15-745

Static Single Assignment

Values ≠ Locations

```
…
for (i=0; i++; i<10) {
  … = … i …;
  …
}
for (i=j; i++; i<20) {
  …  = i …
}
```


Values ≠ Locations

```
…
for (i=0; i++; i<10) {
  … = … i …;
  …
}
for (i=j; i++; i<20) {
  …  = i …
}
```

Def-use chains help solve the problem.

Def-Use chains are expensive

```
foo(int i, int j) {
  …
  switch (i) {
  case 0: x=3;break;
  case 1: x=1; break;
  case 2: x=6; break;
  case 3: x=7; break;
  default: x = 11;
  }
  switch (j) {
  case 0: y=x+7; break;
  case 1: y=x+4; break;
  case 2: y=x-2; break;
  case 3: y=x+1; break;
  default: y=x+9;
  }
  …
```


Def-Use chains are expensive

```
foo(int i, int j) {
  …
  switch (i) {
  case 0: x=3;
  case 1: x=1;
  case 2: x=6;
  case 3: x=7;
  default: x = 11;
  }
  switch (j) {
  case 0: y=x+7;
  case 1: y=x+4;
  case 2: y=x-2;
  case 3: y=x+1;
  default: y=x+9;
  }
  …
```

In general,
N defs
M uses
⇒ O(NM) space and time

A solution is to limit each var to ONE def site

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Def-Use chains are expensive

```
foo(int i, int j) {
  …
  switch (i) {
  case 0: x=3; break;
  case 1: x=1; break;
  case 2: x=6;
  case 3: x=7;
  default: x = 11;
  }
  x1 is one of the above x's
  switch (j) {
  case 0: y=x1+7;
  case 1: y=x1+4;
  case 2: y=x1-2;
  case 3: y=x1+1;
  default: y=x1+9;
  }
  …
```

A solution is to limit each var to ONE def site

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Advantages of SSA

- Makes du-chains explicit
- Makes dataflow analysis easier
- Improves register allocation
  - Automatically builds Webs
  - Makes building interference graphs easier
- For most programs reduces space/time requirements

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SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
  - assign to a fresh variable at each stmt.
  - each use uses the most recently defined var.
  - (Similar to Value Numbering)

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Straight-line SSA

```
a ← x + y
b ← a + x
a ← b + 2
c ← y + 1
a ← c + a
```


Straight-line SSA

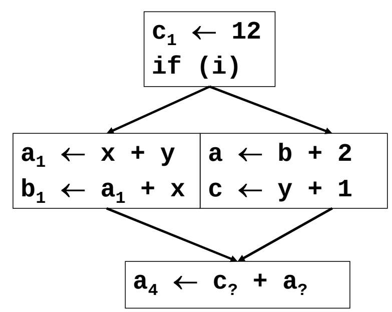
$a \leftarrow x + y$	→	$a_1 \leftarrow x + y$
$b \leftarrow a + x$		$b_1 \leftarrow a_1 + x$
$a \leftarrow b + 2$		$a_2 \leftarrow b_1 + 2$
$c \leftarrow y + 1$		$c_1 \leftarrow y + 1$
$a \leftarrow c + a$		$a_3 \leftarrow c_1 + a_2$

SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
  - assign to a fresh variable at each stmt.
  - each use uses the most recently defined var.
  - (Similar to Value Numbering)
- What about at joins in the CFG?

Merging at Joins

```
c ← 12
if (i) {
  a ← x + y
  b ← a + x
} else {
  a ← b + 2
  c ← y + 1
}
a ← c + a
```



SSA

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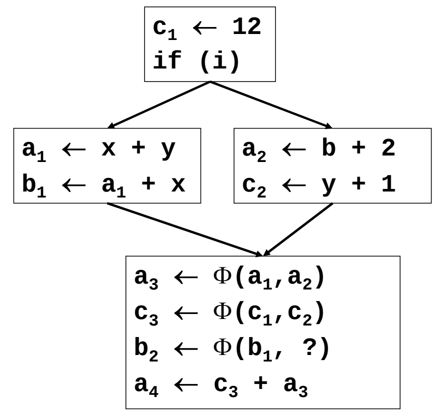
  - Each use uses the most recently defined var.
  - (Similar to Value Numbering)
- What about at joins in the CFG?
  - Use a notional fiction: A Φ function

