15-745

Graph Coloring Register Allocation

Intro to Global Register Allocation

Problem:

- **Allocation of variables (pseudo-registers) to hardware registers in a procedure**

One of the most important optimizations

- **Memory accesses are more costly than register accesses**
  - True even with caches
  - True even with CISC architectures
- **Important for other optimizations**
  - E.g., redundancy elimination assumes old values are kept in registers
- **When it does not work well, the performance impact is noticeable.**

Terminology

Allocation

- **decision to keep a pseudo-register in a hardware register**
- **prior to register allocation, we assume an infinite set of registers**
  - (aka “temps” or “pseudo-registers”).

Spilling

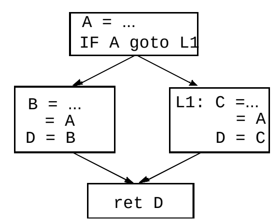
- **when allocation fails...**
- **a pseudo-register is spilled to memory, if not kept in a hardware register**

Assignment

- **decision to keep a pseudo-register in a *specific* hardware register**

What are the Problems?

- For this example:
  - What is the minimum number of registers needed to avoid spilling?
  - Given ***n*** registers in a machine, is spilling necessary?
  - Find an assignment for all pseudo-registers, if possible.
  - If there are not enough registers in the machine, how do we spill to memory?



Abstraction for Reg Alloc & Assignment

Intuitively:

- **Two pseudo-registers *interfere* if at some point in the program they cannot both occupy the same register.**

Interference graph: an undirected graph, where

- **nodes = pseudo-registers**
- **there is an edge between two nodes if their corresponding pseudo-registers interfere**

Register Allocation and Coloring

- **A graph is *n-colorable* if every node in the graph can be colored with one of n colors such that two adjacent nodes do not have the same color.**
- Assigning n registers (without spilling) = Coloring with n colors
  - assign a node to a register (color) such that no two adjacent nodes are assigned same registers(colors)
- Is spilling necessary? = Is the graph n-colorable?
- **To determine if a graph is n-colorable is NP-complete, for n>2**
  - Too expensive
  - Heuristics

Simple Algorithm

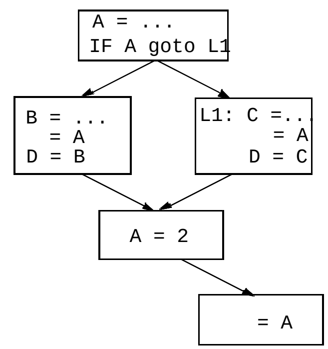
Build an interference graph

- **refining notion of a node**
- **finding the edges**

Coloring

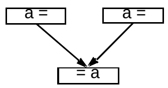
- **use heuristics to try to find an n-coloring**
  - Success ⇒ colorable and we have an assignment
  - Failure ⇒ graph not colorable, or graph is colorable, but we couldn't find a coloring

Nodes in an Interference Graph

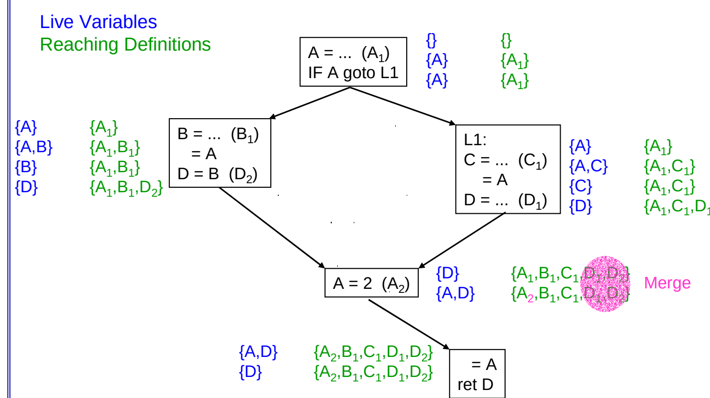


Live Ranges & Merged Live Ranges

- **Motivation:** to create an interference graph that is easier to color
  - Eliminate interference in a variable's "dead" zones.
  - Increase flexibility in allocation: can allocate same variable to different registers
- **A *live range* consists of a definition and all the points in a program (e.g. end of an instruction) in which that definition is live.**
  - How to compute a live range?
- **Two overlapping live ranges for the same variable must be merged**



