Cache Memories

15-213/15-513: Introduction to Computer Systems 10th Lecture, June 6, 2024

Instructors:

Brian Railing

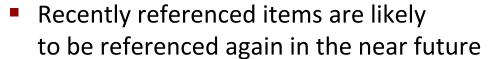
Today

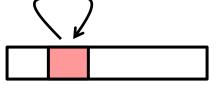
- Cache memory organization and operation
- Performance impact of caches
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

Recall: Locality

 Principle of Locality: Programs tend to use data and instructions with addresses near or equal to those they have used recently

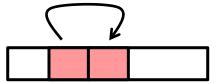


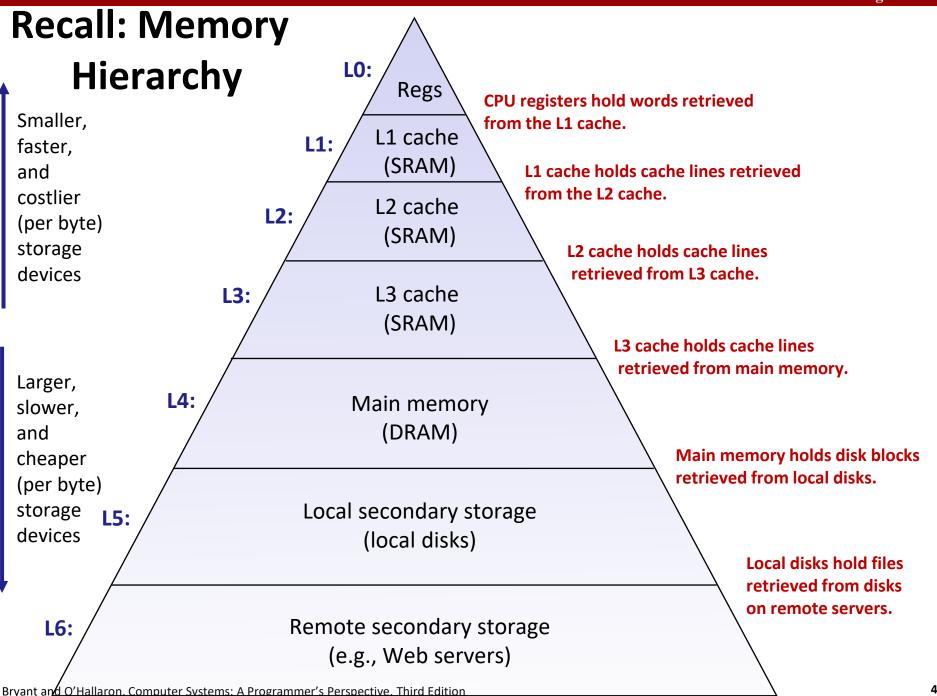




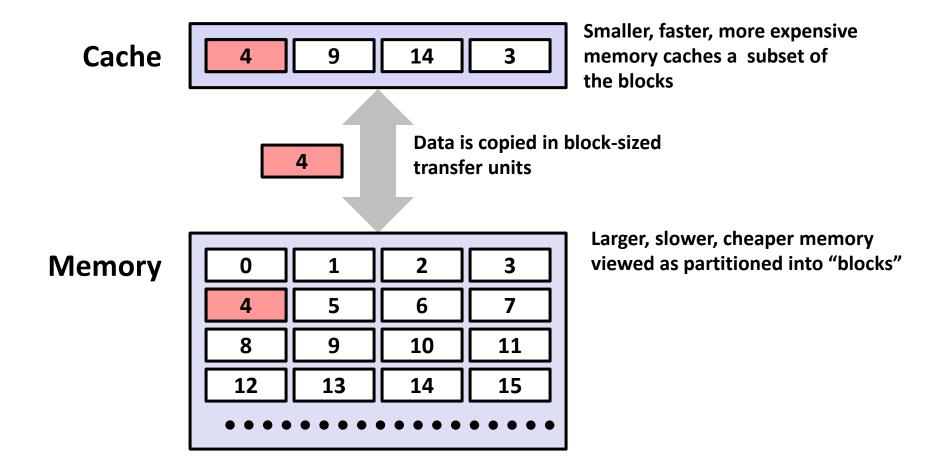


 Items with nearby addresses tend to be referenced close together in time

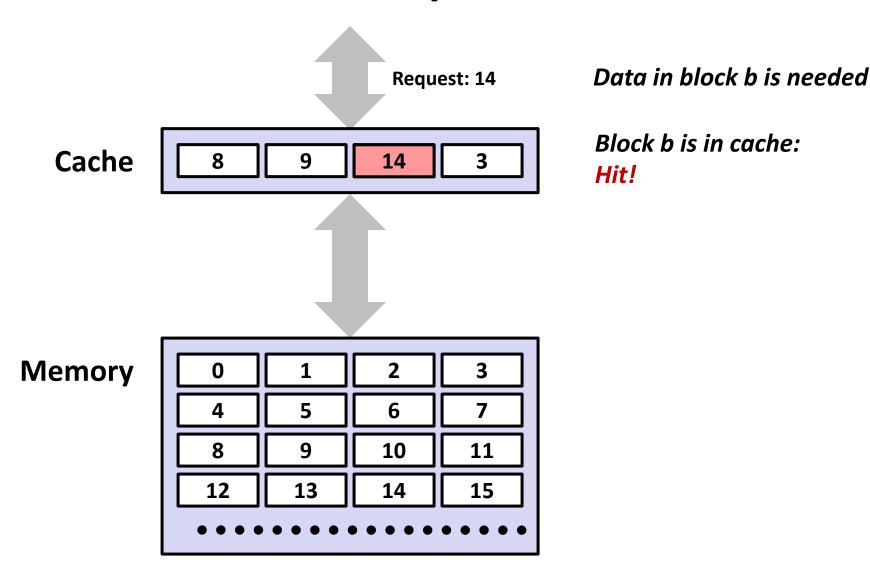




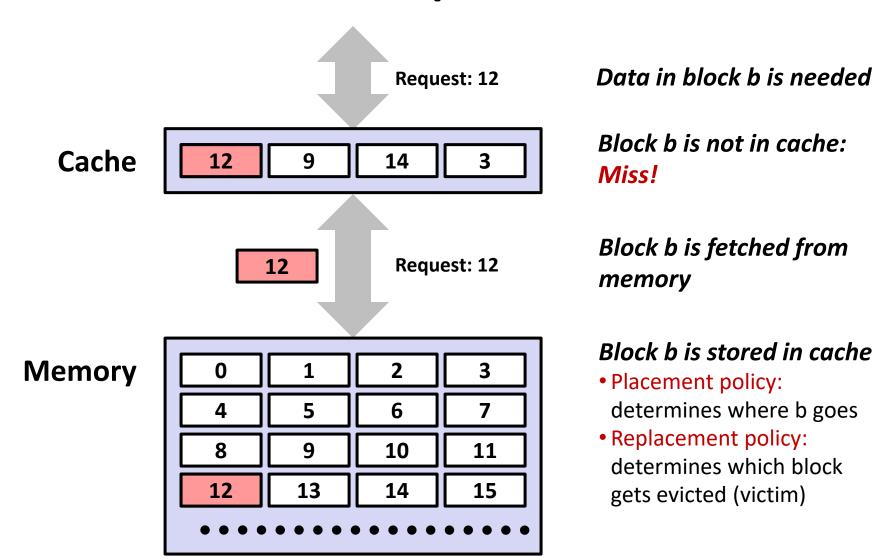
Recall: General Cache Concepts



General Cache Concepts: Hit



General Cache Concepts: Miss



Recall: General Caching Concepts: 3 Types of Cache Misses

■ Cold (compulsory) miss

Cold misses occur because this is the first reference to the block.