Merging at Joins



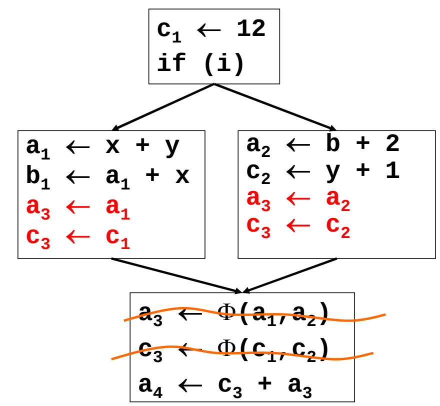
The Φ function

- Φ merges multiple definitions along multiple control paths into a single definition.
- At a BB with p predecessors, there are p arguments to the Φ function.

  $x_{new} \leftarrow \Phi(x_1, x_1, x_1, \ldots, x_p)$

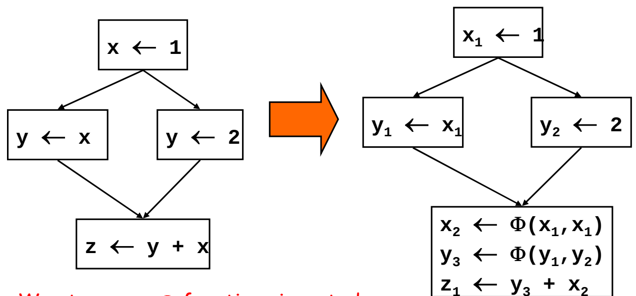
- How do we choose which x_i to use?
  - We don't really care!
  - If we care, use moves on each incoming edge

"Implementing" Φ



Trivial SSA

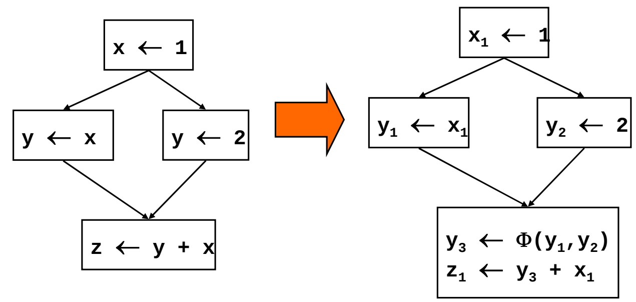
- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables.



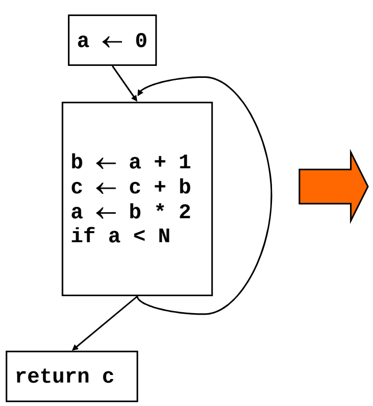
Way too many Φ functions inserted.

Minimal SSA

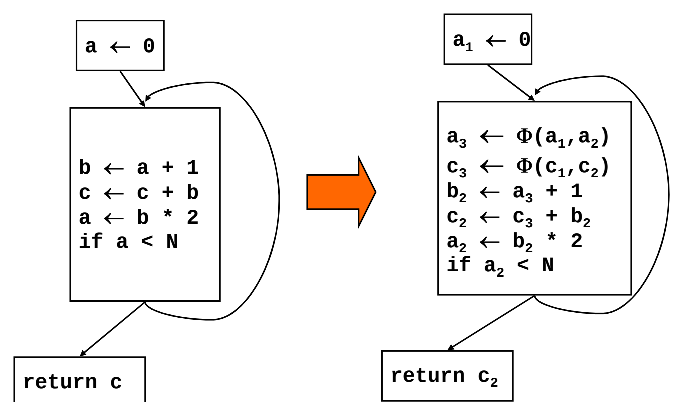
- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables with multiple outstanding defs.