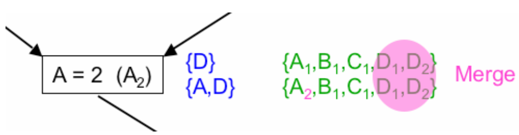
Example (Revisited)



Merging Live Ranges

Merging definitions into equivalence classes:

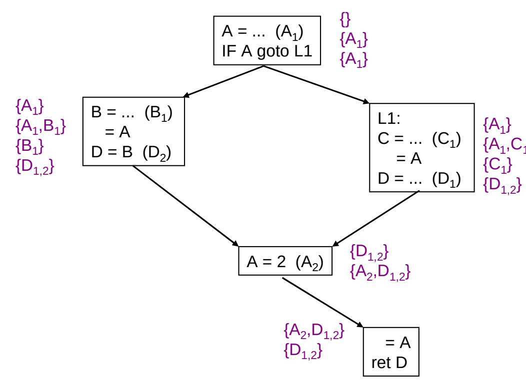
- **Start by putting each definition in a different equivalence class**
- **For each point in a program**
  - if variable is live, and there are multiple reaching definitions for the variable
  - merge the equivalence classes of all such definitions into a one equivalence class



From now on, refer to merged live ranges simply as live range

- Merged live ranges are also known as "webs"

Example: Merged Live Ranges



Edges of Interference Graph

Intuitively:

- **Two live ranges (necessarily of different variables) may interfere if they overlap at some point in the program.**
- **Algorithm:**
  - At each point in program, enter an edge for every pair of live ranges at that point

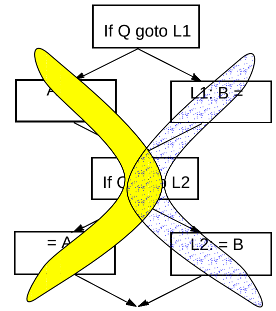
An optimized definition & algorithm for edges:

```
For each inst i
        Let x be live range of definition at inst i
        For each live range y present at end of inst i
                insert an edge between x and y
```

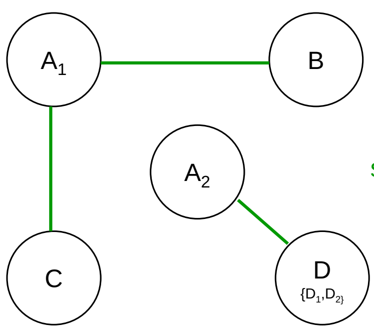
- Faster
- Better quality?

A = 2 (A_2) $\{D_{1,2}\}$ $\{A_2,D_{1,2}\}$ → Edge between A_2 and $D_{1,2}$

Example 2



Example: Interference Graph



So was it worth it to split the live ranges?

Coloring

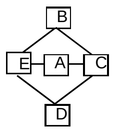
- **Reminder:** coloring for n > 2 is NP-complete
- **Observations**
  - a node with degree < n ⇒
    - ➢ can always color it successfully, given its neighbors' colors
  - a node with degree = n ⇒
  - a node with degree > n ⇒

Coloring Algorithm

Algorithm

- **Iterate until stuck or done**
  - Pick any node with degree < n
  - Remove the node and its edges from the graph
- **If done (no nodes left)**
  - reverse process and add colors

Example (**n = 3**)



- Note: degree of a node may drop in iteration
- Avoids making arbitrary decisions that make coloring fail

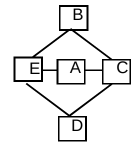
What Does Coloring Accomplish?

