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Bits, Bytes, and Integers

15-213/14-513/15-513:
Introduction to Computer Systems

2nd Lecture, Aug 29, 2024

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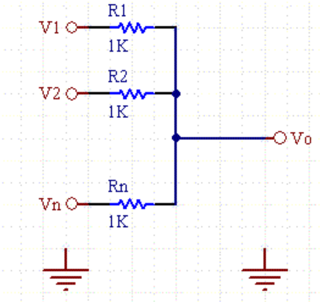
Bits, Bytes, and Integers

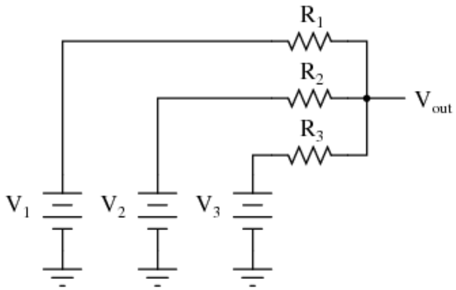
- **Representing information as bits** CSAPP 2.1
- **Bit-level manipulations**
- **Integers** CSAPP 2.2
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting CSAPP 2.3
- **Byte Ordering** CSAPP 2.1.3

Analog Computers

- **Before digital computers there were analog computers.**

- **Consider a couple of simple analog computers:**
  - A simple circuit can allow one to adjust voltages using variable resistors and measure the output using a volt meter:
  - A simple network of adjustable parallel resistors can allow one to find the average of the inputs.





https://www.daycounter.com/Calculators/Voltage-Summer/Voltage-Summer-Calculator.phtml

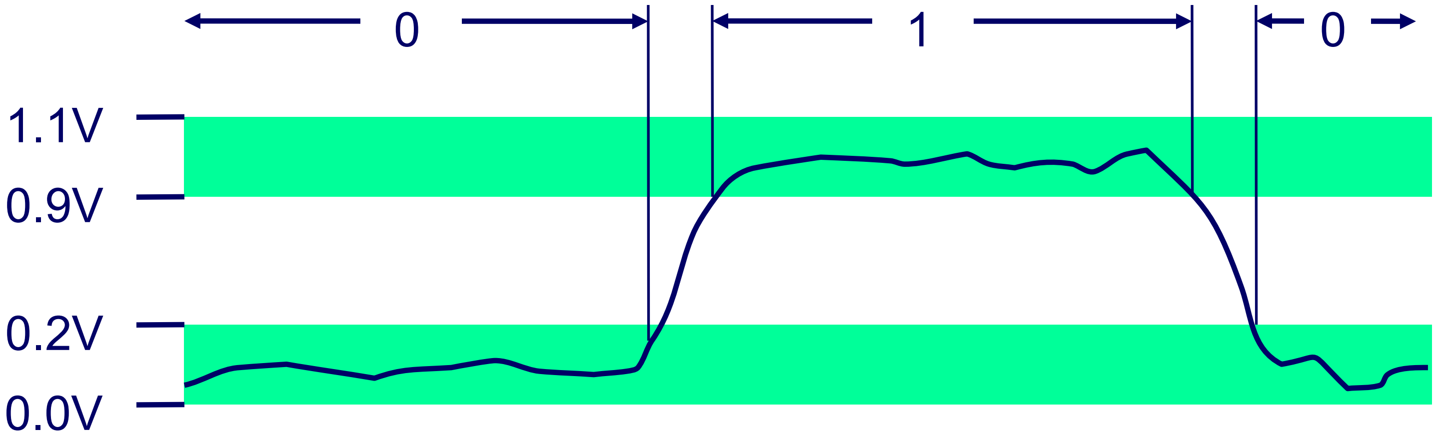
https://www.quora.com/What-is-the-most-basic-voltage-adder-circuit-without-a-transistor-op-amp-and-any-external-supply

Needing Less Accuracy, Precision is Better

- **We don’t try to measure exactly**
  - We just ask, is it high enough to be “On”, or
  - Is it low enough to be “Off”.

- **We have two states, so we have a binary, or 2-ary, system.**
  - We represent these states as 0 and 1

- **Now we can easily interpret, communicate, and duplicate signals well enough to know what they mean.**



Binary Representation

- **Binary representation leads to a simple binary, i.e. base-2, numbering system**
  - 0 represents 0
  - 1 represents 1
  - Each “place” represents a power of two, exactly as each place in our usual “base 10”, 10-ary numbering system represents a power of 10
- **By encoding/interpreting sets of bits in various ways, we can represent different things:**
  - Operations to be executed by the processor, numbers, enumerable things, such as text characters
- **As long as we can assign it to a discrete number, we can represent it in binary**

Binary Representation: Simple Numbers

- **For example, we can count in binary, a base-2 numbering system**
  - 000, 001, 010, 011, 100, 101, 110, 111, …
    - 000 = $0*2^2 + 0*2^1 + 0*2^0 = 0$ (in decimal)
    - 001 = $0*2^2 + 0*2^1 + 1*2^0 = 1$ (in decimal)
    - 010 = $0*2^2 + 1*2^1 + 0*2^0 = 2$ (in decimal)
    - 011 = $0*2^2 + 1*2^1 + 1*2^0 = 3$ (in decimal)
    - Etc.
- **For reference, consider some base-10 examples:**
    - 000 = $0*10^2 + 0*10^1 + 0*10^0$
    - 001 = $0*10^2 + 0*10^1 + 1*10^0$
    - 357 = $3*10^2 + 5*10^1 + 7*10^0$

Hexadecimal and Octal

- **Writing out numbers in binary takes too many digits**
- **We want a way to represent numbers more densely such that fewer digits are required**
  - But also such that it is easy to get at the bits that we want
- **Any power-of-two base provides this property**
  - Octal, e.g. base-8, and hexadecimal, e.g. base-16 are the closest to our familiar base-10.
  - Each has been used by “computer people” over time
  - Hexadecimal is often preferred because it is denser.

Hexadecimal

- **Hexadecimal 00_{16} to FF_{16}**
  - Base 16 number representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
- **Consider 1A2B in Hexadecimal:**
  - $1*16^3 + A*16^2 + 2*16^1 + B*16^0$
  - $1*16^3 + 10*16^2 + 2*16^1 + 11*16^0$ = 6699 (decimal)
  - The C Language prefixes hexadecimal numbers with “0x” so they aren’t confused with decimal numbers
  - Write $FA1D37B_{16}$ in C as
    - 0xFA1D37B
    - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$$15213:\ \underbrace{0011}_{3}\ \underbrace{1011}_{B}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D}$$

Today: Bits, Bytes, and Integers

- Representing information as bits
- **Bit-level manipulations**
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
- Byte Ordering

Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Exclusive-Or (Xor)

- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- **Operate on Bit Vectors**
  - Operations applied bitwise

```
   01101001      01101001      01101001
 & 01010101    | 01010101    ^ 01010101    ~ 01010101
   01000001      01111101      00111100      10101010
```

- **All of the Properties of Boolean Algebra Apply**

Example: Representing & Manipulating Sets

Representation

- Width w bit vector represents subsets of {0, …, w–1}
- $a_j = 1$ if $j \in A$
  - 01101001 { 0, 3, 5, 6 }
  - *76543210*
  - 01010101 { 0, 2, 4, 6 }
  - *76543210*

Operations

&	Intersection	01000001	{ 0, 6 }
|	Union	01111101	{ 0, 2, 3, 4, 5, 6 }
^	Symmetric difference	00111100	{ 2, 3, 4, 5 }
~	Complement	10101010	{ 1, 3, 5, 7 }

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any "integral" data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - ~0x41 → 0xBE
  - ~0x00 → 0xFF
  - 0x69 & 0x55 → 0x41
  - 0x69 | 0x55 → 0x7D

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit-Level Operations in C

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  - Apply to any “integral” data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - ~0x41 → 0xBE
    - ~01000001_2 → 10111110_2
  - ~0x00 → 0xFF
    - ~00000000_2 → 11111111_2
  - 0x69 & 0x55 → 0x41
    - 01101001_2 & 01010101_2 → 01000001_2
  - 0x69 | 0x55 → 0x7D
    - 01101001_2 | 01010101_2 → 01111101_2

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Contrast: Logic Operations in C

- **Contrast to Bit-Level Operators**
  - **Logic Operations: &&, ||, !**
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - !0x41 →
  - !0x00 →
  - !!0x41→

  - 0x69 && 0x55 →
  - 0x69 || 0x55 →

Contrast: Logic Operations in C

- **Contrast to Bit-Level Operators**
  - **Logic Operations: &&, ||, !**
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - !0x41 → 0x00
  - !0x00 → 0x01
  - !!0x41→ 0x01
  - 0x69 && 0x55 → 0x01
  - 0x69 || 0x55 → 0x01
  - p && *p (avoids null pointer access)

Watch out for && vs. & (and || vs. |)…
Super common C programming pitfall!