Another Example

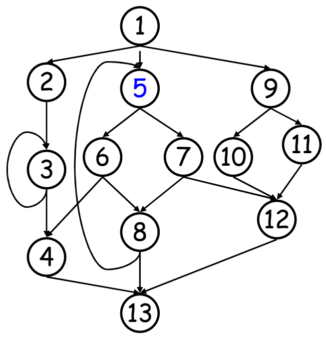


Another Example



Notice use of c_1

When do we insert Φ?



CFG

If there is a def of **a** in block 5, which nodes need a $\Phi()$?

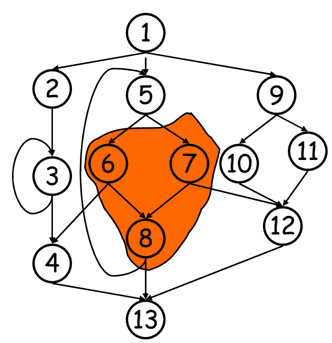
When do we insert Φ?

- We insert a Φ function for variable **A** in block Z iff:
  - **A** was defined more than once before (i.e., **A** defined in X and Y AND $X \neq Y$)
  - There exists a non-empty path from x to z, P_{xz}, and a non-empty from y to z, P_{yz} s.t.
    - $P_{xz} \cap P_{yz} = \{ z \}$
    - $z \notin P_{xq}$ or $z \notin P_{yr}$ where $P_{xz} = P_{xq} \rightarrow z$ and $P_{yz} = P_{yr} \rightarrow z$
- Entry block contains an implicit def of all vars
- Note: $A = \Phi(\ldots)$ is a def of A

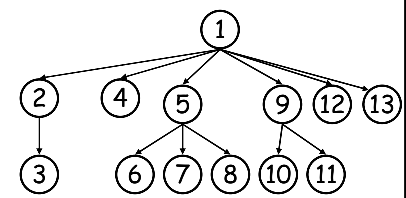
Dominance Property of SSA

- In SSA, definitions dominate uses.
  - If x_i is used in $x \leftarrow \Phi(\ldots, x_i, \ldots)$, then $BB(x_i)$ dominates i^{th} predecessor of BB(PHI)
  - If x is used in $y \leftarrow \ldots x \ldots$, then BB(x) dominates BB(y)
- We can use this for an efficient algorithm to convert to SSA

Dominance



CFG

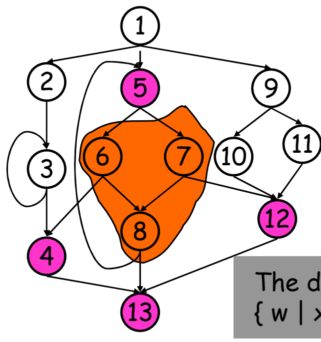


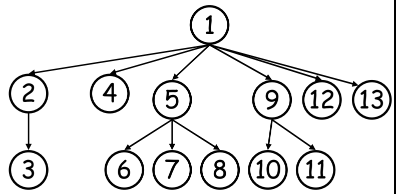
If there is a def of a in block 5, which nodes need a $\Phi()$?

D-Tree

x strictly dominates w (x sdom w) iff x dom w AND $x \neq w$

Dominance Frontier



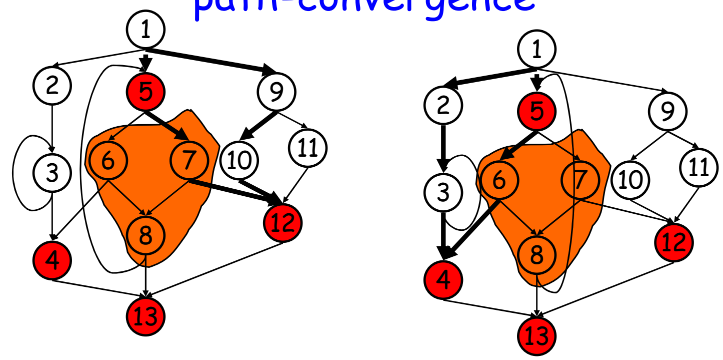


The dominance Frontier of a node x =
{ w | x dom pred(w) AND !(x sdom w)}

CFG D-Tree

x strictly dominates w (x sdom w) iff x dom w AND x ≠ w

Dominance Frontier & path-convergence



Using DF to compute SSA

- place all Φ()
- Rename all variables

Using DF to Place Φ()