Done:

- **colorable**
- **also obtained an assignment (colors correspond to registers)**

Stuck (n = 2):

- **colorable or not?**



- **One solution: optimistically remove nodes and hope we get lucky...**

Checkpoint

Problems:

- **Given n registers in a machine, is spilling avoided?**
- **Find an assignment for all pseudo-registers, whenever possible.**

Solution:

- **Abstraction: an interference graph**
  - nodes: (merged) live ranges
  - edges: presence of live range at time of definition
- **Register Allocation and Assignment problems**
  = n-colorability of interference graph
  ⇒ NP-complete
- **Heuristics to find an assignment for n colors**
  - successful: colorable, and finds assignment
  - unsuccessful: colorability unknown & no assignment

Discussion

What about when we can't k-color?

- **spill to memory: next time**

Is the minimum coloring always what we want?

- **Hint: no**

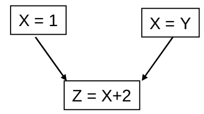
What about architecture strangeness?

- **subword registers (x86, 68k, ColdFire...)**
- **register pairing (HP PA-RISC, SPARC, x86)**
- **register classes (x86, 68k, ColdFire...)**

An Improvement: Move Coalescing

Basic idea:

- **eliminate moves by assigning the src and dest to the same register**
- **copy propagation and dead code elimination can't eliminate all unnecessary moves**



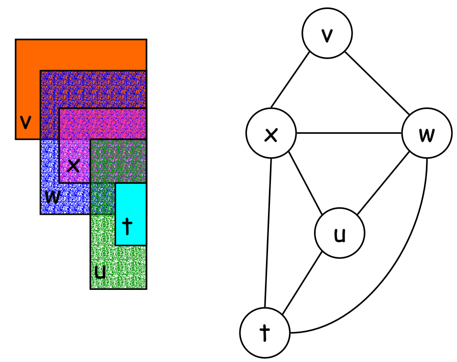
If we allocate X and Y to the same register we can eliminate X = Y

(copy prop couldn't)

How can we modify our interference graph to do this?

An Exciting New Example

```
v <-  1
w <-  v + 3
x <-  w + v
u <-  v
t <-  u + x
  <-  w
  <-  t
  <-  u
```



First compute live ranges...

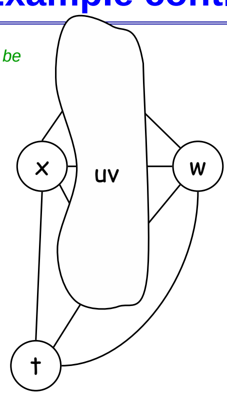
...then construct interference graph

An Exciting New Example cont.

```
v <-  1
w <-  v + 3
x <-  w + v
u <-  v
t <-  u + x
  <-  w
  <-  t
  <-  u
```

Want u and v to be assigned same color...

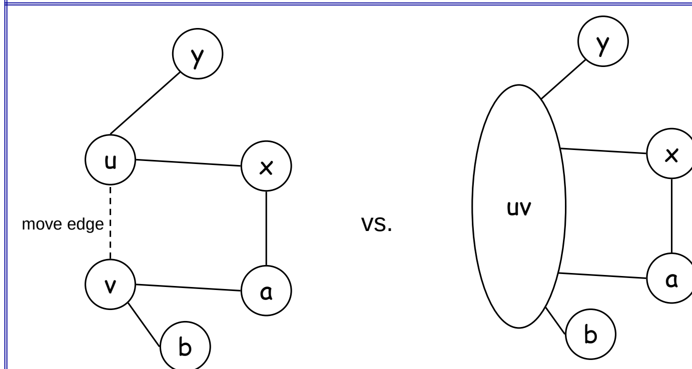
...merge u and v to form a single node



u and v are special:
A move whose source is not live-out of the move is a candidate for coalescing

That is, if the src and dest don't interfere

Is Coalescing Always Good?



2 colorable

And the winner is?

3 colorable

When should we coalesce?

