative)
- Many possible plans
- Each robobee (agent) has a numerical evaluation (utility) for each alternative
- Want to maximize sum of utilities = **social welfare**
- Communication is restricted



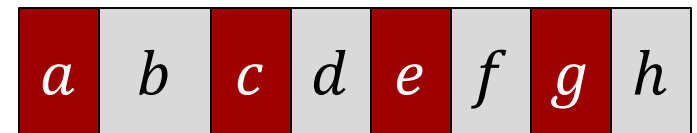


EXAMPLE: ROBOBEEES

- Approach 1:
communicate utilities
 - May be infeasible
- Approach 2: each agent votes for favorite alternative (plurality)
 - $\log m$ bits per agent
 - May select a bad alternative



$n/2 - 1$ agents



$n/2 + 1$ agents

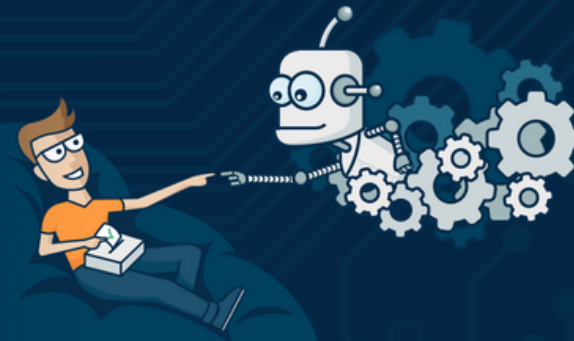
EXAMPLE: ROBOBEEES

- Approach 3: each agent votes for an alternative with probability proportional to its utility
- Theorem [Caragiannis & P 2011]:
if $n = \omega(m \log m)$ then this approach gives almost optimal social welfare in expectation



AI-Driven Decisions

RoboVote is a free service that helps users combine their preferences or opinions into optimal decisions. To do so, RoboVote employs state-of-the-art voting methods developed in artificial intelligence research. [Learn More](#)



Poll Types

RoboVote offers two types of polls, which are tailored to different scenarios; it is up to users to indicate to RoboVote which scenario best fits the problem at hand.



Objective Opinions

In this scenario, some alternatives are objectively better than others, and the opinion of a participant reflects an attempt to estimate the correct order. RoboVote's proposed outcome is guaranteed to be as close as possible — based on the available information — to the best outcome. Examples include deciding which product prototype to develop, or which company to invest in, based on a metric such as projected revenue or market share. [Try the demo.](#)



Subjective Preferences

In this scenario participants' preferences reflect their subjective taste; RoboVote proposes an outcome that mathematically makes participants as happy as possible overall. Common examples include deciding which restaurant or movie to go to as a group, which destination to choose for a family vacation, or whom to elect as class president. [Try the demo.](#)

Ready to get started?

CREATE A POLL

SUMMARY

- Terminology:
 - Voting rules: plurality, Borda, plurality with runoff, STV, Copeland, Maximin, Dodgson
 - Axioms: Majority consistency, Condorcet consistency
- Big ideas:
 - When we build voting systems, we are not constrained by politics and tradition!