- Gather all the defsites of every variable
- Then, for every variable
  - foreach defsite
    - foreach node in DF(defsite)
      - if we haven't put Φ() in node put one in
      - If this node didn't define the variable before: add this node to the defsites
- This essentially computes the Iterated Dominance Frontier on the fly, inserting the minimal number of Φ() neccesary

Using DF to Place Φ()

```
foreach node n {
  foreach variable v defined in n {
    orig[n] ∪= {v}
    defsites[v] ∪= {n}
  }
}
foreach variable v {
  W = defsites[v]
  while W not empty {
    n = remove node from W
    foreach y in DF[n]
    if y ∉ PHI[v] {
      insert "v ← Φ(v,v,…)" at top of y
      PHI[v] = PHI[v] ∪ {y}
      if v ∉ orig[y]: W = W ∪ {y}
    }
  }
}
```

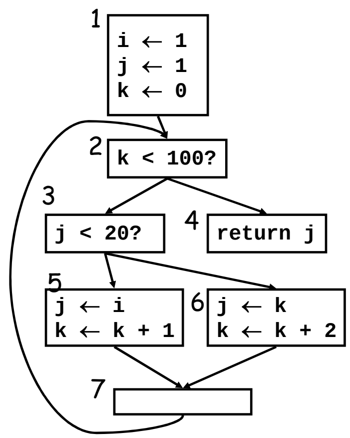

Renaming Variables

- Walk the D-tree, renaming variables as you go
- Replace uses with more recent renamed def
  - For straight-line code this is easy
  - If there are branches and joins?

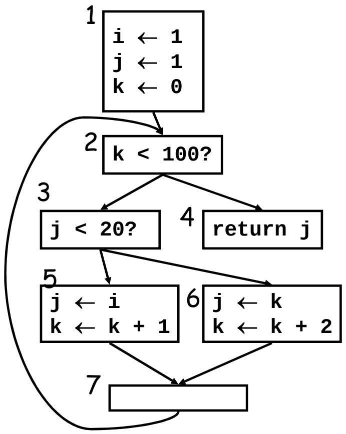
Renaming Variables

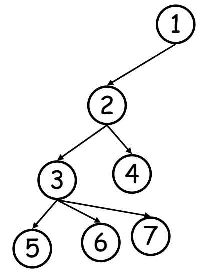
- Walk the D-tree, renaming variables as you go
- Replace uses with more recent renamed def
  - For straight-line code this is easy
  - If there are branches and joins use the closest def such that the def is above the use in the D-tree
- Easy implementation:
  - for each var: rename (v)
  - rename(v): replace uses with top of stack
    at def: push onto stack
    call rename(v) on all children in D-tree
    for each def in this block pop from stack

Compute D-tree



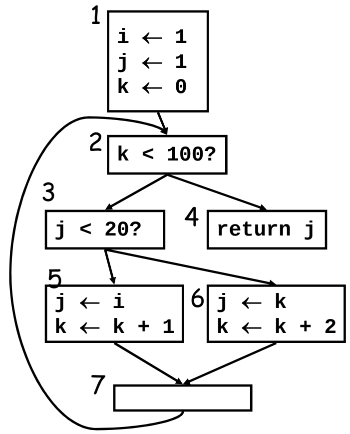
Compute D-tree

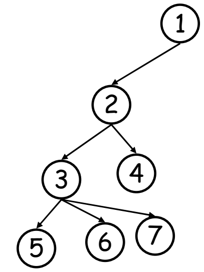




D-tree

Compute Dominance Frontier

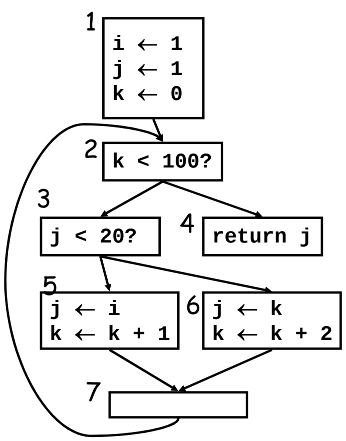




1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

DFs

Insert Φ()



