Lecture 18

Instruction Scheduling

I. Hardware Support for Parallel Execution
II. Constraints on Scheduling
III. List Scheduling

[ALSU 10.1-10.3]

Optimization: *What's the Point?* (A Quick Review)

Machine-Independent Optimizations:

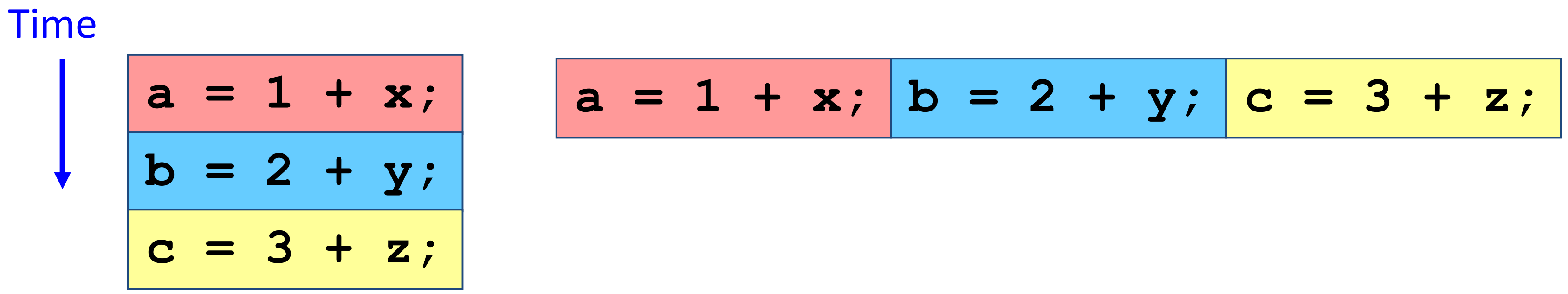
- e.g., constant propagation & folding, redundancy elimination, dead-code elimination, etc.
- Goal: *eliminate work*

Machine-Dependent Optimizations:

- register allocation, locality optimizations
  - Goal: *reduce cost of accessing data*
- instruction scheduling
  - Goal: *???*

The Goal of Instruction Scheduling

- Assume that the remaining instructions are all essential
  - (otherwise, earlier passes would have eliminated them)
- How can we perform this fixed amount of work in less time?
  - Answer: *execute the instructions in parallel*



I. Hardware Support for Parallel Execution

- Three forms of parallelism are found in modern machines:
  - Pipelining
  - Superscalar Processing
  - Multicore

} ← Instruction Scheduling (Pipelining, Superscalar Processing)

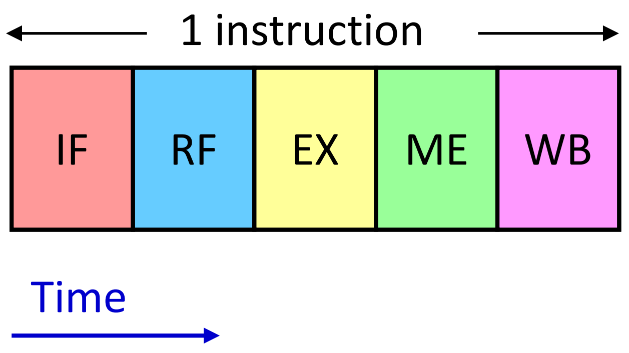
← Automatic Parallelization (Multicore)
[future lecture]

Pipelining

Basic idea:

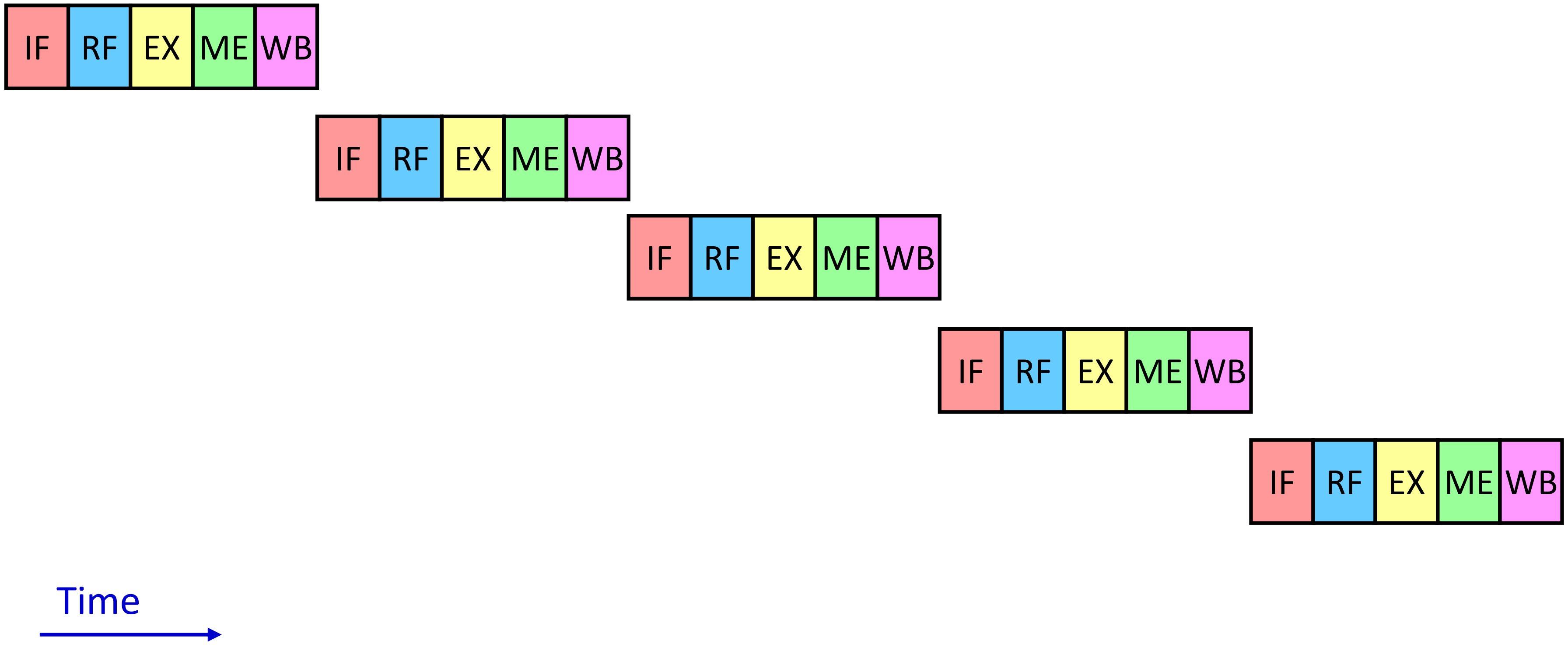
– break instruction into *stages* that can be overlapped