Shift Operations

- **Left Shift:** `x << y`
  - Shift bit-vector **`x`** left **`y`** positions
    - – Throw away extra bits on left
    - Fill with 0's on right
- **Right Shift:** `x >> y`
  - Shift bit-vector **`x`** right **`y`** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

Argument **`x`**	`01100010`
<< 3	`00010`*`000`*
Log. >> 2	*`00`*`011000`
Arith. >> 2	*`00`*`011000`

Argument **`x`**	`10100010`
<< 3	`00010`*`000`*
Log. >> 2	*`00`*`101000`
Arith. >> 2	*`11`*`101000`

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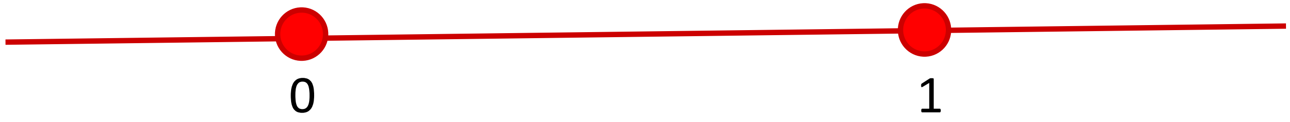
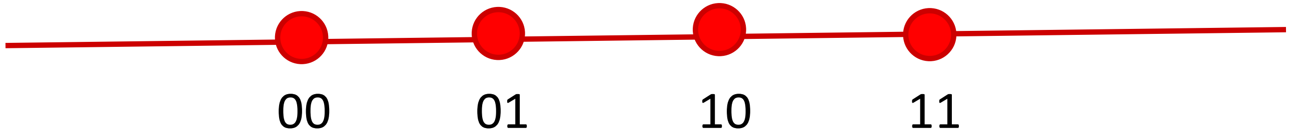
Binary Number Lines

- **In binary, the number of bits in the data type size determines the number of points on the number line.**
  - We can assign the points any meaning we'd like

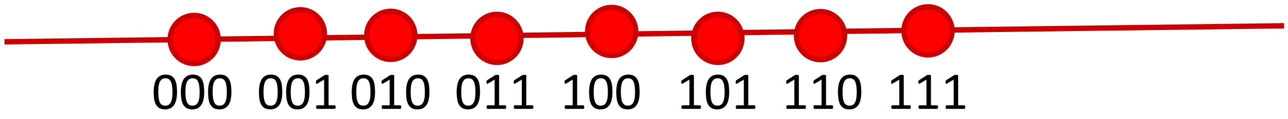
- **Consider the following examples:**
  - 1 bit number line



  - 2 bit number line

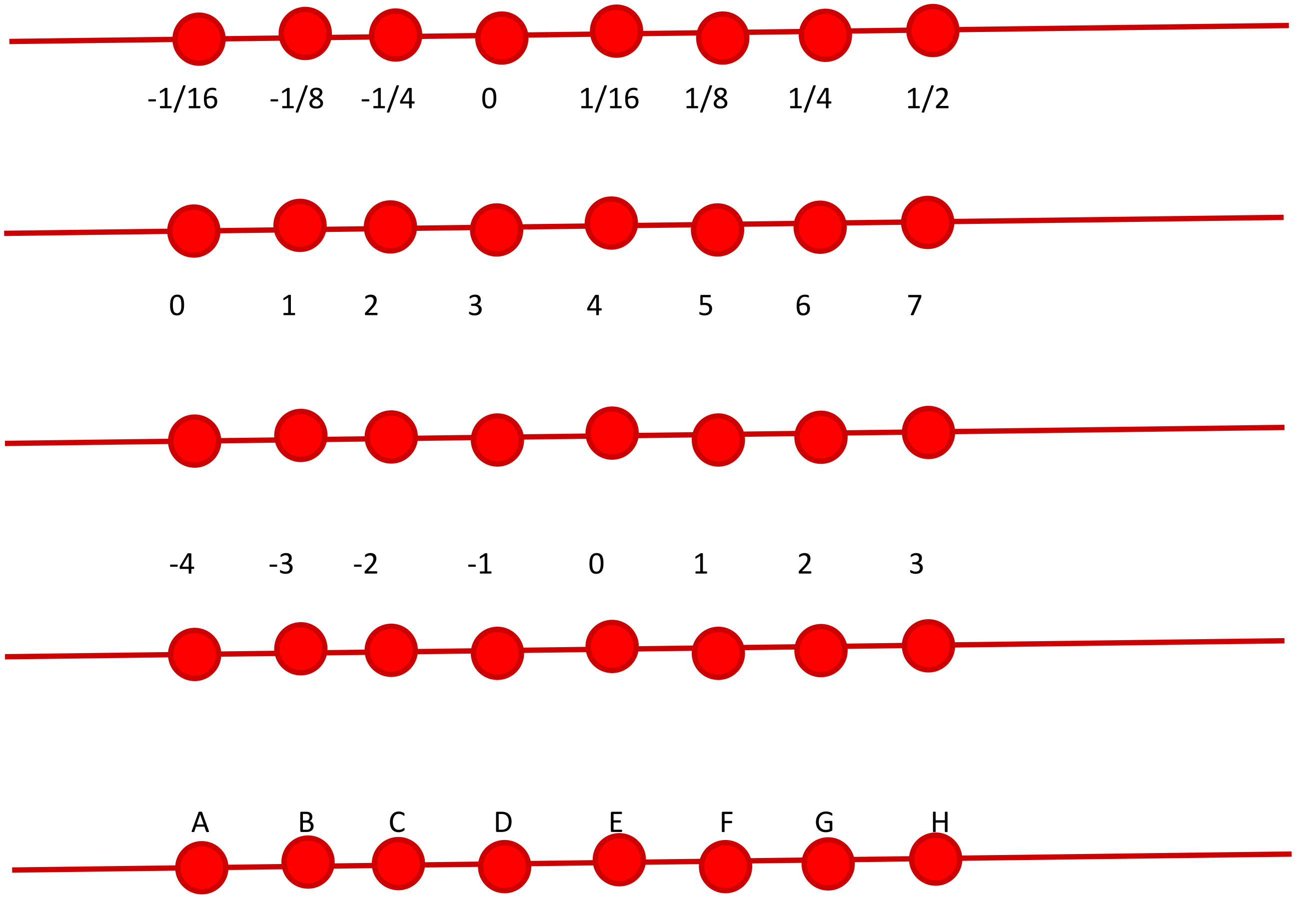


  - 3 bit number line



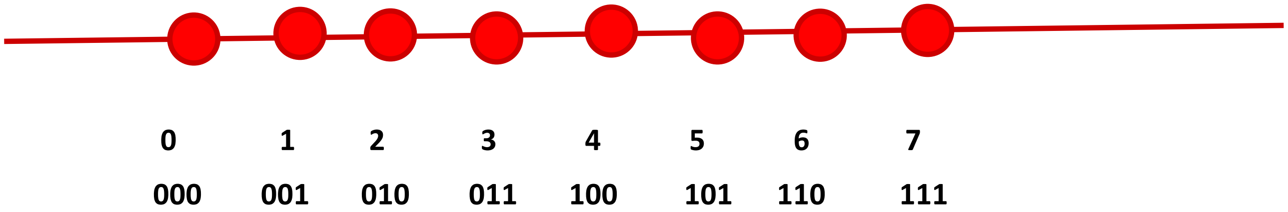
Some Purely Imaginary Examples

- **3 bit number line**



Overflow

- **Let’s consider a simple 3 digit number line:**



0 1 2 3 4 5 6 7

000 001 010 011 100 101 110 111

- **What happens if we add 1 to 7?**
  - In other words, what happens if we add 1 to 111?

- **111+ 001 = 1 000**
  - But, we only get 3 bits – so we lose the leading-1.
  - This is called overflow

- **The result is 000**

Modulus Arithmetic

- **Let's explore this idea of overflow some more**
  - 111 + 001 = 1 000 = 000
  - 111 + 010 = 1 001 = 001
  - 111 + 011 = 1 010 = 010
  - 111 + 100 = 1 011 = 011
  - …
  - 111 + 110 = 1 101 = 101
  - 111 + 111 = 1 110 = 110
- **So, arithmetic "wraps around" when it gets "too positive"**

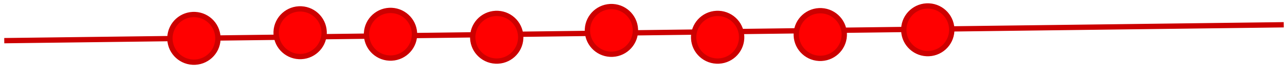
Unsigned and Non-Negative Integers

- **We'll use the term "ints" to mean the finite set of integer numbers that we can represent on a number line enumerated by some fixed number of bits, i.e. *bit width*.**

- **We normally represent unsigned and non-negative int using simple binary as we have already discussed**
  - An "unsigned" int is any int on a number line, e.g. of a data type, that doesn't contain any negative numbers
  - A non-negative number is a number greater than or equal to (>=) 0 on a number line, e.g. of a data type, that does contain negative numbers

How represent negative Numbers?