1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

DFs

orig[n]

1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

defsites[v]

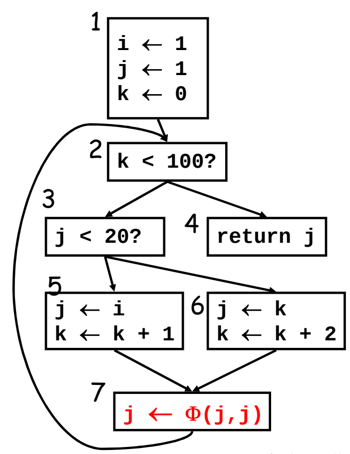
i	{1}
j	{1,5,6}
k	{1,5,6}

var i: W={1}

var j: W={1,5,6}

DF{1}, DF{5}

Insert Φ()



1	{}
2	{2}
3	{2}
4	{}
5	{7}
6	{7}
7	{2}

DFs

orig[n]

1	{ i,j,k}
2	{}
3	{}
4	{}
5	{j,k}
6	{j,k}
7	{}

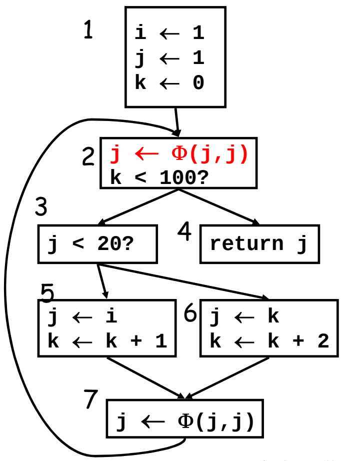
defsites[v]

i	{1}
j	{1,5,6}
k	{1,5,6}

var j: W={1,5,6}

DF{1}, DF{5}

Insert Φ()



n	DFs	orig[n]
1	{}	{ i,j,k}
2	{2}	{}
3	{2}	{}
4	{}	{}
5	{7}	{j,k}
6	{7}	{j,k}
7	{2}	{}

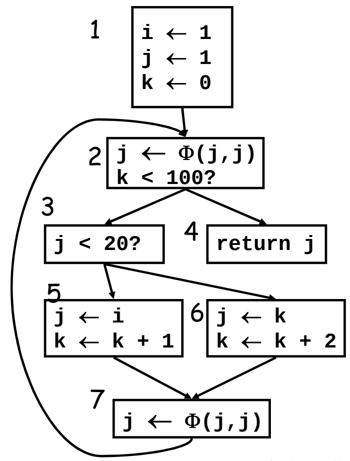
defsites[v]	
i	{1}
j	{1,5,6}
k	{1,5,6}

DFs

var j: W={1,5,6}

DF{1}, DF{5}

Insert Φ()



n	DFs	orig[n]
1	{}	{ i,j,k}
2	{2}	{}
3	{2}	{}
4	{}	{}
5	{7}	{j,k}
6	{7}	{j,k}
7	{2}	{}

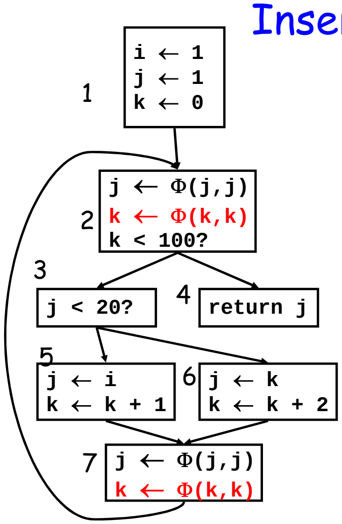
defsites[v]	
i	{1}
j	{1,5,6}
k	{1,5,6}

DFs

var j: W={1,5,6}

DF{1}, DF{5}, DF{6}

Insert Φ()