Capacity miss

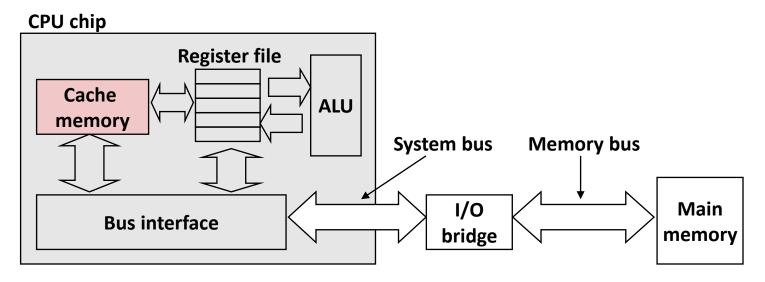
 Occurs when the set of active cache blocks (working set) is larger than the cache.

Conflict miss

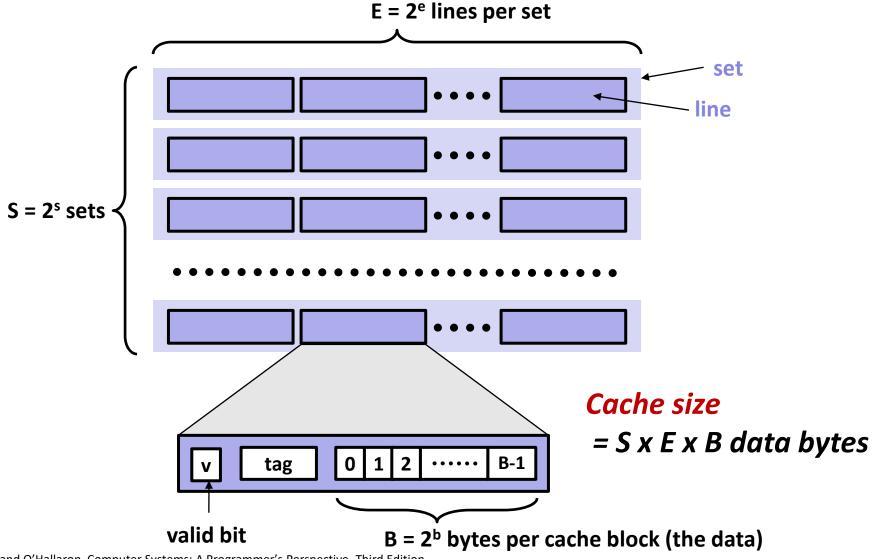
- Most caches limit blocks at level k+1 to a small subset (sometimes a singleton) of the block positions at level k.
 - E.g. Block i at level k+1 must be placed in block (i mod 4) at level k.
- Conflict misses occur when the level k cache is large enough, but multiple data objects all map to the same level k block.
 - E.g. Referencing blocks 0, 8, 0, 8, 0, 8, ... would miss every time.

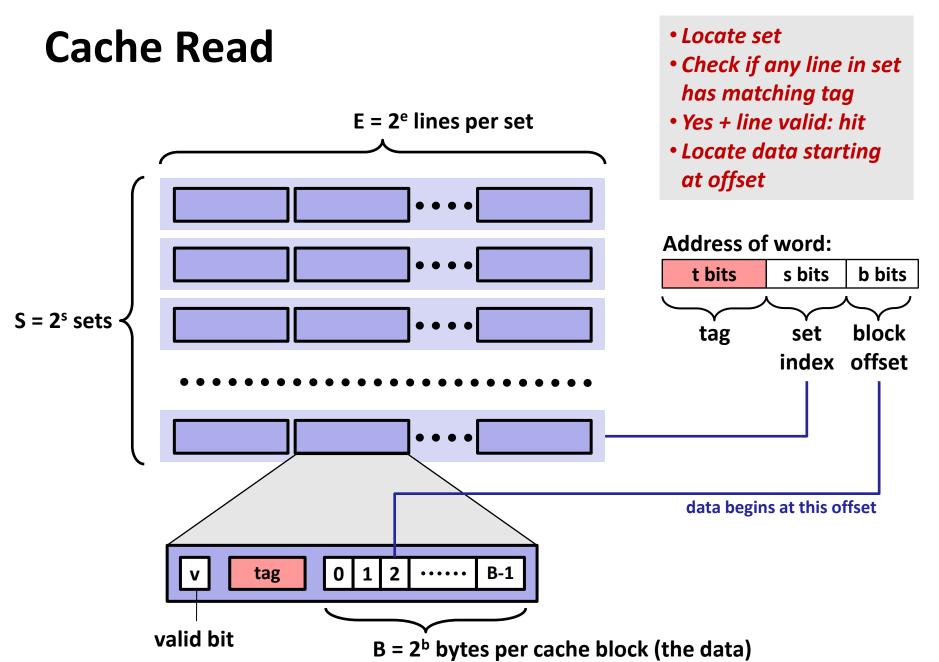
Cache Memories

- Cache memories are small, fast SRAM-based memories managed automatically in hardware
 - Hold frequently accessed blocks of main memory
- CPU looks first for data in cache
- Typical system structure:



General Cache Organization (S, E, B)

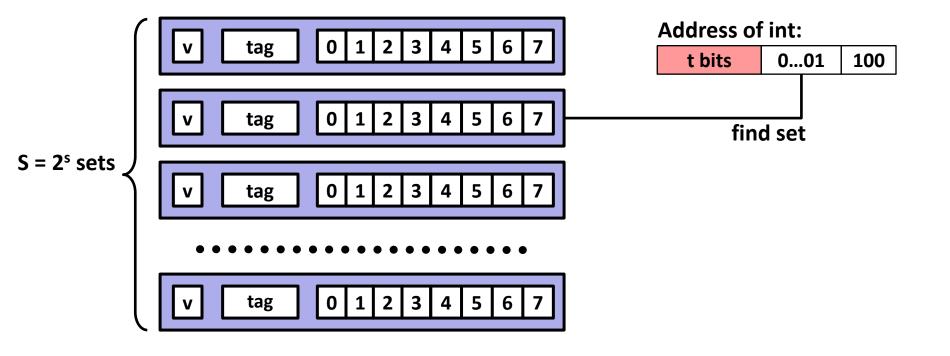




Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

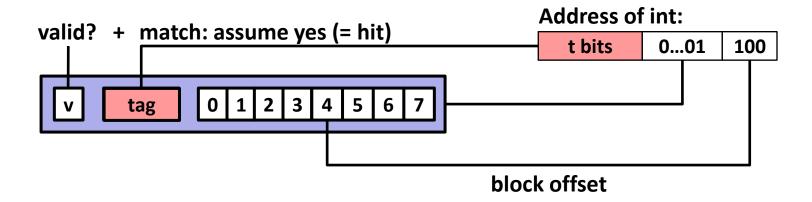
Assume: cache block size B=8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