- **We could use the leading bit as a *sign bit*:**
  - 0 means non-negative
  - 1 means negative



000	001	010	011	100	101	110	111
0	1	2	3	-0	-1	-2	-3

- **This has some benefits**
  - It lets us represent negative and non-negative numbers
  - 0 represents 0

- **It also has some drawbacks**
  - There is a -0, which is the same as 0, except that it is different
  - How to add such numbers 1 + -1 should equal 0
    - But, by simple math, 001 + 101 = 110, which is -2?

A Magic Trick!

- **Let's just start with three ideas:**
  - 1 should be represented as 1
  - -1 + 1 = 0
  - We want addition to work in the familiar way, with simple rules.

- **We want a situation where "-1" + 1 = 0**

- **Consider a 3 bit number:**
  - 001 + "-1" = 0
  - 001 + 111 = 0
    - Remember 001 + 111 = 1 000, and the leading one is lost to overflow.

- **"-1" = 111**
  - Yep!

Negative Numbers

- **Well, if 111 is -1, what is -2?**
  - -1 - 1
  - 111 – 001 = 110

- **Does that really work?**
  - If it does -2 + 2 = 0
  - 110 + 010 = 1 000 = 000

- **-2 + 5 should be 3, right?**
  - 110 + 101 = 1 011 = 011

Finding –x the easy way

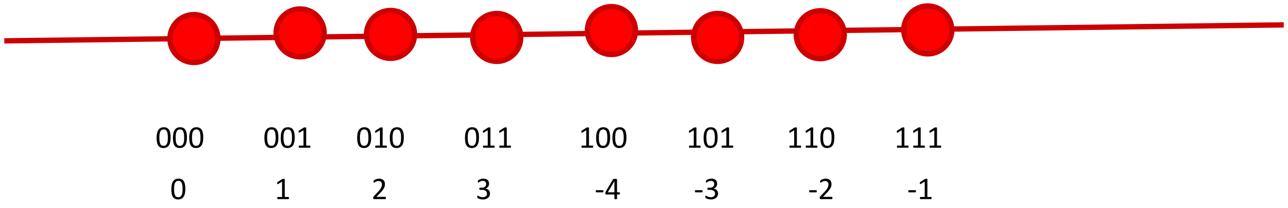
- **Given a non-negative number in binary, e.g. 5, represented with a fixed bit width, e.g. 4**
  - 0101
- **We can find its negative by flipping each bit and adding 1**
  - 0101 This is 5
  - 1010 This is the “ones complement of 5”, e.g. 5 with bits flipped
  - 1011 This is the “twos complement of 5”, e.g. 5 with the bits flipped and 1 added
  - 0101 + 1011 = 1 0000 = 0000
  - -x = ~x+1
- **Because of the fixed width, the “two’s complement” of a number can be used as its negative.**

Why Does –x = ~x + 1 Work?

- Consider any number and its (ones) complement:
  - 0101
  - 1010
- They are called complements because complementary bits are set. As a result, if they are added, all bits are necessarily set:
  - 0101 + 1010 = 1111
- Adding 1 to the sum of a number and its complement necessarily results in a 0 due to overflow
  - (0101 + 1010) + 1 = 1111 + 1 = 1 0000 = 0000
- And if x + y = 0, y must equal –x

Visualizing Two’s Complement

- **Numbers “wrap around” with -1 at the very end**



000	001	010	011	100	101	110	111
0	1	2	3	-4	-3	-2	-1

- **A few things to note:**
  - All negative numbers start with a ”1”
    - E.g. 100 is “-4”
  - You can view the leading “1” as introducing a “-4”
    - E.g. 101 = 1*-4+0*2+1*1= -3
    - But 010 = 0*-4+1*2+0*1 = 2
  - -4 is missing a positive partner

Complement & Increment Examples

x = 0

	Decimal	Hex	Binary
0	0	00 00	00000000 00000000
~0	-1	FF FF	11111111 11111111
~0+1	0	00 00	00000000 00000000

x = Tmin (The most negative two's complement number)

	Decimal	Hex	Binary
x	-32768	80 00	10000000 00000000
~x	32767	7F FF	01111111 11111111
~x+1	-32768	80 00	10000000 00000000

Canonical counter example

Encoding Integers: Dense Form

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- **C does not mandate using two's complement**
  - But, most machines do, and we will assume so
- **C `short` (2 bytes long)**

	Decimal	Hex	Binary
`x`	15213	`3B 6D`	`00111011 01101101`
`y`	-15213	`C4 93`	`11000100 10010011`

- **Sign Bit**
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative, 1 for negative

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0

    000…0
  - *UMax* = $2^w - 1$

    111…1

- **Two's Complement Values**
  - *TMin* = -2^{w-1}

    100…0
  - *TMax* = $2^{w-1} - 1$

    011…1
  - Minus 1

    111…1

Values for *W* = 16

	Decimal	Hex	Binary
UMax	**65535**	**FF FF**	**11111111 11111111**
TMax	**32767**	**7F FF**	**01111111 11111111**
TMin	**-32768**	**80 00**	**10000000 00000000**
-1	**-1**	**FF FF**	**11111111 11111111**
0	**0**	00 00	00000000 00000000

Quiz Time!

Check out:

Day 2

https://canvas.cmu.edu/courses/42532/quizzes/127206

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Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