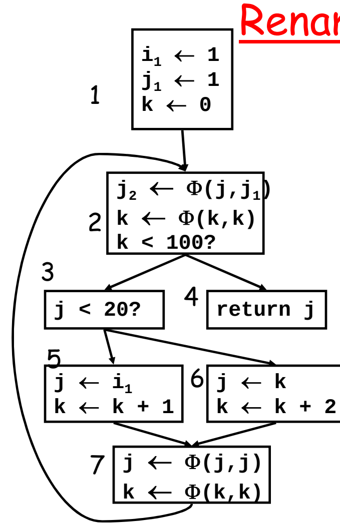
n	DFs	orig[n]
1	{}	{ i,j,k}
2	{2}	{}
3	{2}	{}
4	{}	{}
5	{7}	{j,k}
6	{7}	{j,k}
7	{2}	{}

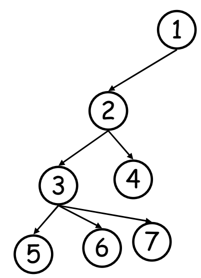
defsites[v]	
i	{1}
j	{1,5,6}
k	{1,5,6}

DFs

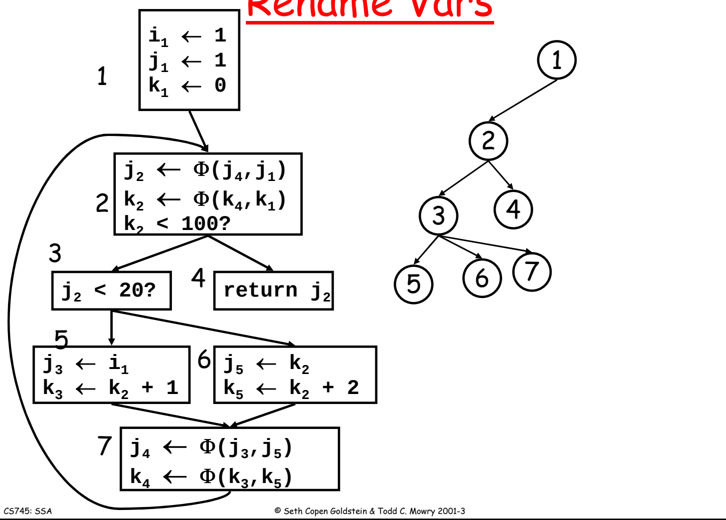
var k: W={1,5,6}

Rename Vars

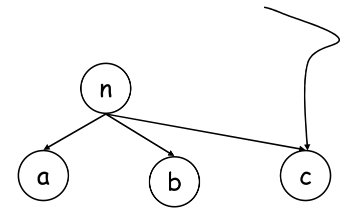




Rename Vars

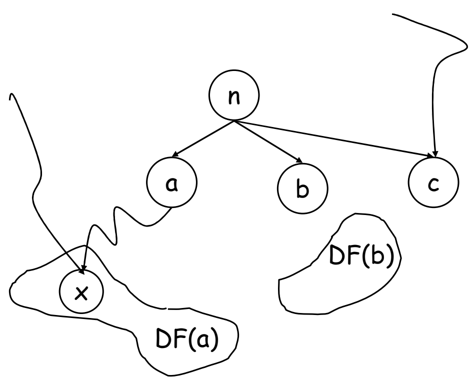


Computing DF(n)



n dom a
n dom b
!n dom c

Computing DF(n)



n dom a
n dom b
!n dom c

Computing the Dominance Frontier

The dominance Frontier of a node x =
{ w | x dom pred(w) AND !(x sdom w)}

```
compute-DF(n)
    S = {}
    foreach node y in succ[n]
        if idom(y) ≠ n
            S = S ∪ { y }
    foreach child of n, c, in D-tree
        compute-DF(c)
        foreach w in DF[c]
            if !n dom w
                S = S ∪ { w }
    DF[n] = S
```

SSA Properties

- Only 1 assignment per variable
- definitions dominate uses