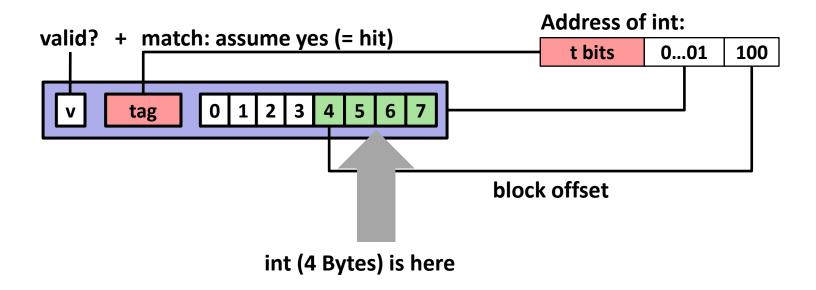
Assume: cache block size B=8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

Assume: cache block size B=8 bytes



If tag doesn't match (= miss): old line is evicted and replaced

Direct-Mapped Cache Simulation

t=1	s=2	b=1
X	XX	X

4-bit addresses (address space size M=16 bytes) S=4 sets, E=1 Blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

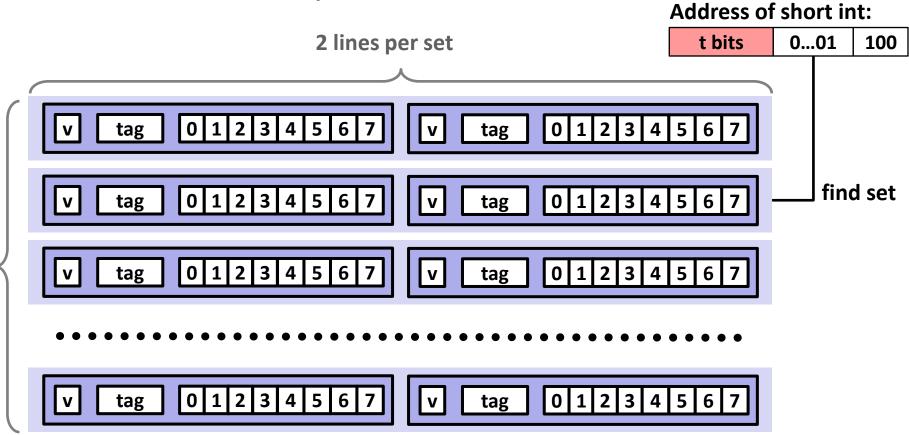
0	$[0000_2],$	miss
1	$[0001_{2}^{-}],$	hit
7	$[0111_2],$	miss
8	$[1000_{2}^{-}],$	miss
0	[00002]	miss

	V	Tag	Block
Set 0	1	0	M[0-1]
Set 1	0		
Set 2	0		
Set 3	1	0	M[6-7]

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size B=8 bytes



S sets

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size B=8 bytes

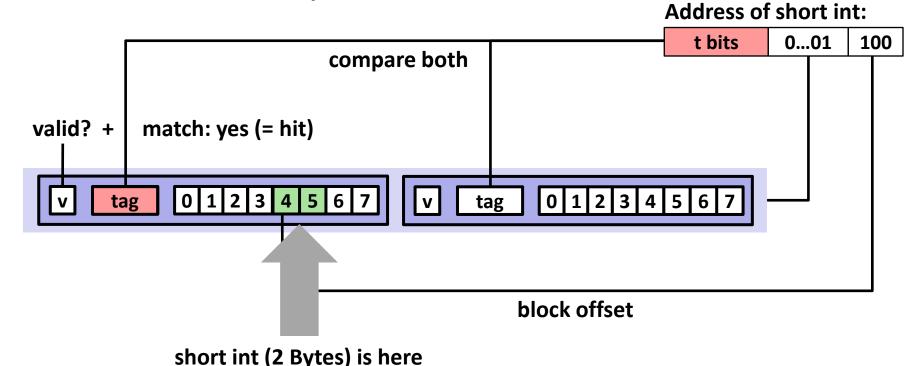
Address of short int: t bits 0...01 100 compare both valid? + match: yes (= hit) v tag 0 1 2 3 4 5 6 7

block offset

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size B=8 bytes



No match or not valid (= miss):

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

2-Way Set Associative Cache Simulation

t=2	s=1	b=1
XX	X	Х

4-bit addresses (M=16 bytes) S=2 sets, E=2 blocks/set, B=2 bytes/block

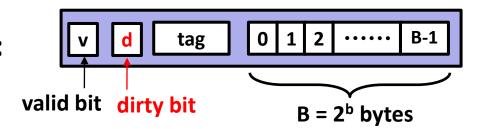
Address trace (reads, one byte per read):

0	$[00\underline{0}0_2],$	miss
1	$[0001_2],$	hit
7	$[01\underline{1}1_2],$	miss
8	$[1000_2],$	miss
0	$[0000_2]$	hit

	V	Tag	Block
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

What about writes?

- Multiple copies of data exist:
 - L1, L2, L3, Main Memory, Disk



What to do on a write-hit?

- Write-through (write immediately to memory)
- Write-back (defer write to memory until replacement of line)
 - Each cache line needs a dirty bit (set if data has been written to)

What to do on a write-miss?

- Write-allocate (load into cache, update line in cache)
 - Good if more writes to the location will follow
- No-write-allocate (writes straight to memory, does not load into cache)

Typical

- Write-through + No-write-allocate
- Write-back + Write-allocate

Practical Write-back Write-allocate

- A write to address X is issued
- valid bit dirty bit $B = 2^b$ bytes

- If it is a hit
 - Update the contents of block
 - Set dirty bit to 1 (bit is sticky and only cleared on eviction)

If it is a miss

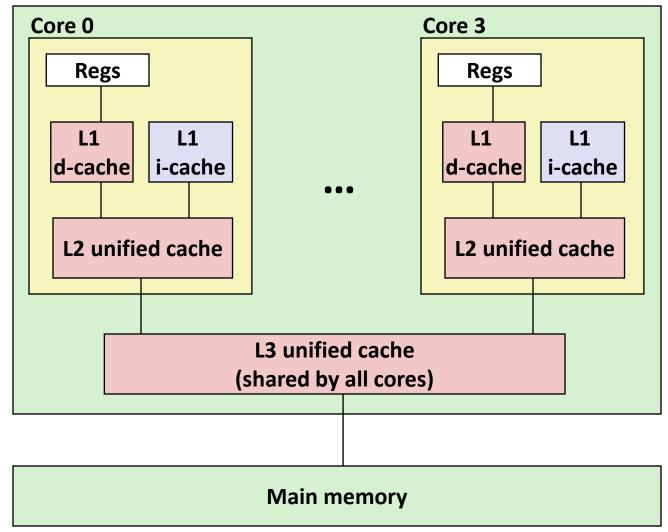
- Fetch block from memory (per a read miss)
- The perform the write operations (per a write hit)

If a line is evicted and dirty bit is set to 1

- The entire block of 2^b bytes are written back to memory
- Dirty bit is cleared (set to 0)
- Line is replaced by new contents

Intel Core i7 Cache Hierarchy

Processor package



L1 i-cache and d-cache:

32 KB, 8-way, Access: 4 cycles

L2 unified cache:

256 KB, 8-way, Access: 10 cycles

L3 unified cache:

8 MB, 16-way, Access: 40-75 cycles

Block size: 64 bytes for

all caches.