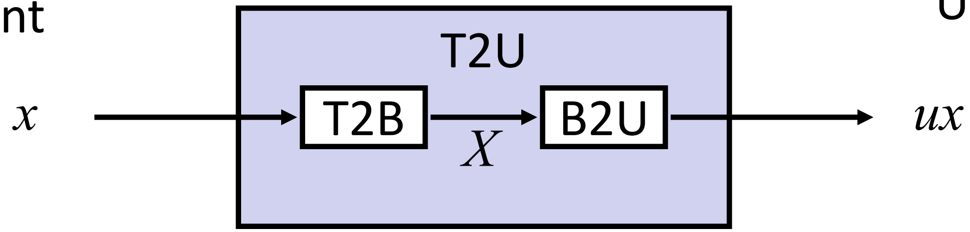
=

+/- 16

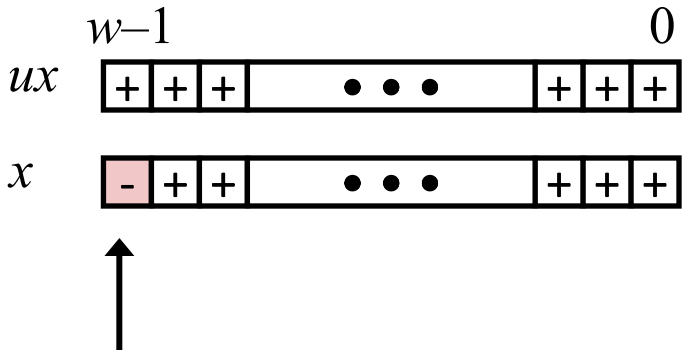
Relation between Signed & Unsigned

Two's Complement

Unsigned



Maintain Same Bit Pattern



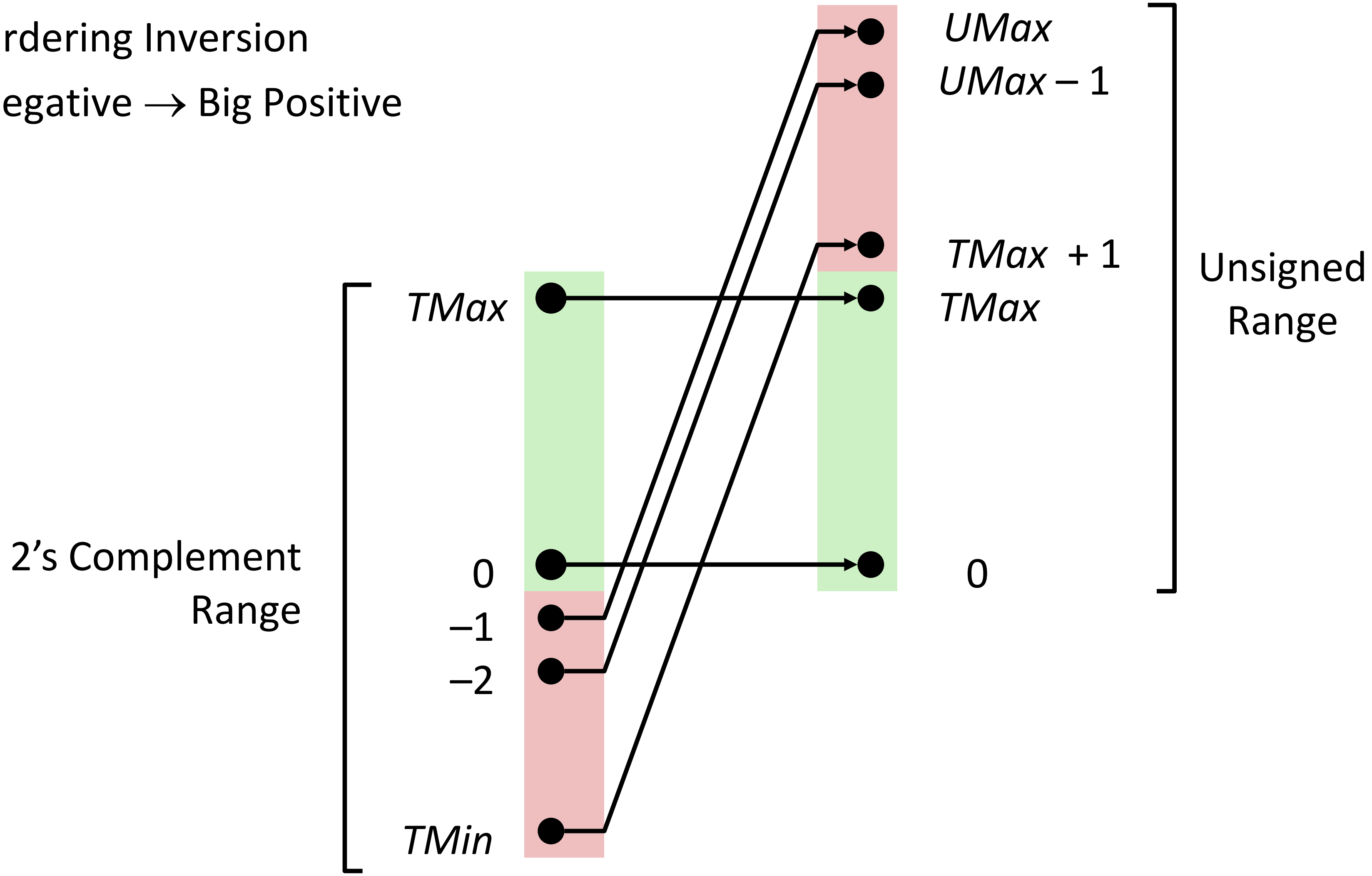
Large negative weight

becomes

Large positive weight

Conversion Visualized

- **2’s Comp. → Unsigned**
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

- **Constants**
  - By default are considered to be signed integers
  - Unsigned if have “U” as suffix

```
0U, 4294967259U
```