Example: simple 5-stage pipeline from early RISC machines



IF = Instruction Fetch
RF = Decode & Register Fetch
EX = Execute on ALU
ME = Memory Access
WB = Write Back to Register File

Pipelining Illustration



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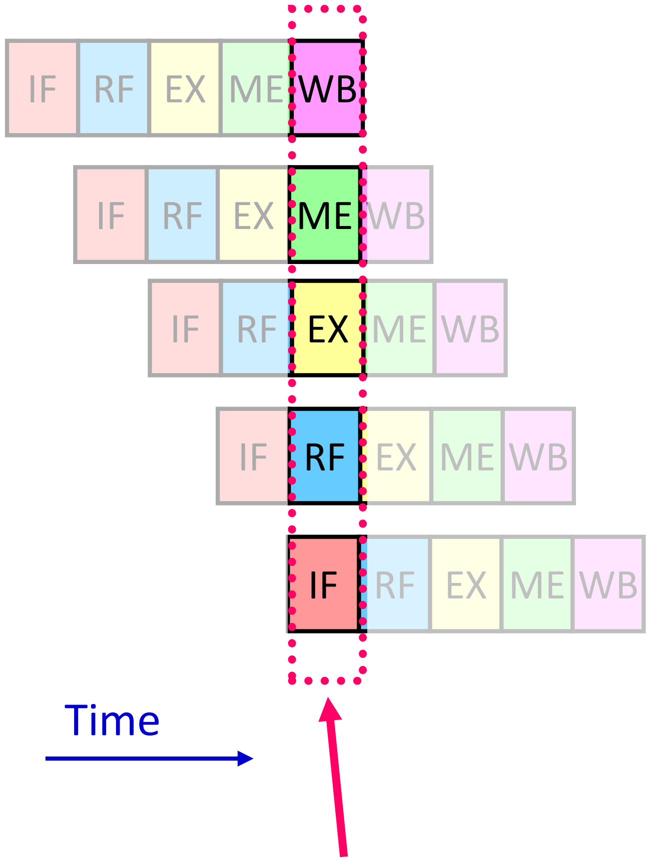
Pipelining Illustration



- In a given cycle, each instruction is in a different stage

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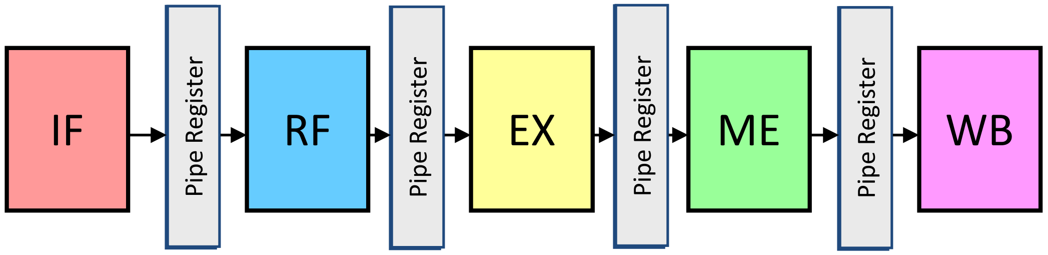
Beyond 5-Stage Pipelines: Even More Parallelism

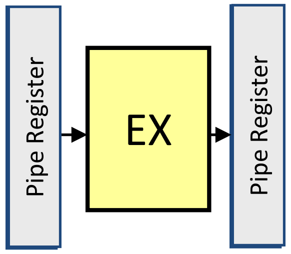
- Should we simply make pipelines deeper and deeper?



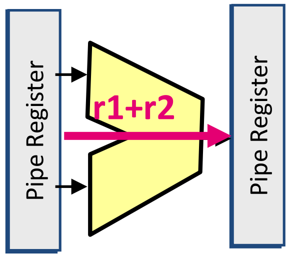
  - registers between pipeline stages have fixed overheads
    - hence diminishing returns with more stages (Amdahl's Law)
  - value of pipe stage unclear if it takes less time than an integer add
- However, many consumers think "performance = clock rate"
  - perceived need for higher clock rates -> deeper pipelines
  - e.g., Pentium 4 processor had a 20-stage pipeline [2000-2008]

Beyond Pipelining: "Superscalar" Processing

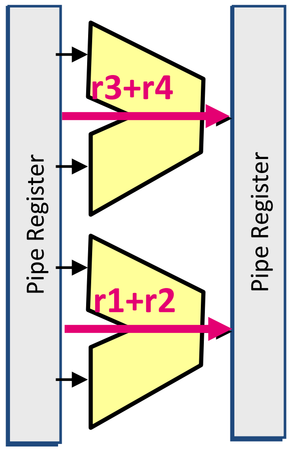
- Basic Idea:
  - multiple (independent) instructions can proceed simultaneously through the same pipeline stages
- Requires additional hardware
  - example: "Execute" stage