Always

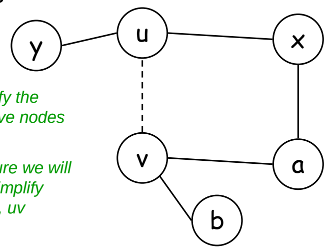
- **If we run into trouble start un-coalescing**
  - no nodes with degree < k, see if breaking up coalesced nodes fixes
- **yuck**

Only if we can prove it won't cause problems

- **Briggs: Conservative Coalescing**
- **George: Iterated Coalescing**

When we simplify the graph, we remove nodes of degree < k...

want to make sure we will still be able to simplify coalesced node, uv



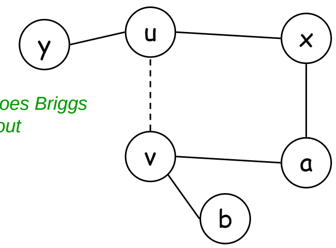
Briggs: Conservative Coalescing

- Can coalesce u and v if:
  - (# of neighbors of uv with degree ≥ k) < k
- Why?
  - *Simplify* pass removes all nodes with degree < k
  - # of remaining nodes < k
  - Thus, uv can be simplified

What does Briggs say about

k = 3?

k = 2?



George: Iterated Coalescing

Can coalesce u and v if

foreach neighbor t of u

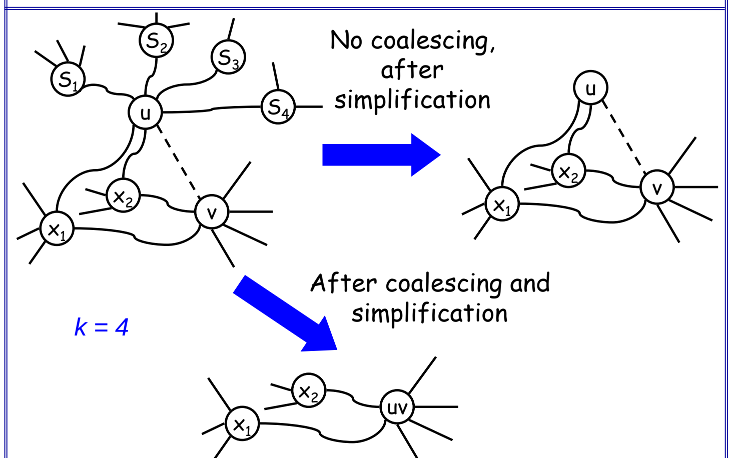
- t interferes with v, or, *doesn't change degree*
- degree of t < k *removed by simplification*

Resulting node uv will (after simplification) have degree equal to degree of v

Why?

- **let S be set of neighbors of u with degree < k**
- **If no coalescing, simplify removes all nodes in S, call that graph G^1**
- **If we coalesce we can still remove all nodes in S, call that graph G^2**
- **G^2 is a subgraph of G^1**

George: Iterated Coalescing



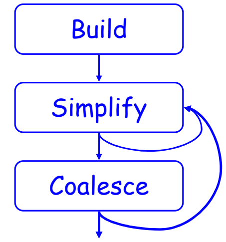
Why Two Methods?

- **Why not?**
- **With Briggs, one needs to look at all neighbors of a & b**
- **With George, only need to look at neighbors of a.**

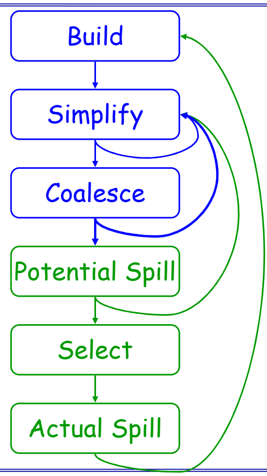
So:

- **Use George if one of a & b has very large degree**
- **Use Briggs otherwise**

Where We Are



Where We're Going



plus a bunch of important details...

Proto-Professor Algarth Zag, pioneer in fire research