- **Casting**
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;                    int fun(unsigned u);
uy = ty;                    uy = fun(tx);
```

Casting Surprises

- **Expression Evaluation**
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 , TMAX = 2,147,483,647**

Constant$_1$	**Constant$_2$**	**Relation**	**Evaluation**
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w

- Expression containing signed and unsigned int
  - `int` is cast to `unsigned`!!

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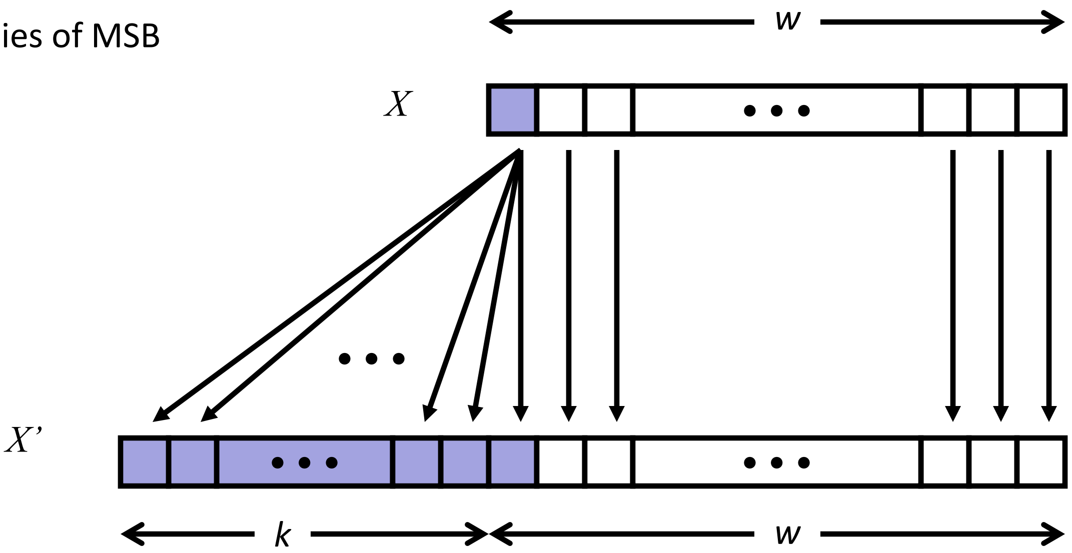
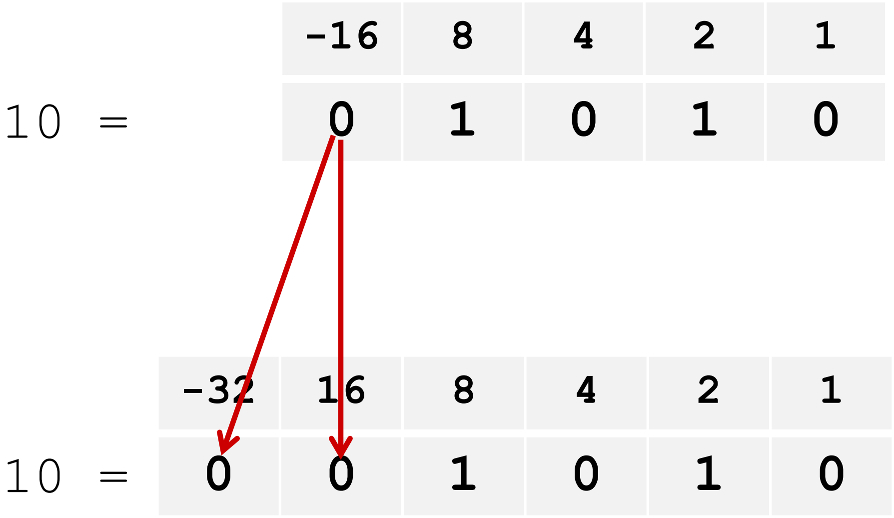
Sign Extension

- **Task:**
  - Given *w*-bit signed integer *x*
  - Convert it to *w*+*k*-bit integer with same value
- **Rule:**
  - Make *k* copies of sign bit:
  - $X' = \underbrace{x_{w-1},\ldots,x_{w-1}}_{k\text{ copies of MSB}},x_{w-1},x_{w-2},\ldots,x_0$



Sign Extension: Simple Example

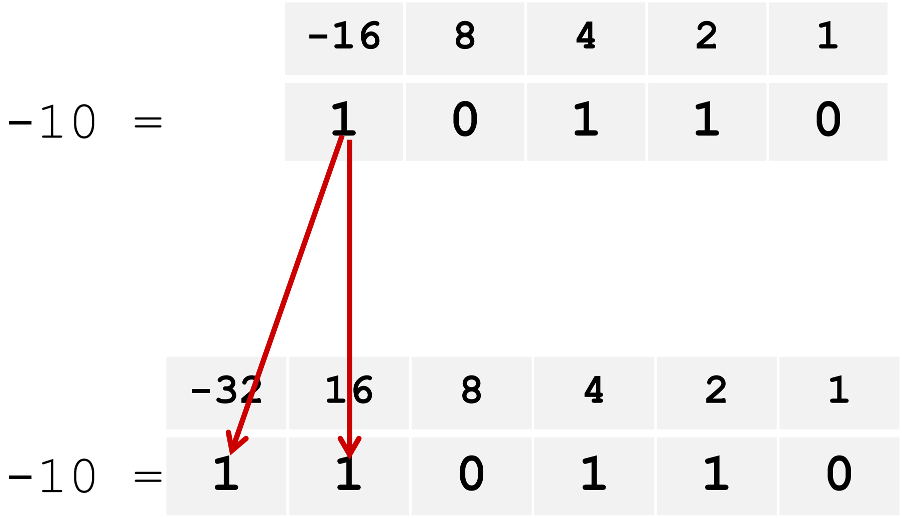
Positive number



	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

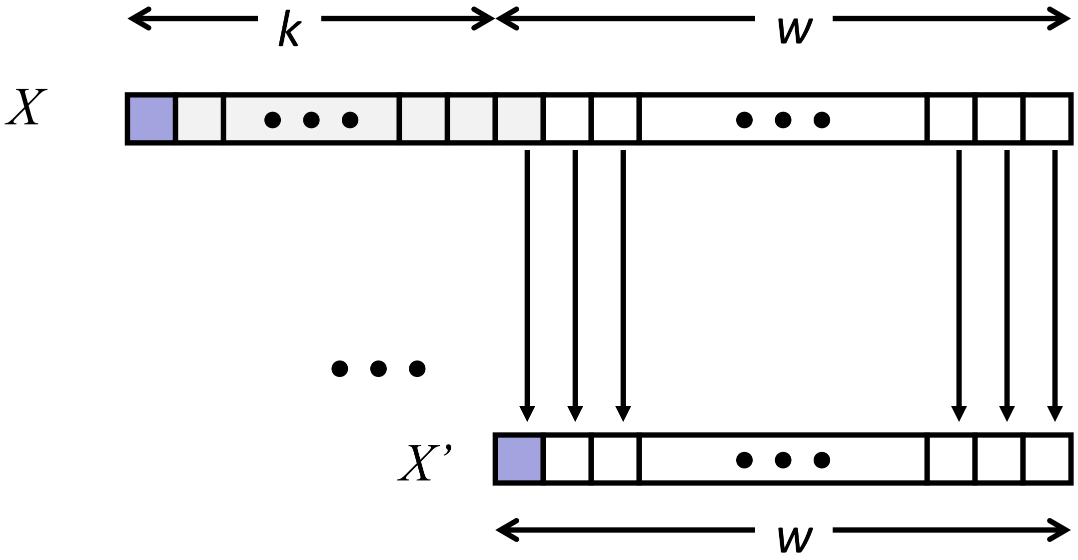


	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Truncation

- **Task:**
  - Given k+*w*-bit signed or unsigned integer *X*
  - Convert it to *w*-bit integer X' with same value for "small enough" X
- **Rule:**
  - Drop top *k* bits:
  - $X' = x_{w-1}, x_{w-2}, \ldots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

`2 mod 16 = 2`

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

`-6 mod 16 = 26U mod 16 = 10U = -6`

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

`10 mod 16 = 10U mod 16 = 10U = -6`

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

`-10 mod 16 = 22U mod 16 = 6U = 6`

Summary:
Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small (in magnitude) numbers yields expected behavior

Today: Bits, Bytes, and Integers

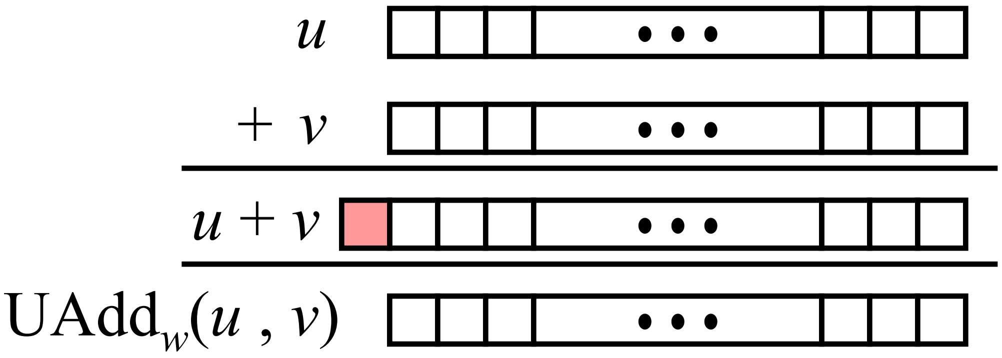
- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - **Addition, negation, multiplication, shifting**
- Byte Ordering

Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