Abstract Representation

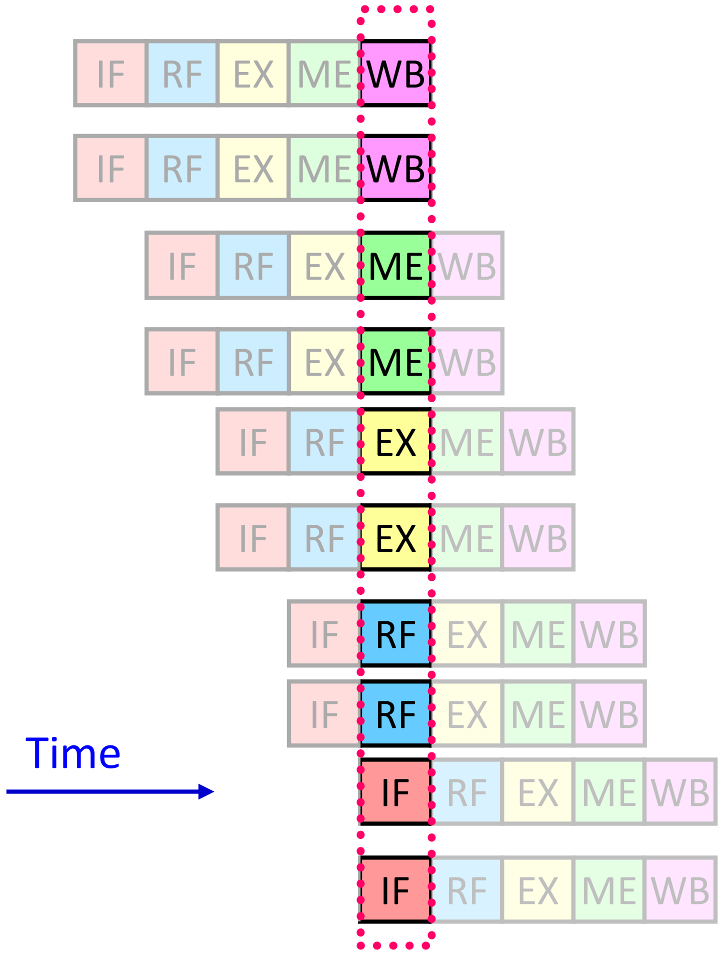


Hardware for Scalar Pipeline: 1 ALU



Hardware for 2-way Superscalar: 2 ALUs

Superscalar Pipeline Illustration



Original (scalar) pipeline:

- Only one instruction in a given pipe stage at a given time

Superscalar pipeline:

- Multiple instructions in the same pipe stage at the same time
- Unlike SIMD/vector instructions, instructions of different types can be in the same pipe stage at same time

II. Constraints on Scheduling

1. Hardware Resources
2. Data Dependences
3. Control Dependences

Constraint #1: Hardware Resources

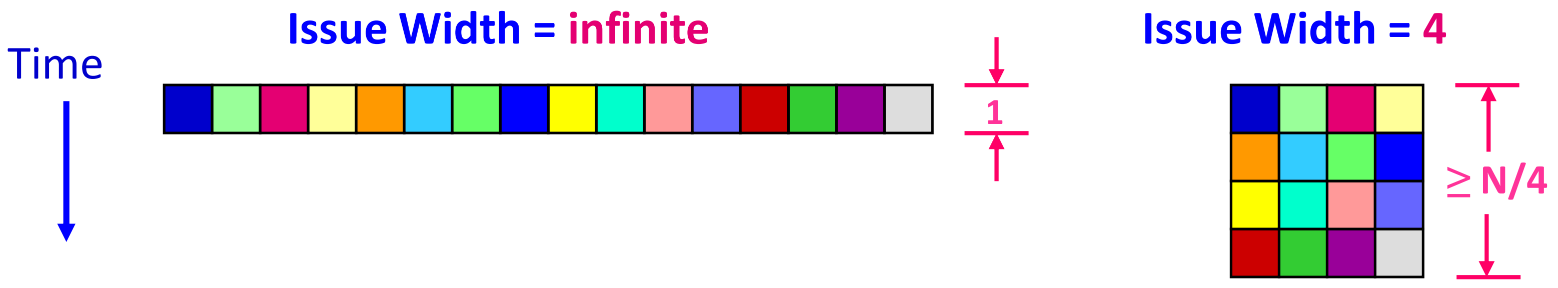
- Processors have finite resources, and there are often constraints on how these resources can be used.

Examples:

- Finite issue width
- Limited functional units (FUs) per given instruction type
- Limited pipelining within a given functional unit (FU)

