Recitation 5

Randomization

5.1 Announcements

- *RandomLab* has been released, and is due **Friday afternoon**. It's worth 100 points.
- FingerLab will be released this Friday. However, it will be due in two weeks to allow time to study for Exam I, which is going to be on **Friday, Feb 24**.

5.2 Rock, Paper, Scissors, Shoot!

You and a friend are playing Rock, Paper, Scissors. Despite humans actually being remarkably bad at generating randomness, assume that on each round, both you and your friend will uniformly randomly produce one of Rock, Paper, or Scissors (each has probability $1/3$).

Task 5.1. *Determine the probability of winning this game.*

Task 5.2. *How many rounds do we expect to play before someone wins?*

5.3 Flipping Coins

Task 5.3. *Describe an algorithm which flips a fair coin an expected constant number of times in order to simulate a coin with bias* p*, where* $0 < p < 1$ *(that is, we're simulating a coin which flips heads with probability* p*). It may be helpful to consider writing* p *as a binary number* $0.p_1p_2p_3\ldots$*, where each* $p_i \in \{0, 1\}$.

5.4 High Probability

Task 5.4. *Umut has a secret algorithm which has* $O(\log n)$ *span with high probability, and* $O(n)$ *span in the worst case. Specifically, Umut's algorithm has* $O(\log n)$ *span with probability at least* $1 - \frac{1}{n^3}$. *Prove that Umut's algorithm has* $O(\log n)$ *span in expectation.*

5.5 The Birthday Problem

Task 5.5. *Suppose there are D days in the year, and assume that all babies are born uniformly randomly on any one of these days. If there are n people in a room, what is the expected number of them to share a birthday with at least one other person in the same room?*

5.6 Other Exercises

Exercise 5.6. *Prove that the expected value of a geometric random variable X with probability of success p is $1/p$ in two separate ways:*

1. *by directly solving using the definition of expected value, and*
2. *by writing a recurrence and solving it.*

In the second approach, use the law of total expectation, i.e.

$$\mathbf{E}\left[X\right] = \sum_i \mathbf{E}\left[X \mid Y_i\right] \mathbf{Pr}\left[Y_i\right]$$

where the Y_i's form a partition of the sample space.

Exercise 5.7. *Suppose you are given a coin with unknown but fixed bias p, where $0 < p < 1$ (in your analysis, treat p as a constant). Describe an algorithm which flips this mysterious coin an expected constant number of times in order to simulate a fair coin.*