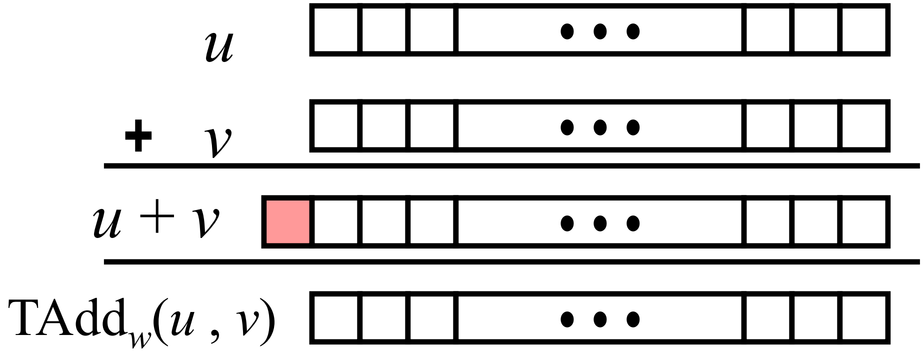
- **Standard Addition Function**
  - Ignores carry output
- **Implements Modular Arithmetic**

$$s \quad = \quad \text{UAdd}_w(u\ ,\ v) \quad = \quad u + v \mod 2^w$$

`unsigned char`	1110 1001	E9	223
	+ 1101 0101	+ D5	+ 213
	1 1011 1110	1BE	446
	1011 1110	BE	190

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Two's Complement Addition



Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits

- **TAdd and UAdd have Identical Bit-Level Behavior**
  - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

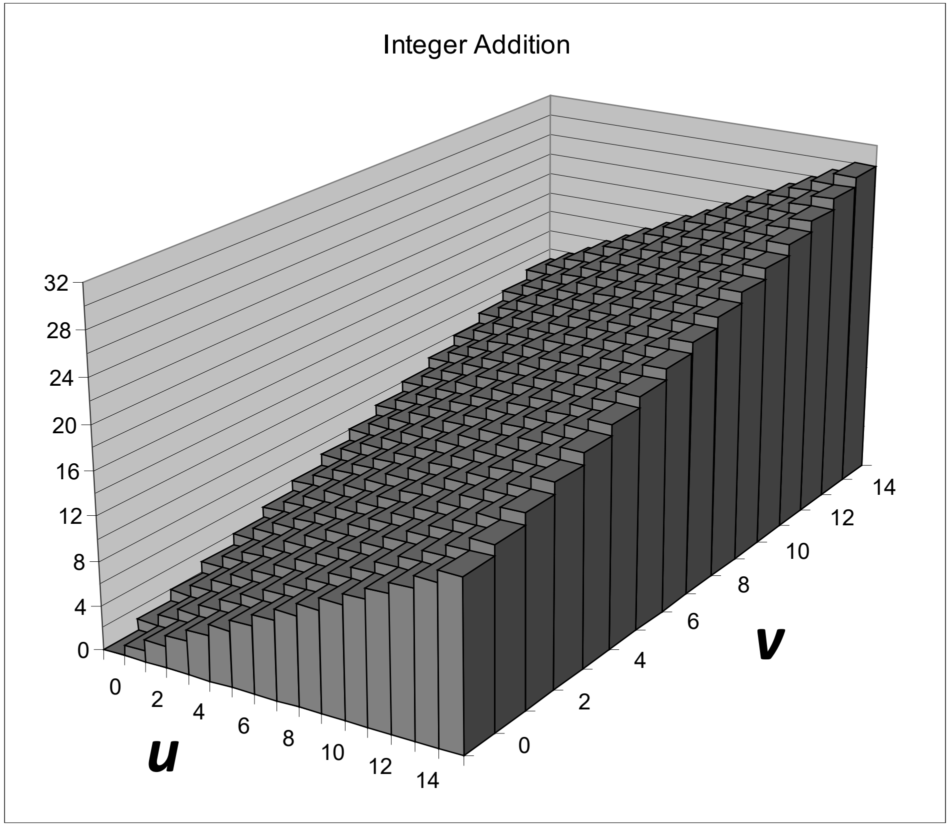
  - Will give `s == t`

1110 1001	E9	-23
+ 1101 0101	+ D5	+ -43
1 1011 1110	1BE	-66
1011 1110	BE	-66

Visualizing "True Sum" Integer Addition

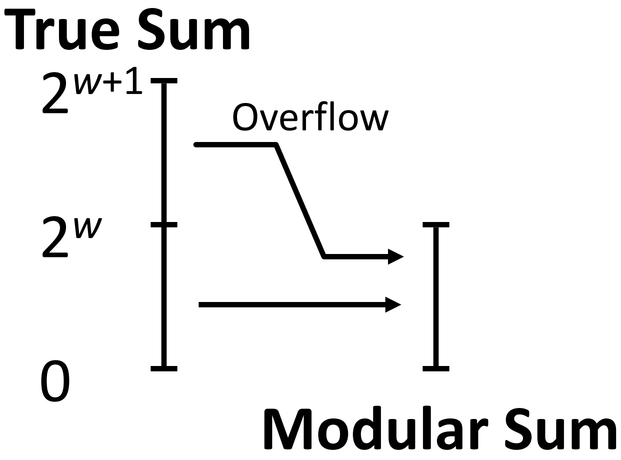
- **Integer Addition**
  - 4-bit integers *u*, *v*
  - Compute true sum $\mathrm{Add}_4(u, v)$
  - Values increase linearly with *u* and *v*
  - Forms planar surface

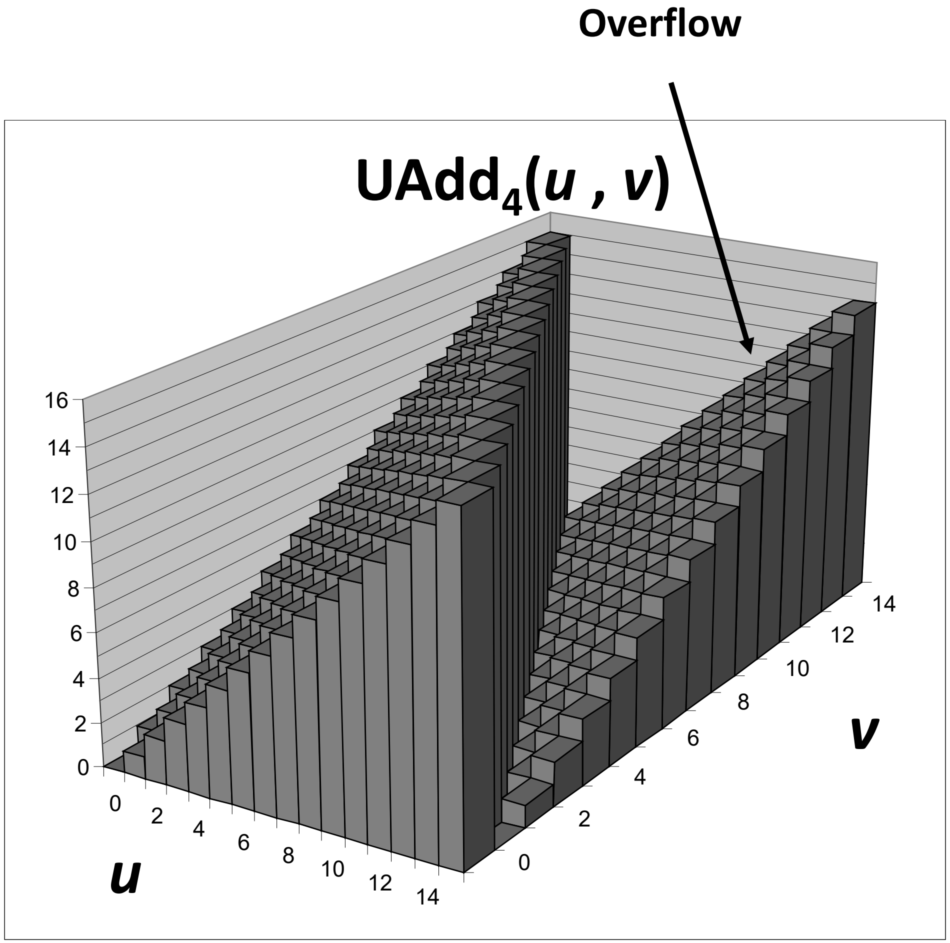
$\mathrm{Add}_4(u, v)$



Visualizing Unsigned Addition

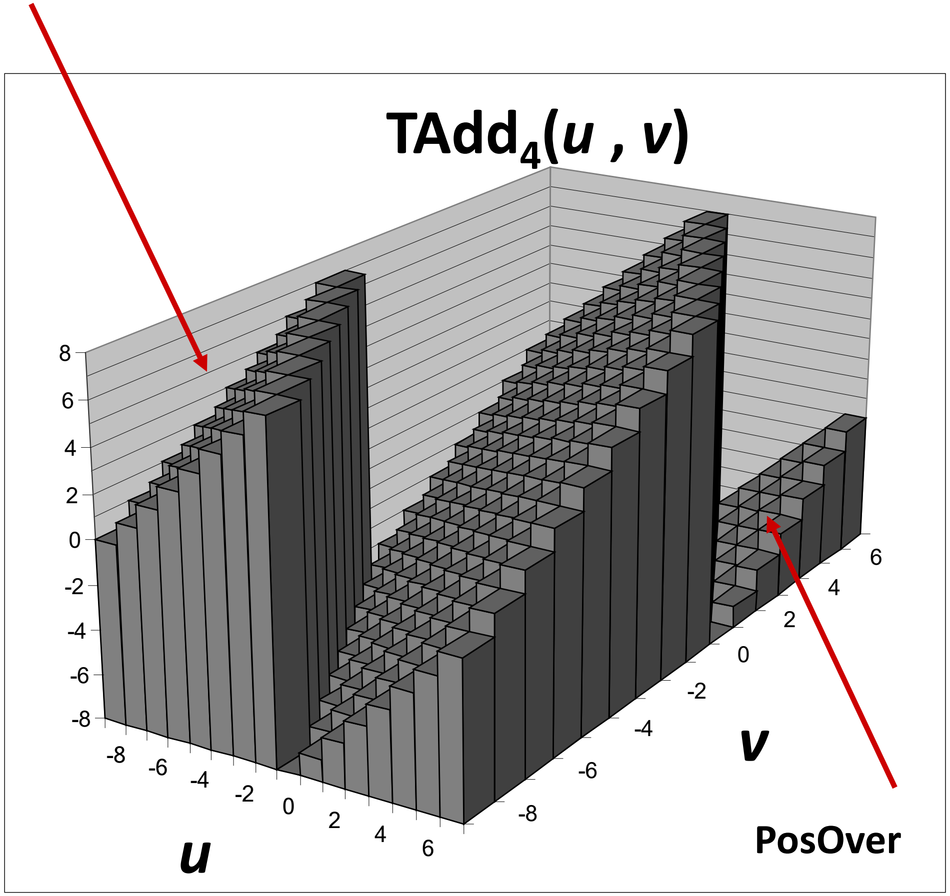
- **Wraps Around**
  - If true sum ≥ 2^w
  - At most once





Visualizing 2’s Complement Addition

- **Values**
  - 4-bit two’s comp.
  - Range from -8 to +7
- **Wraps Around**
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once



NegOver

PosOver

Multiplication

- **Goal: Computing Product of *w*-bit numbers *x*, *y***
  - Either signed or unsigned
- **Result: Same as computing ideal, exact result *x*y* and keeping *w* lower bits.**
- **Ideal,exact results can be bigger than *w* bits**
  - Worst case is up to 2w bits
    - Unsigned, because all bits are magnitude
    - Signed, but only for Tmin*Tmin, because anything added to Tmin reduces its magnitude and Tmax is less than Tmin.
- **So, maintaining exact results…**
  - would need to keep expanding word size with each product computed
  - Impossible in hardware (at least without limits), as all resources are finite
  - In practice, is done in software, if needed
    - e.g., by “arbitrary precision” arithmetic packages