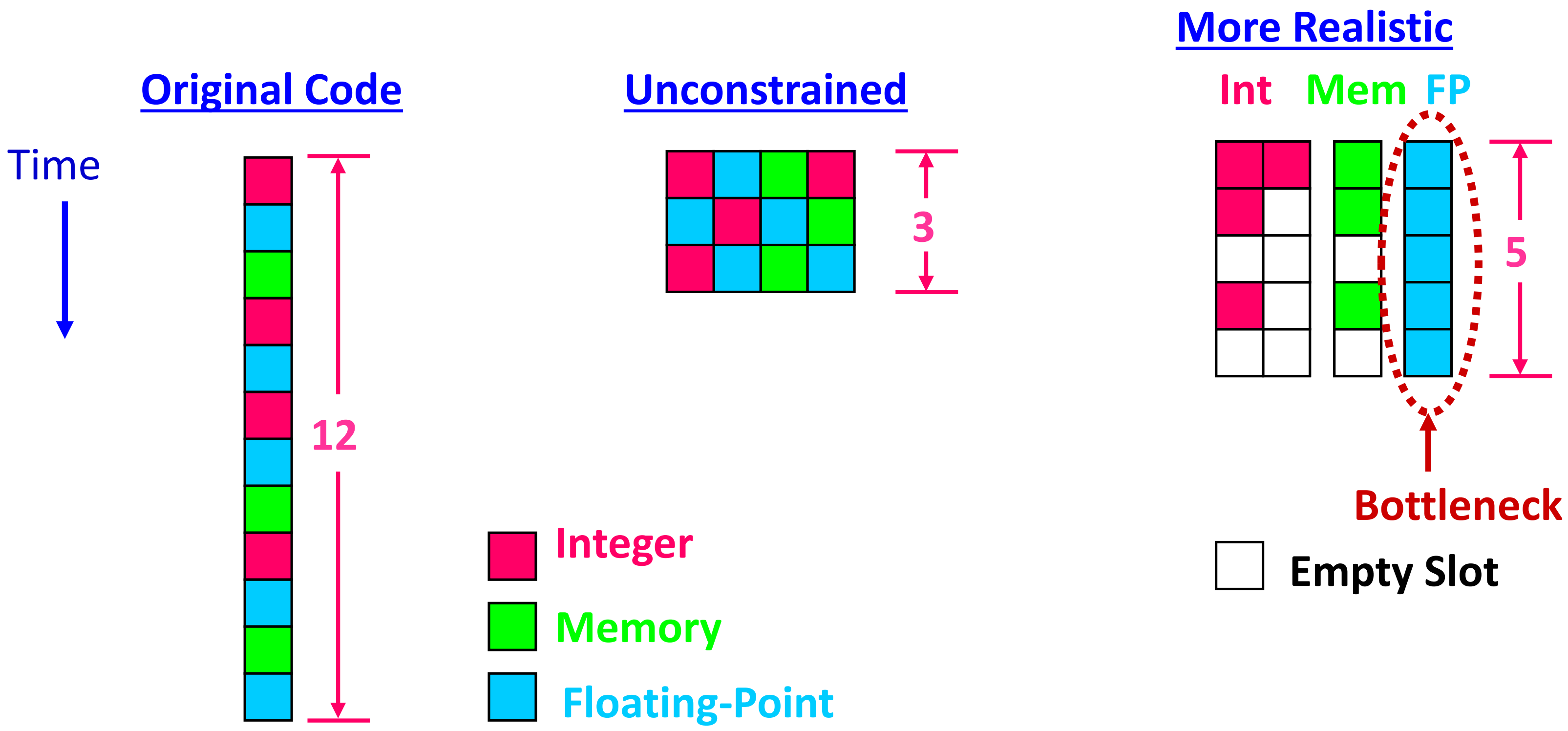
Finite Issue Width

- Prior to superscalar processing:
  - processors only “issued” one instruction per cycle
- Even with superscalar processing:
  - limit on total # of instructions issued per cycle



Limited FUs per Instruction Type

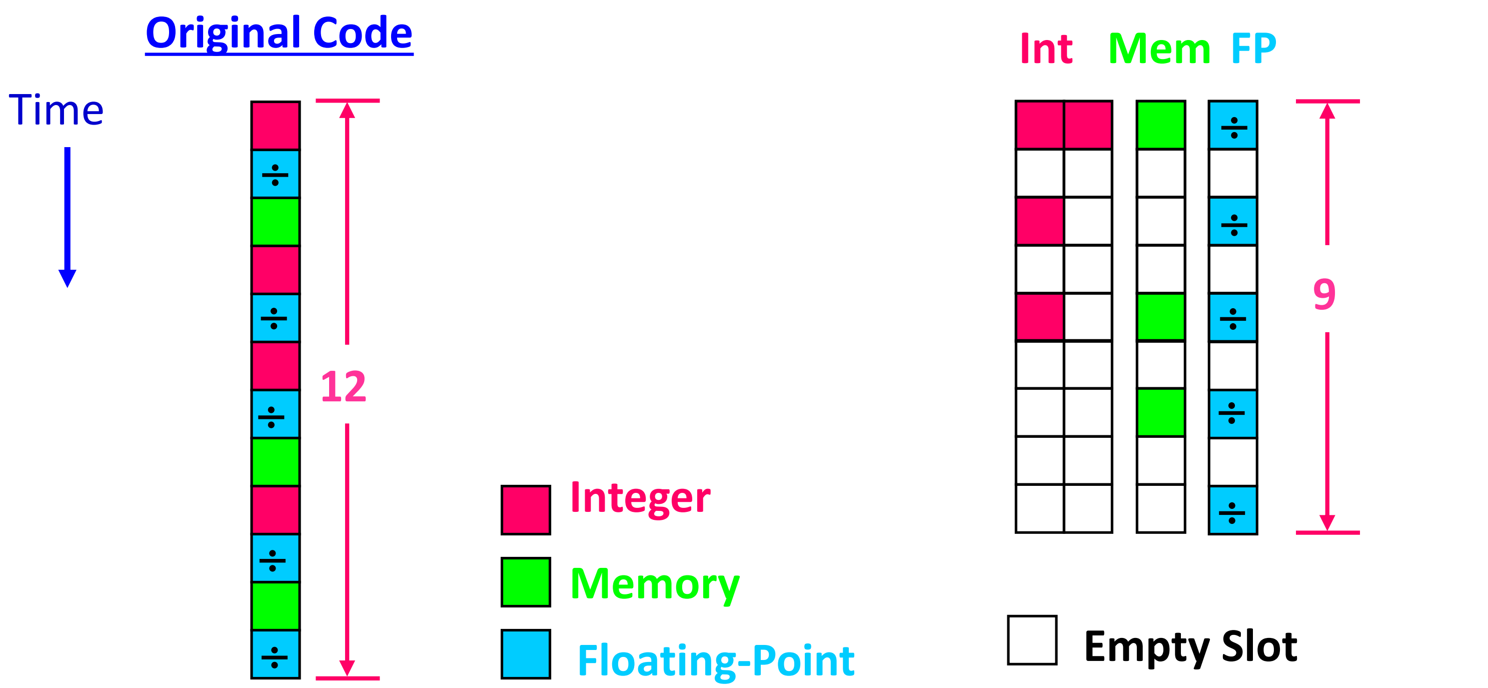
- e.g., a 4-way superscalar might only be able to issue up to 2 integer, 1 memory, and 1 floating-point insts per cycle