Cache Performance Metrics

Miss Rate

- Fraction of memory references not found in cache (misses / accesses)
 = 1 hit rate
- Typical numbers (in percentages):
 - 3-10% for L1
 - can be quite small (e.g., < 1%) for L2, depending on size, etc.

Hit Time

- Time to deliver a line in the cache to the processor
 - includes time to determine whether the line is in the cache
- Typical numbers:
 - 4 clock cycle for L1
 - 10 clock cycles for L2

Miss Penalty

- Additional time required because of a miss
 - typically 50-200 cycles for main memory (Trend: increasing!)

Let's think about those numbers

- Huge difference between a hit and a miss
 - Could be 100x, if just L1 and main memory
- Would you believe 99% hits is twice as good as 97%?
 - Consider this simplified example: cache hit time of 1 cycle miss penalty of 100 cycles
 - Average access time:

97% hits: 1 cycle + 0.03 x 100 cycles = 4 cycles

99% hits: 1 cycle + 0.01 x 100 cycles = 2 cycles

■ This is why "miss rate" is used instead of "hit rate"

Writing Cache Friendly Code

- Make the common case go fast
 - Focus on the inner loops of the core functions
- Minimize the misses in the inner loops
 - Repeated references to variables are good (temporal locality)
 - Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories

Quiz

https://canvas.cmu.edu/courses/40739/quizzes/123417

Today

- Cache organization and operation
- Performance impact of caches
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

The Memory Mountain

- Read throughput (read bandwidth)
 - Number of bytes read from memory per second (MB/s)
- Memory mountain: Measured read throughput as a function of spatial and temporal locality.
 - Compact way to characterize memory system performance.

Memory Mountain Test Function

```
long data[MAXELEMS]; /* Global array to traverse */
/* test - Iterate over first "elems" elements of
          array "data" with stride of "stride",
          using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;
    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {</pre>
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
       acc3 = acc3 + data[i+sx3];
    }
    /* Finish any remaining elements */
    for (; i < length; i++) {</pre>
        acc0 = acc0 + data[i];
    return ((acc0 + acc1) + (acc2 + acc3));
                               mountain/mountain.c
```

Call test() with many combinations of elems and stride.

For each elems and stride:

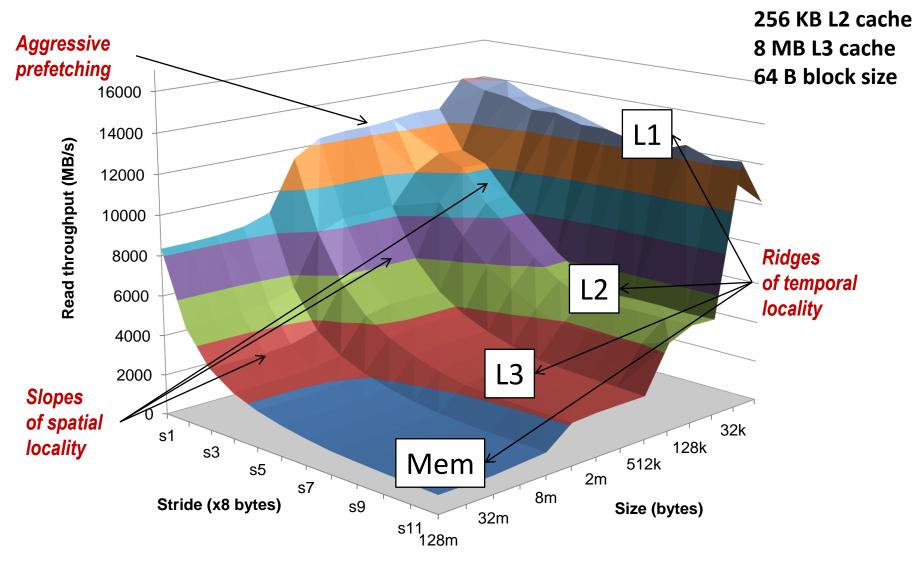
- 1. Call test() once to warm up the caches.
- 2. Call test() again and measure the read throughput(MB/s)

Core i7 Haswell

32 KB L1 d-cache

2.1 GHz

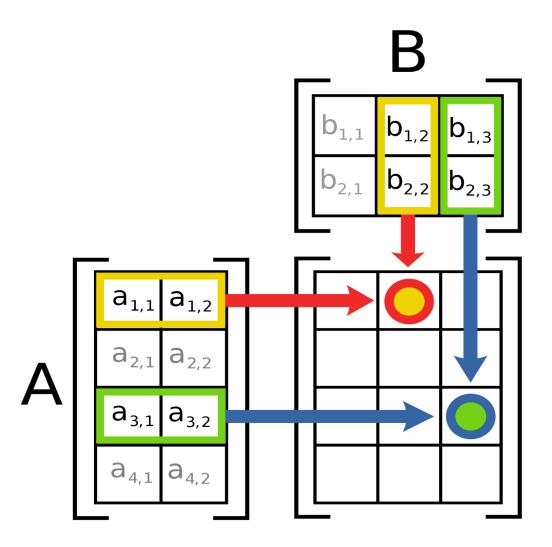
The Memory Mountain



Today

- Cache organization and operation
- Performance impact of caches
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

Remember matrix multiplication



Matrix Multiplication Example

Description:

- Multiply N x N matrices
- Matrix elements are doubles (8 bytes)
- $O(N^3)$ total operations
- N reads per source element
- N values summed per destination
 - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
       sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}

matmult/mm.c
```

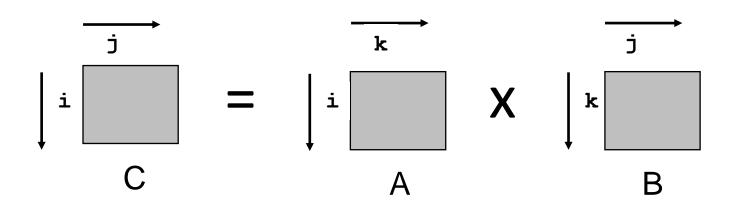
Miss Rate Analysis for Matrix Multiply

Assume:

- Block size = 32B (big enough for four doubles)
- Matrix dimension (N) is very large
 - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
 - each row in contiguous memory locations
 - a[i][j] = a[i*N + j] where N is the number of columns
- Stepping through columns in one row:

```
for (i = 0; i < N; i++)
sum += a[0][i];</pre>
```

- accesses successive elements
- if block size (B) > sizeof(a_{ii}) bytes, exploit spatial locality
 - miss rate = sizeof(a_{ii}) / B
- Stepping through rows in one column:

```
for (i = 0; i < n; i++)
sum += a[i][0];</pre>
```

- accesses distant elements
- no spatial locality!
 - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
       sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}

matmult/mm.c</pre>
```

```
Inner loop:

(*,j)

(i,*)

B

C

↑

Row-wise Column-
wise
```

Miss rate for inner loop iterations:

<u>A</u>

<u>B</u>

<u>C</u>

Block size = 32B (four doubles)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
       sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
matmult/mm.c</pre>
```

```
Inner loop:

(*,j)

(i,*)

B

C

↑

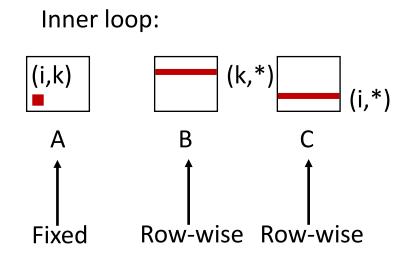
Row-wise Column-
wise
```

Miss rate for inner loop iterations:

<u>A</u> <u>B</u> <u>C</u> 0.25 1.0 0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
        c[i][j] += r * b[k][j];
  }
}
matmult/mm.c</pre>
```



Miss rate for inner loop iterations:

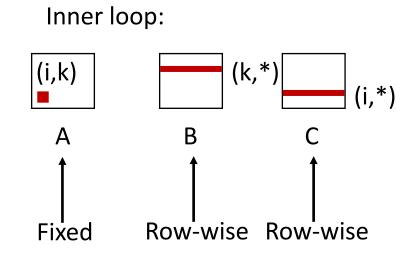
<u>A</u>

<u>B</u>

<u>C</u>

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
        c[i][j] += r * b[k][j];
  }
}
matmult/mm.c</pre>
```

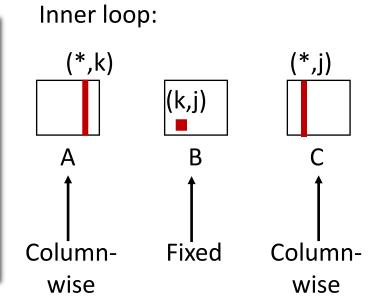


Miss rate for inner loop iterations:

<u>A</u> <u>B</u> <u>C</u> 0.0 0.25 0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
        c[i][j] += a[i][k] * r;
  }
}
    matmult/mm.c</pre>
```



Miss rate for inner loop iterations:

<u>A</u>

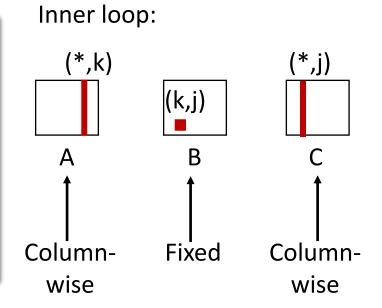
<u>B</u>

<u>C</u>

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
        c[i][j] += a[i][k] * r;
  }
}

matmult/mm.c</pre>
```



Miss rate for inner loop iterations:

<u>A</u> <u>B</u> <u>C</u> 1.0 0.0 1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;
  for (k=0; k<n; k++)
    sum += a[i][k] * b[k][j];
  c[i][j] = sum;
}
</pre>
```

```
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
  for (j=0; j<n; j++)
    c[i][j] += r * b[k][j];
}</pre>
```

```
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
  for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
}</pre>
```

ijk (& jik):

- 2 loads, 0 stores
- avg misses/iter = **1.25**

kij (& ikj):

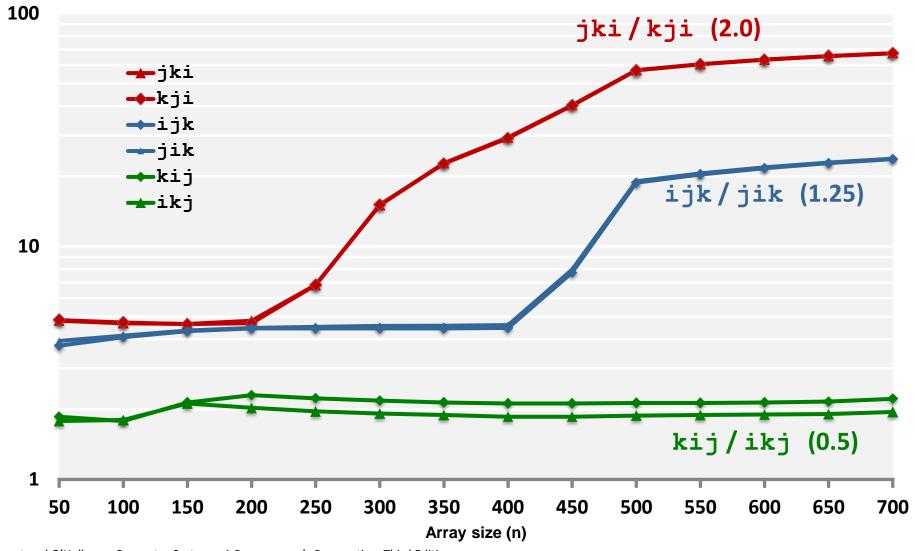
- 2 loads, 1 store
- avg misses/iter = **0.5**

jki (& kji):

- 2 loads, 1 store
- avg misses/iter = 2.0

Core i7 Matrix Multiply Performance

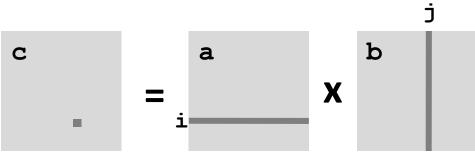
Cycles per inner loop iteration



Today

- Cache organization and operation
- Performance impact of caches
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

Example: Matrix Multiplication



n

Cache Miss Analysis

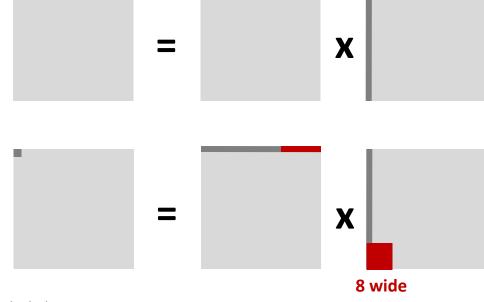
Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size C << n (much smaller than n)

First iteration:

• n/8 + n = 9n/8 misses

Afterwards in cache: (schematic)



n

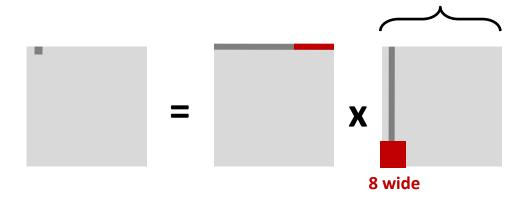
Cache Miss Analysis

Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size C << n (much smaller than n)

Second iteration:

• Again: n/8 + n = 9n/8 misses



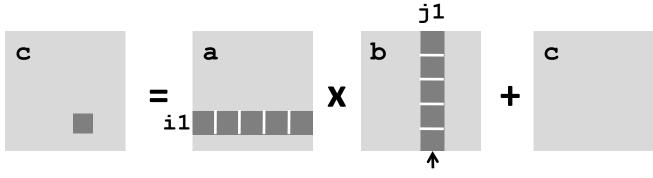
Total misses:

 $9n/8 n^2 = (9/8) n^3$

48

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);
/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
       for (j = 0; j < n; j+=B)
             for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                  for (i1 = i; i1 < i+B; i1++)
                      for (j1 = j; j1 < j+B; j1++)
                          for (k1 = k; k1 < k+B; k1++)
                              c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
                                                         matmult/bmm.c
```



Cache Miss Analysis

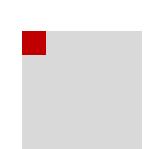
Assume:

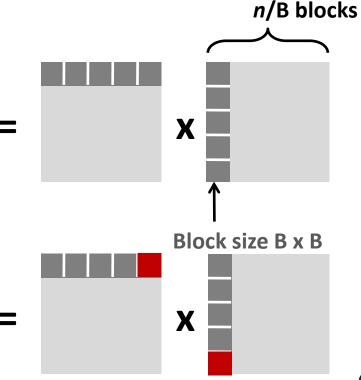
- Cache block = 8 doubles
- Cache size C << n (much smaller than n)
- Three blocks \blacksquare fit into cache: $3B^2 < C$

First (block) iteration:

- B*B/8 misses for each block
- $2n/B \times B^2/8 = nB/4$ (omitting matrix c)

Afterwards in cache (schematic)





n/B blocks

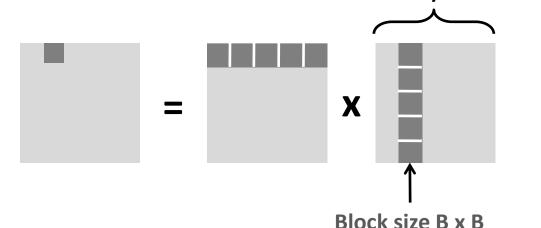
Cache Miss Analysis

Assume:

- Cache block = 8 doubles
- Cache size C << n (much smaller than n)
- Three blocks fit into cache: 3B² < C</p>

Second (block) iteration:

- Same as first iteration
- $2n/B \times B^2/8 = nB/4$



Total misses:

• $nB/4 * (n/B)^2 = n^3/(4B)$

Blocking Summary

- No blocking: (9/8) n³ misses
- Blocking: $(1/(4B)) n^3$ misses
- Use largest block size B, such that B satisfies 3B² < C</p>
 - Fit three blocks in cache! Two input, one output.
- Reason for dramatic difference:
 - Matrix multiplication has inherent temporal locality:
 - Input data: $3n^2$, computation $2n^3$
 - Every array elements used O(n) times!
 - But program has to be written properly

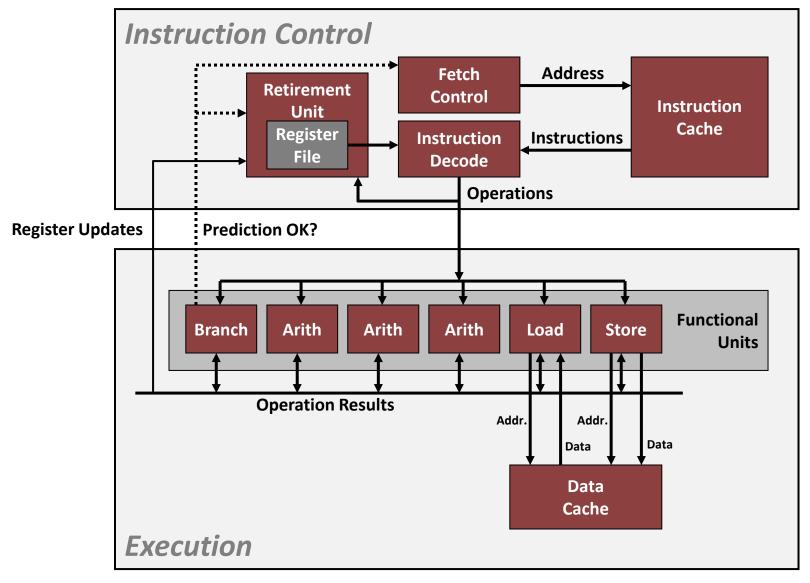
Cache Summary

Cache memories can have significant performance impact

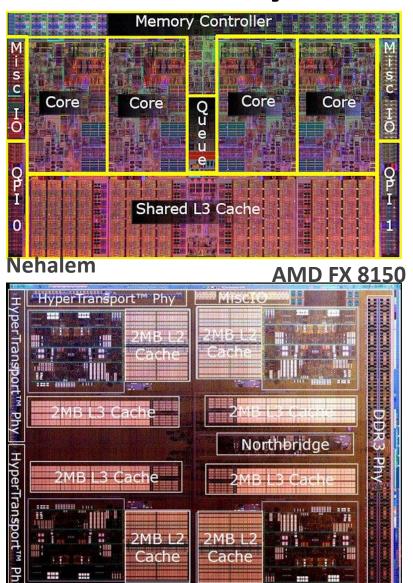
- You can write your programs to exploit this!
 - Focus on the inner loops, where bulk of computations and memory accesses occur.
 - Try to maximize spatial locality by reading data objects sequentially with stride 1.
 - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

Supplemental slides