Power-of-2 Multiply with Shift

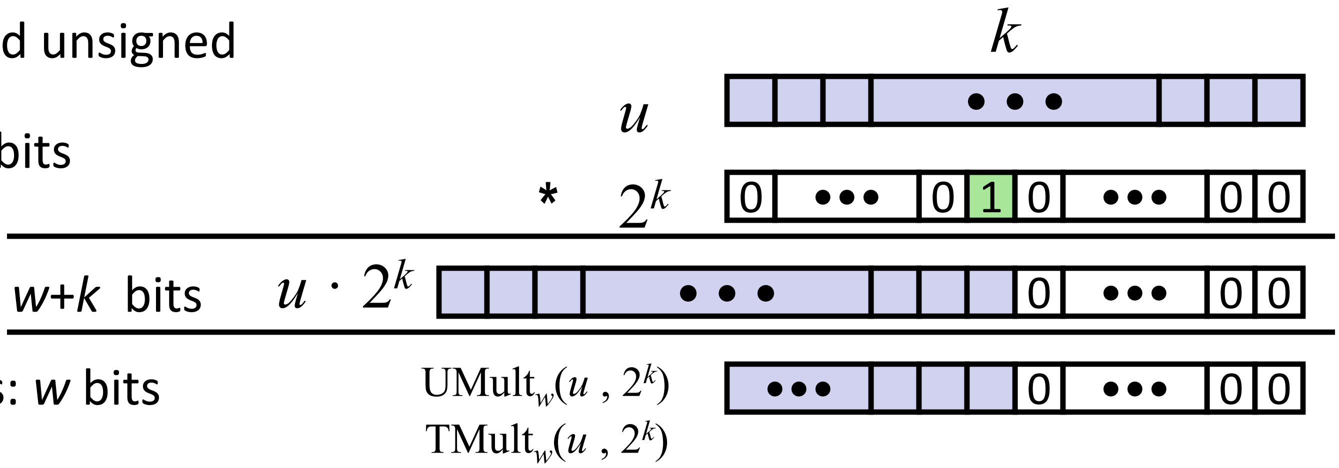
Operation

- `u << k` gives `u *` 2^k
- Both signed and unsigned

Operands: w bits

True Product: $w+k$ bits

Discard k bits: w bits

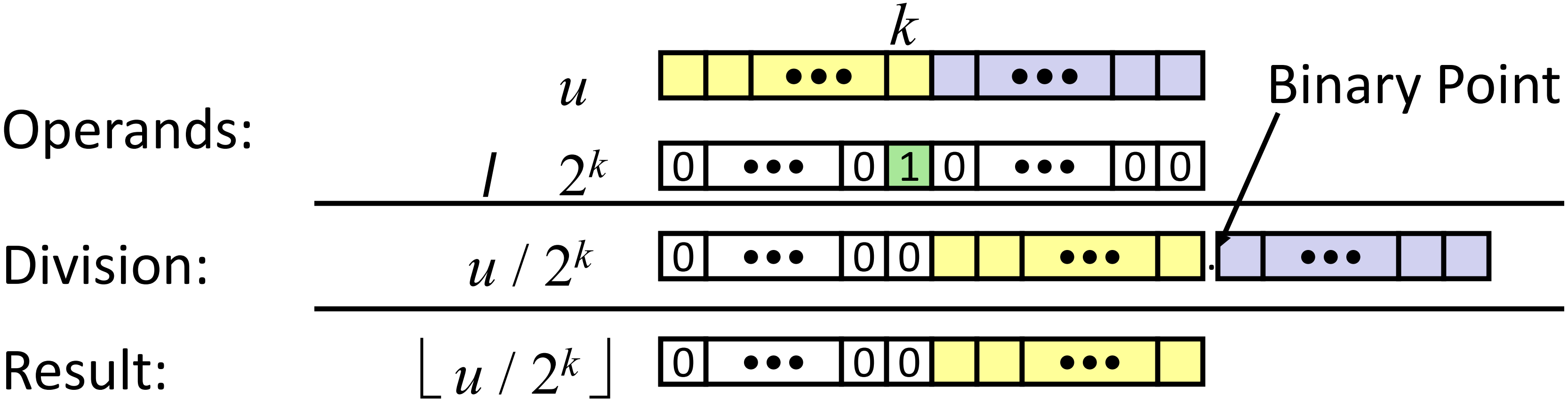


Examples

- `u << 3                ==    u * 8`
- `(u << 5) – (u << 3)  ==     u * 24`
- Most machines shift and add faster than multiply
  - Compiler generates this code automatically

Unsigned Power-of-2 Divide with Shift

- **Quotient of Unsigned by Power of 2**
  - `u >> k` gives $\lfloor u / 2^k \rfloor$
    - **Uses logical shift**



Operands: u $/ \quad 2^k$

Division: $u / 2^k$

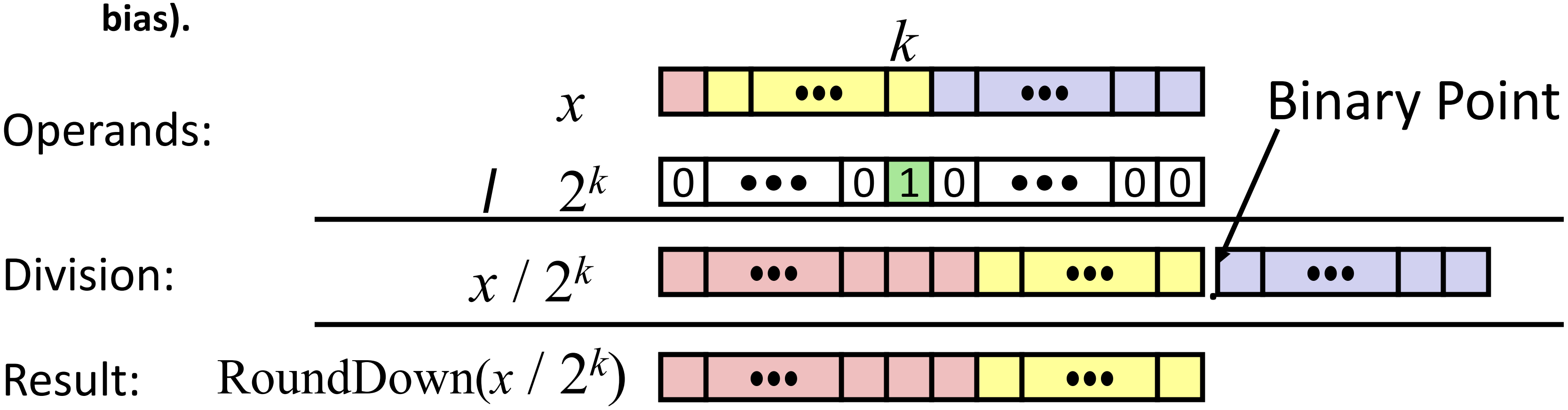
Result: $\lfloor u / 2^k \rfloor$

Binary Point

	Division	Computed	Hex	Binary
`x`	15213	15213	`3B 6D`	`00111011 01101101`
`x >> 1`	7606.5	7606	`1D B6`	`00011101 10110110`
`x >> 4`	950.8125	950	`03 B6`	`00000011 10110110`
`x >> 8`	59.4257813	59	`00 3B`	`00000000 00111011`

Signed Power-of-2 Divide with Shift

- **Quotient of Signed by Power of 2**
  - `x >> k` gives $\lfloor \mathtt{x} / 2^k \rfloor$
    - **Uses arithmetic shift**
    - **Rounds to the left, not towards zero (Unlikely to be what is expected, introduces a bias).**

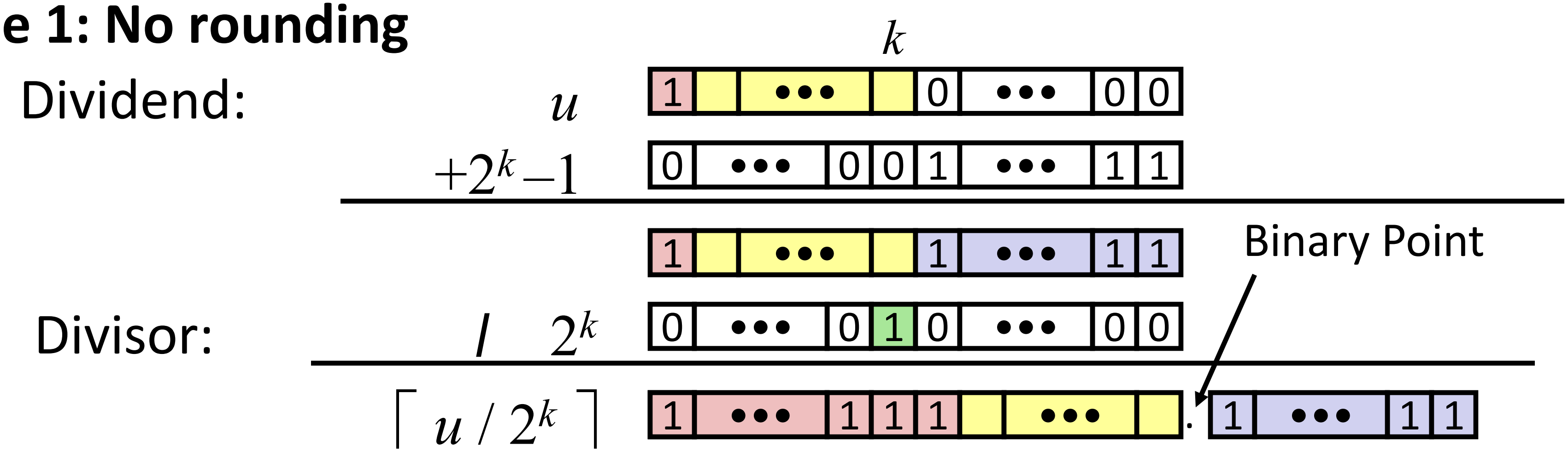


	Division	Computed	Hex	Binary
`x`	-15213	-15213	`C4 93`	`11000100 10010011`
`x >> 1`	-7606.5	-7607	`E2 49`	`11100010 01001001`
`x >> 4`	-950.8125	-951	`FC 49`	`11111100 01001001`
`x >> 8`	-59.4257813	-60	`FF C4`	`11111111 11000100`

Round-toward-0 Divide

- **Quotient of Negative Number by Power of 2**
  - Want $\lceil x / 2^k \rceil$ (Round Toward 0)
  - Compute as $\lfloor (x+(2^k-1)) / 2^k \rfloor$
    - In C: `(x + (1<<k)-1) >> k`
    - Biases dividend toward 0

Case 1: No rounding



Dividend: u 1 ••• 0 ••• 0 0

$+2^k-1$ 0 ••• 0 0 1 ••• 1 1

1 ••• 1 ••• 1 1

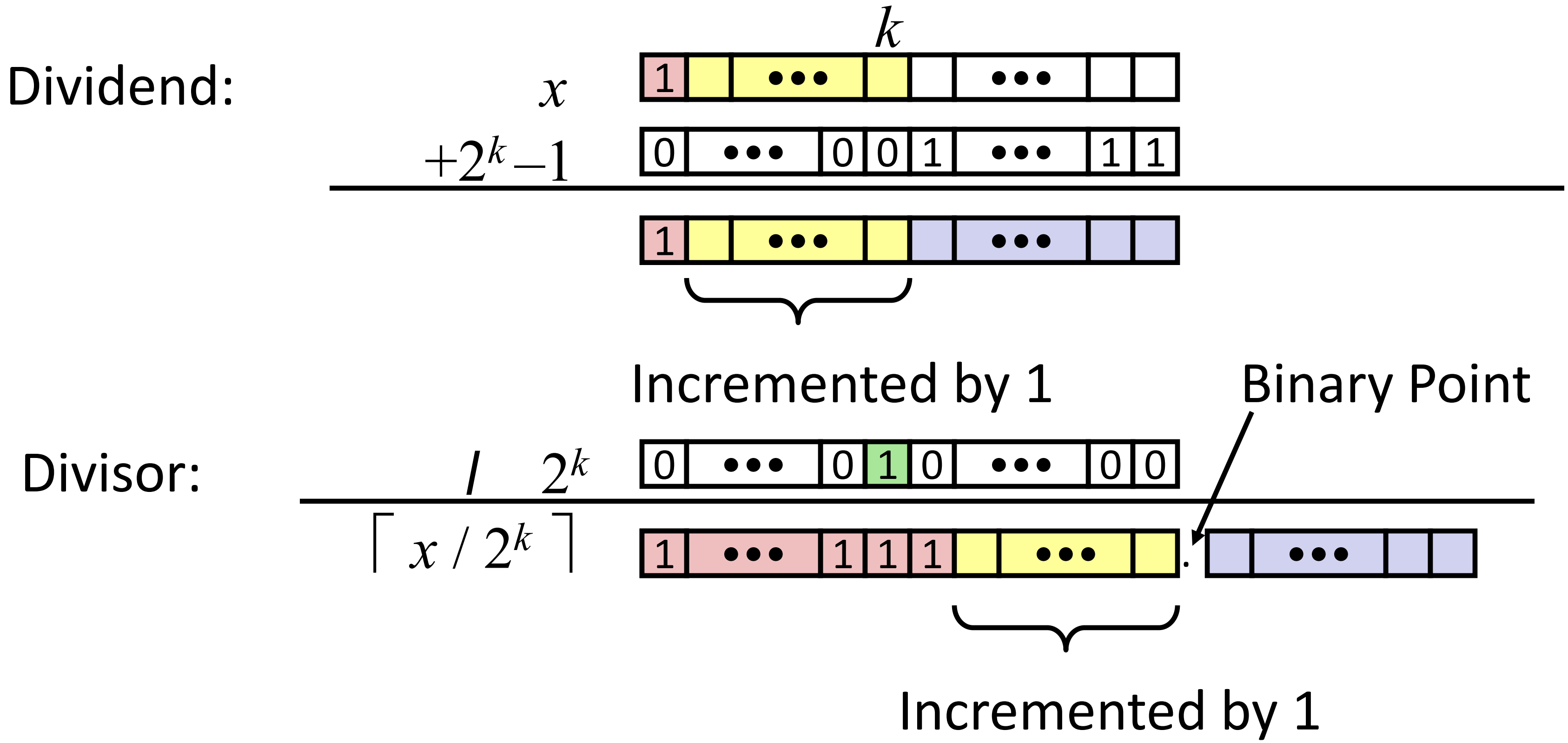
Divisor: $/ \quad 2^k$ 0 ••• 0 1 0 ••• 0 0

$\lceil u / 2^k \rceil$ 1 ••• 1 1 1 ••• . 1 ••• 1 1

Biasing has no effect

Correct Power-of-2 Divide (Cont.)

Case 2: Rounding


Dividend:

Divisor:

Incremented by 1

Binary Point

Incremented by 1

Biasing adds 1 to final result

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- Representing information as bits
- Bit-level manipulations
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  - Representation: unsigned and signed
  - Conversion, casting
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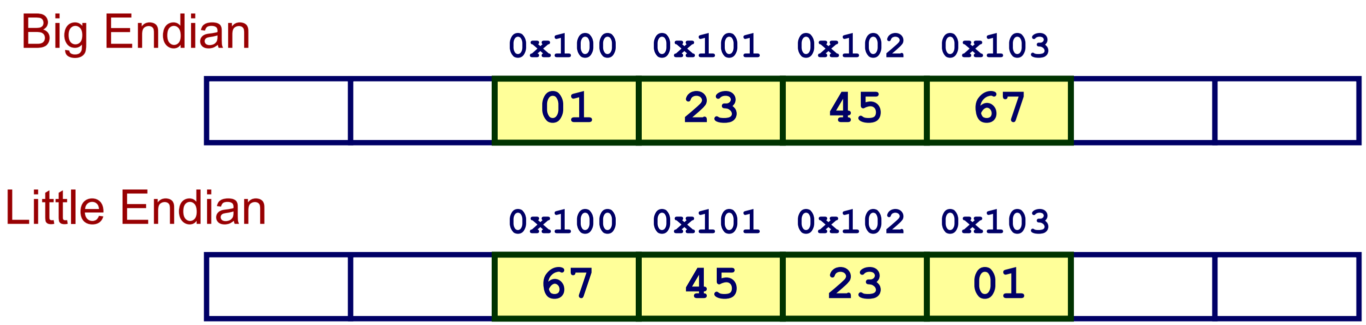
- **Byte Ordering**

Byte Ordering

- **So, how are the bytes within a multi-byte word ordered in memory?**
- **Conventions**
  - Big Endian: Sun (Oracle SPARC), PPC Mac, *Internet*
    - Least significant byte has highest address
  - Little Endian: *x86*, ARM processors running Android, iOS, and Linux
    - Least significant byte has lowest address
- **Becomes a concern when data is communicated**
  - Over a network, via files, etc.
- **Important notes**
  - Bits are not reversed, as the low order bit is the reference point.
  - Doesn’t affect chars, or strings (arrays of chars), as chars are only one byte

Byte Ordering Example

- **Example**
  - Variable x has 4-byte value of 0x01234567
  - Address given by &x is 0x100



Big Endian

`0x100` `0x101` `0x102` `0x103`

		01	23	45	67		

Little Endian

`0x100` `0x101` `0x102` `0x103`

		67	45	23	01		

Reading Byte-Reversed Listings

- **Disassembly**
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- **Example Fragment**

```
     Address        Instruction Code                Assembly Rendition
    8048365                 5b                         pop      %ebx
    8048366     81 c3 ab 12 00 00         add       $0x12ab,%ebx
    804836c    83 bb 28 00 00 00 00       cmpl      $0x0,0x28(%ebx)
```

- **Deciphering Numbers**
  - Value: `0x12ab`
  - Pad to 32 bits: `0x000012ab`
  - Split into bytes: `00 00 12 ab`
  - Reverse bytes: `ab 12 00 00`

Today: Bits, Bytes, and Integers

- **Representing information as bits** CSAPP 2.1
- **Bit-level manipulations**
- **Integers** CSAPP 2.2
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting CSAPP 2.3
- **Byte Ordering** CSAPP 2.1.3

Questions?