🟥 Integer
🟩 Memory
🟦 Floating-Point
☐ Empty Slot

Limited Pipelining within a Functional Unit

- e.g., only 1 new floating-point division once every 2 cycles



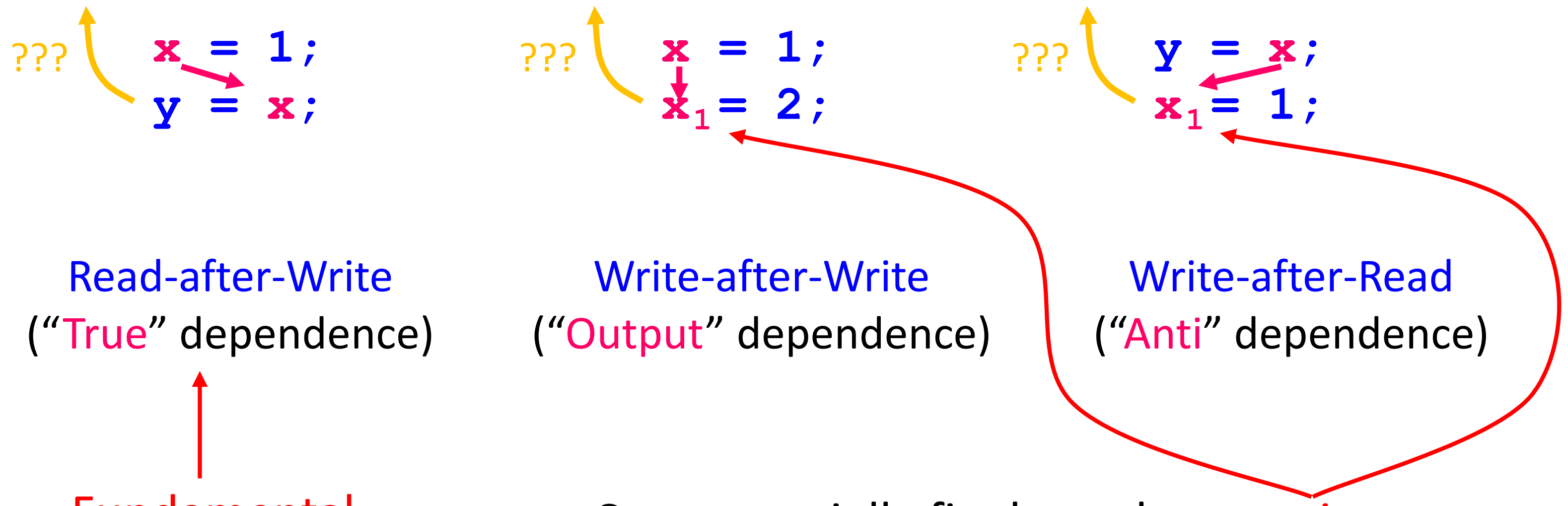
Constraints on Scheduling

1. Hardware Resources
2. Data Dependences
3. Control Dependences

Constraint #2: Data Dependences

- If we read or write a data location "too early", the program may behave incorrectly.

(Assume that initially, `x = 0`*.)*



```
x = 1;
y = x;
```

```
x = 1;
x1= 2;
```

```
y = x;
x1= 1;
```

Read-after-Write
("True" dependence)

Write-after-Write
("Output" dependence)

Write-after-Read
("Anti" dependence)

Fundamental
(no simple fix)

Can potentially fix through *renaming*.

Why Data Dependences are Challenging

- Which of these instructions can be reordered?

```
 x = a[i];
*p = 1;
 y = *q;
*r = z;
```

- *ambiguous data dependences* are very common in practice
  - difficult to resolve, despite fancy pointer analysis [earlier lecture]

Given Ambiguous Data Dependences, What To Do?

```
 x = a[i];
*p = 1;
 y = *q;
*r = z;
```

- Conservative approach: don’t reorder instructions
  - ensures correct execution
  - but may suffer poor performance
- Aggressive approach?
  - is there a way to safely reorder instructions?

Hardware Limitations: Multi-cycle Execution Latencies

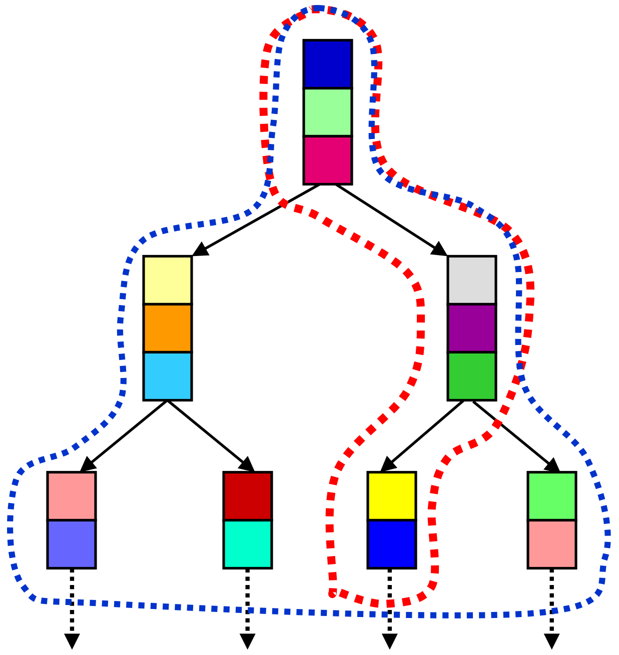
- Simple instructions often "execute" in one cycle
  - (as observed by other instructions in the pipeline)
  - e.g., integer addition
- More complex instructions may require multiple cycles
  - e.g., integer division, square-root
  - cache misses!