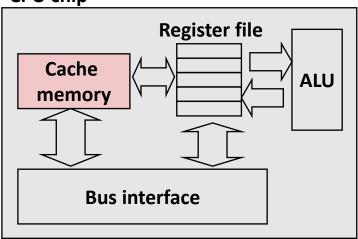
Recall: Modern CPU Design



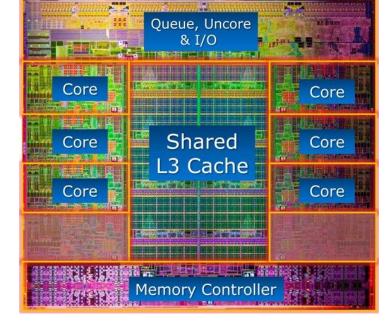
What it Really Looks Like



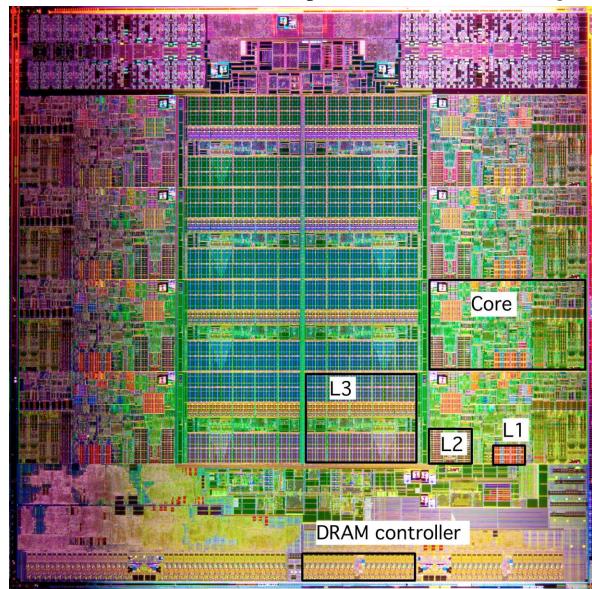
CPU chip



Core i7-3960X



What it Really Looks Like (Cont.)



Intel Sandy Bridge Processor Die

L1: 32KB Instruction + 32KB Data

L2: 256KB

L3: 3-20MB

Why Index Using Middle Bits?

Direct mapped: One line per set
Assume: cache block size 8 bytes

v tag 0 1 2 3 4 5 6 7

tag

tag

 $S = 2^{s}$ sets

Standard Method:
Middle bit indexing

Address of int:

t bits 0...01 100

find set

v tag 0 1 2 3 4 5 6 7

3

3

5

5

High bit indexing

Address of int:

t bits

100

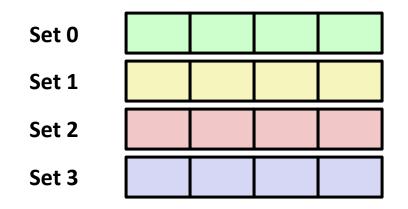
Alternative Method:

1...11

find set

Illustration of Indexing Approaches

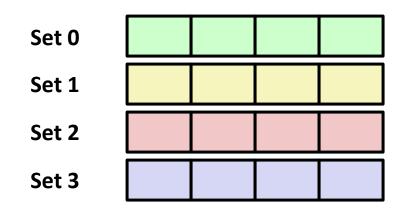
- 64-byte memory
 - 6-bit addresses
- 16 byte, direct-mapped cache
- Block size = 4. (Thus, 4 sets; why?)
- 2 bits tag, 2 bits index, 2 bits offset



		0000xx
		0001xx
		0010xx
		0011xx
		0100xx
		0101xx
		0110xx
		0111xx
		1000xx
		1001xx
		1010xx
		1011xx
		1100xx
		1101xx
		1110xx
		1111xx
) 51

Middle Bit Indexing

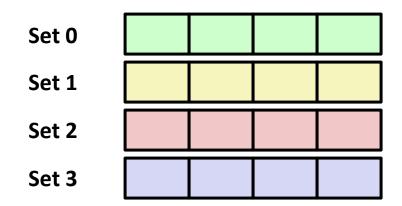
- Addresses of form TTSSBB
 - **TT** Tag bits
 - Set index bits
 - **BB** Offset bits
- Makes good use of spatial locality



		0000xx
		0001xx
		0010xx
		0011xx
		0100xx
		0101xx
		0110xx
		0111xx
		1000xx
		1001xx
		1010xx
		1011xx
		1100xx
		1101xx
		1110xx
		1111xx
		,

High Bit Indexing

- Addresses of form SSTTBB
 - Set index bits
 - **TT** Tag bits
 - **BB** Offset bits
- Program with high spatial locality would generate lots of conflicts



		0000xx
		0001xx
		0010xx
		0011xx
		0100xx
		0101xx
		0110xx
		0111xx
		1000xx
		1001xx
		1010xx
		1011xx
		1100xx
		1101xx
		1110xx
		1111xx
		l •

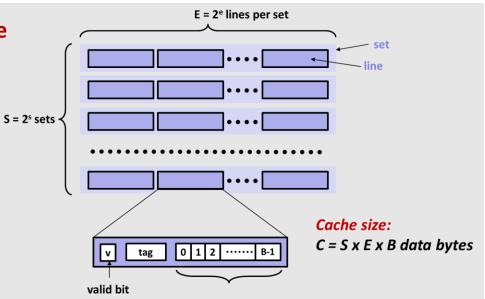
Example: Core i7 L1 Data Cache

32 kB 8-way set associative 64 bytes/block 47 bit address range

$$S = , s =$$

E = , e =

C =



Het Decimal 0 0 00000 1 1 0001 2 2 0010 3 3 0011

_		0
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
В	11	1011
С	12	1100
D	13	1101

14

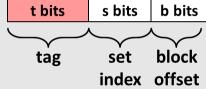
