Recitation 8

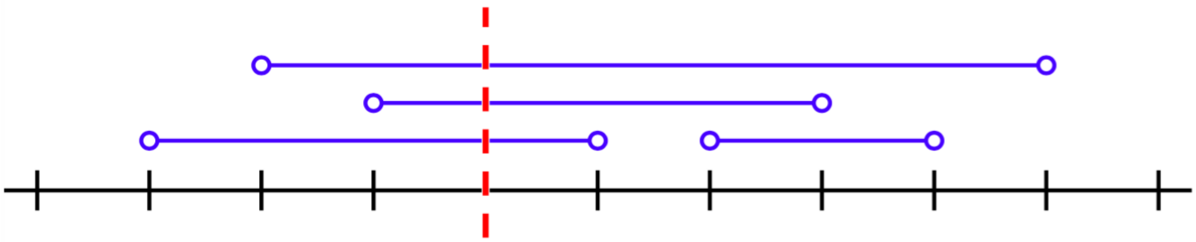
Augmented Tables

8.1 Announcements

- *RangeLab* has been released, and is due *Friday afternoon*.
- *BridgeLab* will be released on Friday. The written portion will be due the following Friday, while the coding portion will be due the Monday after that.

8.2 Interval Checking

Suppose you're given a set of intervals $I \subset \mathbb{Z} \times \mathbb{Z}$ and some $k \in \mathbb{Z}$, and you're interested in determining whether or not there exists $(l, r) \in I$ such that $l < k < r$. For simplicity, let's assume that no two intervals share an endpoint.



Task 8.1. *Implement a function*

val `intervalCheck : (int * int) Seq.t → int → bool`

where (`intervalCheck I k`) *answers the query mentioned above. Your function must be staged such that the line*

val q = `intervalCheck I`

performs $O(|I| \log |I|)$ *work and* $O(\log^2 |I|)$ *span, while each subsequent call* $q(k)$ *only performs* $O(\log |I|)$ *work and span. Try solving this problem with augmented tables.*

We'll store each (l, r) in a table as $(l \mapsto r)$, and augment the table with the function *max*. This allows us to determine the rightmost endpoint of a set of intervals in constant time. To answer the query, we can split I at k to get a set I' of all intervals which begin before k. We then just need to check if any of these have endpoints which are greater than k.

Algorithm 8.2. *Interval Checking with Augmented Tables.*

```
structure Val =
struct
   type t = int
   val f = Int.max
   val I = −∞
   val toString = Int.toString
end

structure Table = MkTreapAugTable (structure Key = IntElt
                                   structure Val = Val)

fun intervalCheck I =
   let
      val T = Table.fromSeq I
      fun query k =
         let val (T′,_,_) = Table.split (T,k)
         in (|T′| > 0) ∧ (Table.reduceVal T′ > k)
         end
   in
      query
   end
```

8.3 Interval Counting

Now suppose you want to solve a more general problem. Given I and k, you want to return $|\{(l, r) \in I \mid l < k < r\}|$. Once again, for simplicity, we'll assume all endpoints are distinct.

Task 8.3. *Implement a function*

```
val intervalCount : (int * int) Seq.t → int → int
```

where (`intervalCheck I k`) *answers the interval counting query as mentioned above. Your function must be staged, just like Task* **??**.

Similar to parentheses matching, we can use a counter which "increments" at the beginning of each interval, and "decrements" at the end. This corresponds to building a table of $(l \mapsto 1)$ and $(r \mapsto -1)$ for each interval (l, r), and augmenting the table with addition. After splitting this table at k, we can determine the number of "unmatched" intervals on the left in $O(1)$ time.

We have to be careful about off-by-one errors, though: if an interval ends at k, we need to subtract 1. This is handled on line **??** below.

Algorithm 8.4. *Interval Counting with Augmented Tables.*

```
structure Val =
struct
  type t = int
  val f = op+
  val I = 0
  val toString = Int.toString
end

structure Table = MkTreapAugTable (structure Key = IntElt
                                   structure Val = Val)

fun intervalCount I =
  let
    val L = Seq.map (fn (l,_) ⇒ (l,1)) I
    val R = Seq.map (fn (_,r) ⇒ (r,−1)) I
    val T = Table.fromSeq (Seq.append (L,R))
    fun query k =
      let val (T′,co,_) = Table.split (T,k)
          val c = case co of SOME −1 ⇒ −1 | _ ⇒ 0
      in Table.reduceVal T′ + c
      end
  in
    query
  end
```