- These latencies, when combined with data dependencies, can result in non-trivial critical path lengths through code

Constraints on Scheduling

1. Hardware Resources
2. Data Dependences
3. Control Dependences

Constraint #3: Control Dependences



- What do we do when we reach a conditional branch?
  - choose a “frequently-executed” path?
  - choose multiple paths?

Scheduling Constraints: Summary

- Hardware Resources
  - finite set of FUs with instruction type, bandwidth, and latency constraints
  - cache hierarchy also has many constraints
- Data Dependences
  - can’t consume a result before it is produced
  - ambiguous dependences create many challenges
- Control Dependences
  - impractical to schedule for all possible paths
  - choosing an “expected” path may be difficult
    - recovery costs can be non-trivial if you are wrong

III. List Scheduling

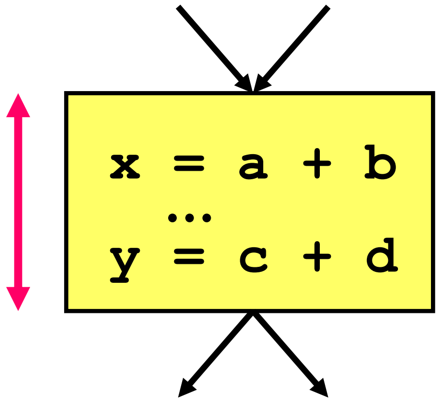
- The most common technique for scheduling instructions within a basic block

Basic block scheduling doesn't need to worry about:

- control flow [topic of future lecture]

Does need to worry about:

- data dependences
- hardware resources



- Even without control flow, the problem is still NP-hard

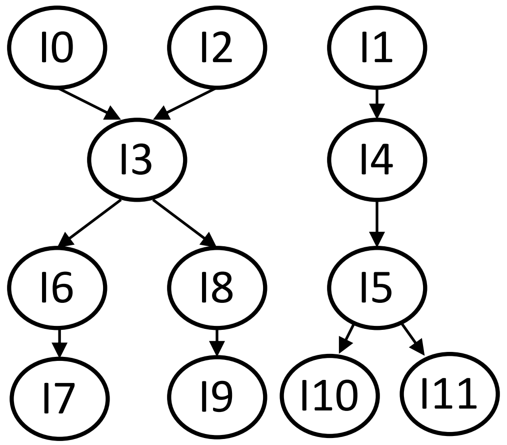
List Scheduling Algorithm: Inputs and Outputs

Algorithm reproduced from:

- *"An Experimental Evaluation of List Scheduling"*, Keith D. Cooper, Philip J. Schielke, and Devika Subramanian. Rice University, Dept of Computer Science Tech. Rep. 98-326, 1998.
- *"Despite the importance of scheduling, we know quite little about the behavior of list scheduling—the most widely used technique for instruction scheduling [1, 3]."*

Inputs:

Data Precedence Graph (DPG)



Machine Parameters

# of FUs:
2 INT, 1 FP
Latencies:
add = 1 cycle, …
Pipelining:
1 add/cycle, …

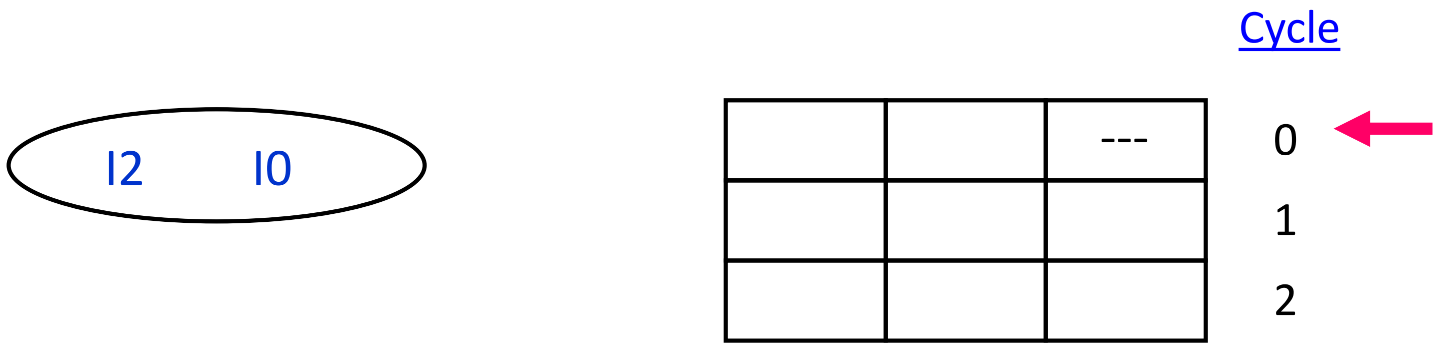
Output:

Scheduled Code

ALU 0	ALU 1	FP	Cycle
I0	I2	---	0
---	I1	I3	1
I4	I8	I6	2
I5	---	I9	3
I7	I10	I11	4

List Scheduling: The Basic Idea

- Maintain a list of instructions that are ready to execute
  - data dependence constraints would be preserved
  - machine resources are available
- Moving cycle-by-cycle through the schedule template:
  - choose instructions from the list & schedule them
  - update the list for the next cycle