15

0x??

1110

1111

Address of word:



Block offset: . bits

Set index: . bits

Tag: . bits

Stack Address:

0x00007f7262a1e010

Block offset:

Set index: 0x??

Tag: 0x??

Example: Core i7 L1 Data Cache

32 kB 8-way set associative 64 bytes/block

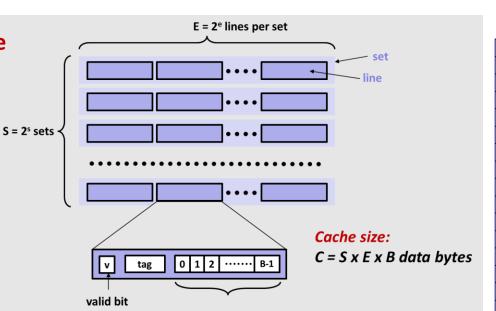
47 bit address range

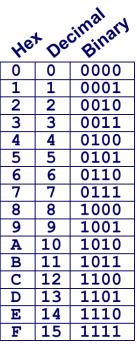
$$B = 64$$

$$S = 64$$
, $s = 6$

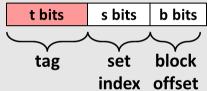
$$E = 8, e = 3$$

 $C = 64 \times 64 \times 8 = 32,768$









Block offset: 6 bits

Set index: 6 bits

Tag: 35 bits



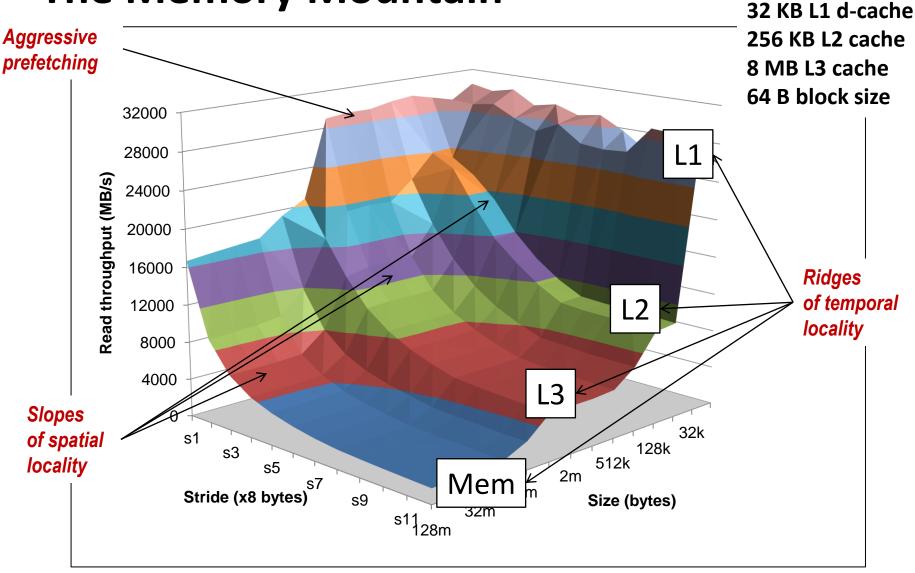
Block offset: 0×10 Set index: 0×0

Tag: 0x7f7262a1e

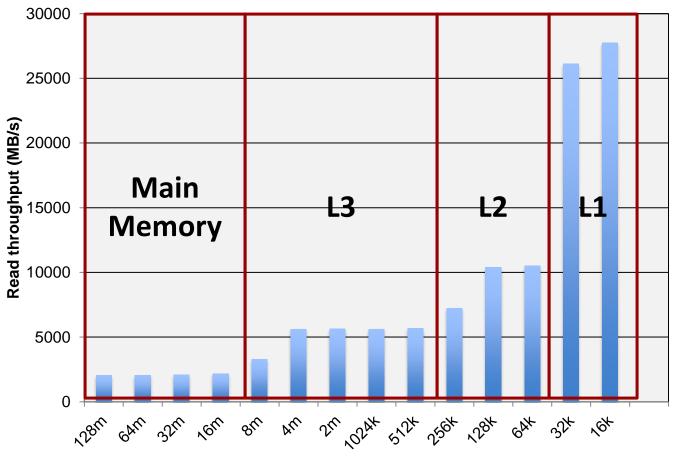
Core i5 Haswell

3.1 GHz

The Memory Mountain



Cache Capacity Effects from Memory Mountain



Core i7 Haswell
3.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

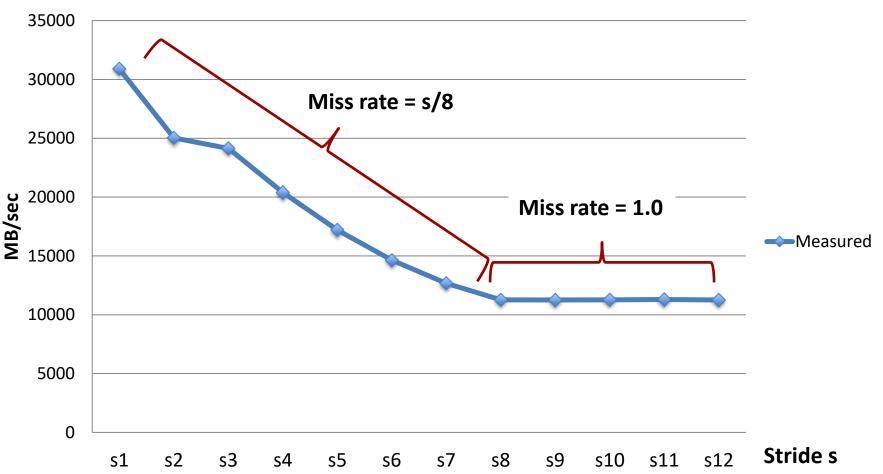
Slice through memory mountain with stride=8

Working set size (bytes)

Cache Block Size Effects from Memory Mountain

Core i7 Haswell 2.26 GHz 32 KB L1 d-cache 256 KB L2 cache 8 MB L3 cache 64 B block size

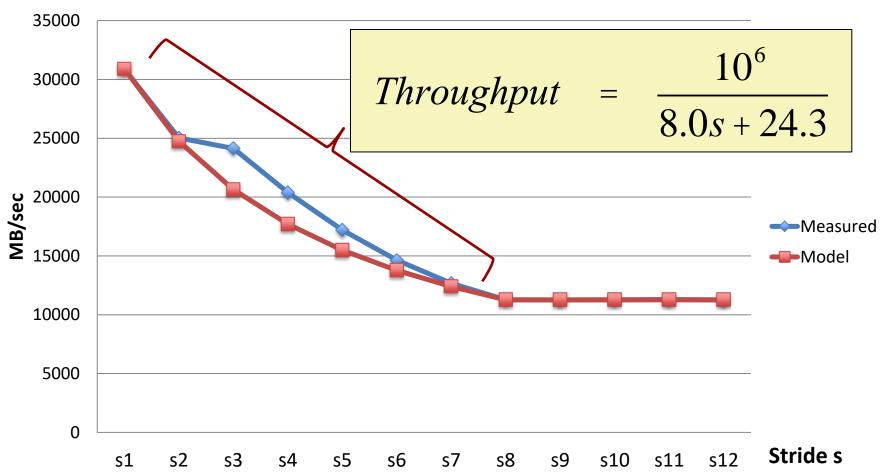


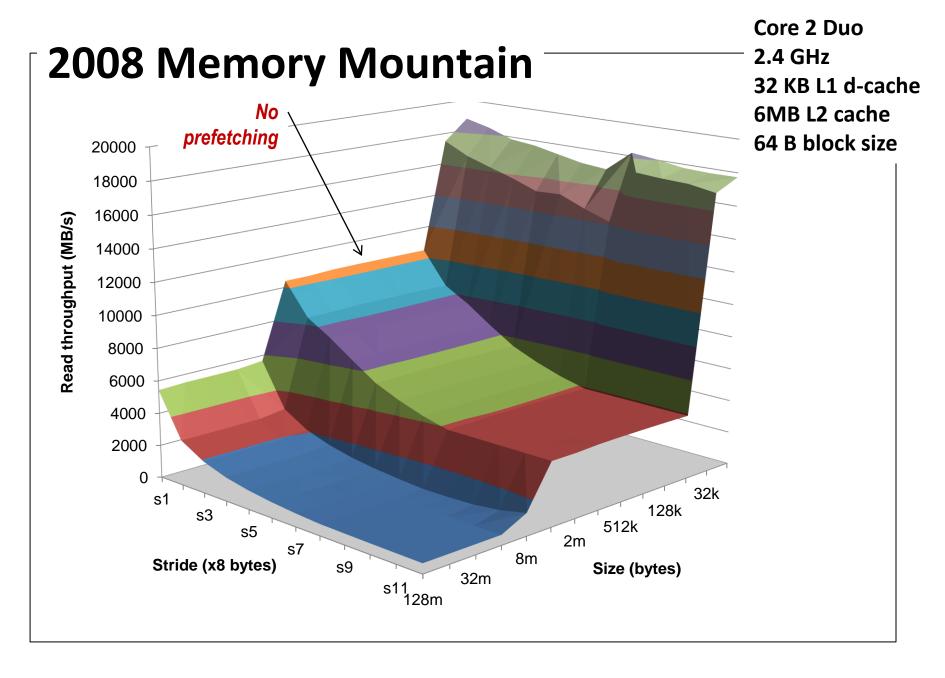


Modeling Block Size Effects from Memory Mountain

Core i7 Haswell
2.26 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

Throughput for size = 128K

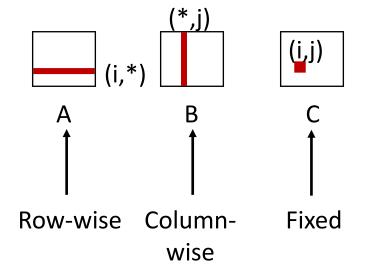




Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
        sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
    matmult/mm.c</pre>
```

Inner loop:



Misses per inner loop iteration:

<u>A</u> <u>B</u> <u>C</u> 0.25 1.0 0.0

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
  for (j=0; j<n; j++)
    c[i][j] += r * b[k][j];
  }
}
matmult/mm.c</pre>
```

```
Inner loop:

(i,k)

A

B

C

↑

↑

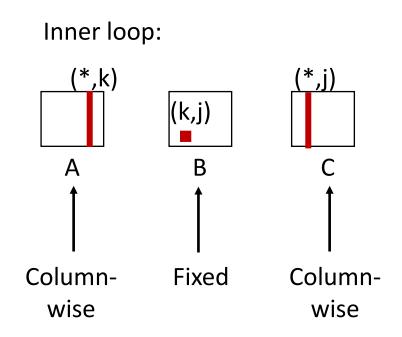
Row-wise Row-wise
```

Misses per inner loop iteration:

<u>A</u> <u>B</u> <u>C</u> 0.0 0.25 0.25

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
        c[i][j] += a[i][k] * r;
  }
}
    matmult/mm.c</pre>
```



Misses per inner loop iteration:

<u>A</u> <u>B</u> <u>C</u> 1.0 0.0 1.0