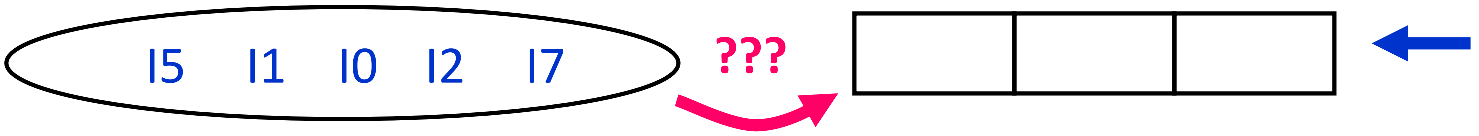
What Makes Life Interesting: Choice

Easy case:

- all ready instructions can be scheduled this cycle

Interesting case:

- we need to pick a subset of the ready instructions

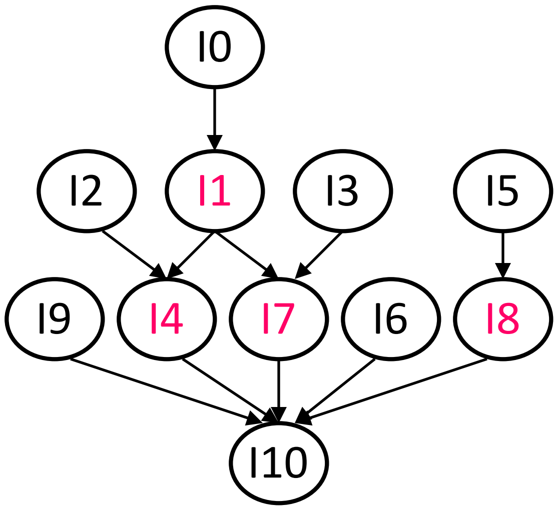


- List scheduling makes choices based upon *priorities*
  - assigning priorities correctly is a key challenge

List Scheduling Example

Suppose: Assign priorities based on instruction number

```
I0:  a = 1
I1:  f = a + x
I2:  b = 7
I3:  c = 9
I4:  g = f + b
I5:  d = 13
I6:  e = 19
I7:  h = f + c
I8:  j = d + y
I9:  z = -1
I10: JMP L1
```

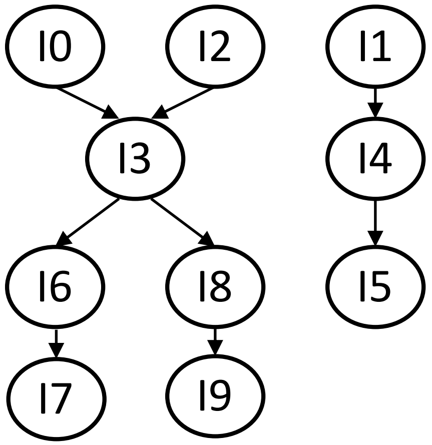


		Cycle
		0
		1
		2
		3
		4
		5
		6

- 2 identical fully-pipelined FUs
- adds take 2 cycles; all other insts take 1 cycle

Intuition Behind Priorities

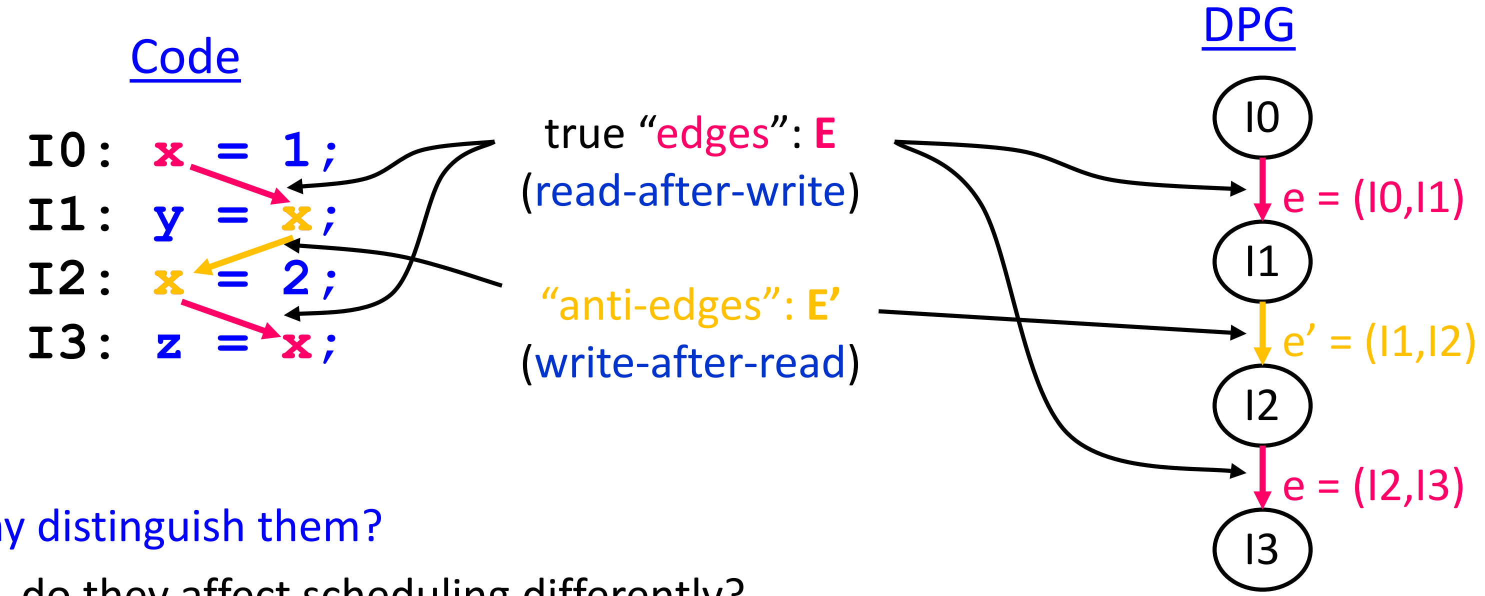
- Intuitively, what should the priority correspond to?
- What factors are used to compute it?
  - data dependences?
  - machine parameters?


$$\begin{tikzcd}
I0 \arrow[dr] & & I2 \arrow[dl] & I1 \arrow[d] \\
& I3 \arrow[dl] \arrow[dr] & & I4 \arrow[d] \\
I6 \arrow[d] & & I8 \arrow[d] & I5 \\
I7 & & I9 &
\end{tikzcd}$$

# of FUs:
    2 INT, 1 FP
Latencies:
    add = 1 cycle, …
Pipelining:
    1 add/cycle, …

Representing Data Dependences: The Data Precedence Graph (DPG)

- Two different kinds of edges:



```
I0: x = 1;
I1: y = x;
I2: x = 2;
I3: z = x;
```

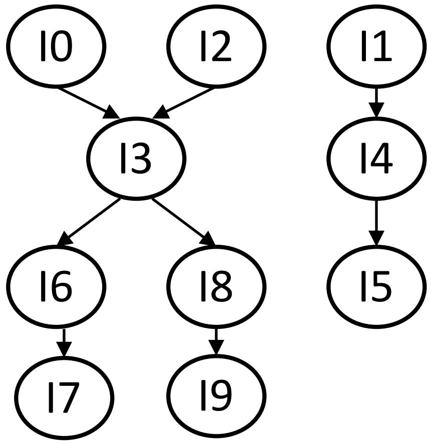
- Why distinguish them?
  - do they affect scheduling differently?

- What about output dependences?

Computing Priorities

- Let’s start with just true dependences (i.e. “edges” in DPG)
- Priority = *latency-weighted depth* in the DPG

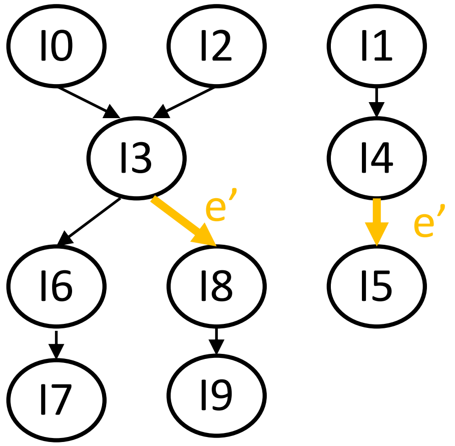
$$priority(x) = max(\forall_{l \in leaves(DPG)} \forall_{p \in paths(x,...,l)} \sum_{p_i = x}^{l} latency(p_i))$$



Computing Priorities (Cont.)

- Now let's also take anti-dependences into account
  - i.e. anti-edges in the set E'

$$priority(x) = \begin{cases} latency(x) & \text{if } x \text{ is a leaf} \\ max(latency(x) + max_{(x,y)\in E}(priority(y)), & \\ \quad max_{(x,y)\in E'}(priority(y))) & otherwise. \end{cases}$$



List Scheduling Algorithm

```
cycle = 0;
ready-list = root nodes in DPG;
inflight-list = {};
                                                          ties?
while (|ready-list|+|inflight-list| > 0) {
    for op = (all nodes in ready-list in decreasing priority order) {
        if (an FU exists for op to start at cycle) {
            remove op from ready-list and add to inflight-list;
            add op to schedule at time cycle;
            if (op has an outgoing anti-edge)
                add all targets of op's anti-edges that are ready to ready-list;
        }
    }
    cycle = cycle + 1;
    for op = (all nodes in inflight-list)
        if (op finishes at time cycle) {
            remove op from inflight-list;
            check nodes waiting for op &
                                add to ready-list if all operands available;
        }
    }
}
```

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List Scheduling Example

$$priority(x) = \begin{cases} latency(x) & \text{if } x \text{ is a leaf} \\ max(latency(x) + max_{(x,y)\in E}(priority(y)), \\ \quad max_{(x,y)\in E'}(priority(y))) & otherwise. \end{cases}$$

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I5:  d = 13
I6:  e = 19
I7:  h = f + c
I8:  j = d + y
I9:  z = -1
I10: JMP L1
```

$$\begin{tikzcd}
 & & I0 \arrow[d] & & \\
 & I2 \arrow[d] & I1 \arrow[dl] \arrow[d] & I3 \arrow[dl] & I5 \arrow[d] \\
I9 \arrow[drr] & I4 \arrow[dr] & I7 \arrow[d] & I6 \arrow[dl] & I8 \arrow[dll] \\
 & & I10 & &
\end{tikzcd}$$

		Cycle
		0
		1
		2
		3
		4
		5
		6

- 2 identical fully-pipelined FUs
- adds take 2 cycles; all other insts take 1 cycle

Break ties by lower instruction number

What if break ties differently?

$$priority(x) = \begin{cases} latency(x) & \text{if } x \text{ is a leaf} \\ max(latency(x) + max_{(x,y)\in E}(priority(y)), & \\ \quad max_{(x,y)\in E'}(priority(y))) & otherwise. \end{cases}$$

```
I0:  a = 1
I1:  f = a + x
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I6:  e = 19
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I8:  j = d + y
I9:  z = -1
I10: JMP L1
```

$$\begin{tikzcd}
 & & I0^{6} \arrow[d] & & \\
I2^{4} \arrow[dr] & & I1^{5} \arrow[dl] \arrow[dr] & I3^{4} \arrow[dl] & I5^{4} \arrow[d] \\
I9^{2} \arrow[drr] & I4^{3} \arrow[dr] & I7^{3} \arrow[d] & I6^{2} \arrow[dl] & I8^{3} \arrow[dll] \\
 & & I10^{1} & &
\end{tikzcd}$$

		Cycle
		0
		1
		2
		3
		4
		5
		6

- 2 identical fully-pipelined FUs
- adds take 2 cycles; all other insts take 1 cycle

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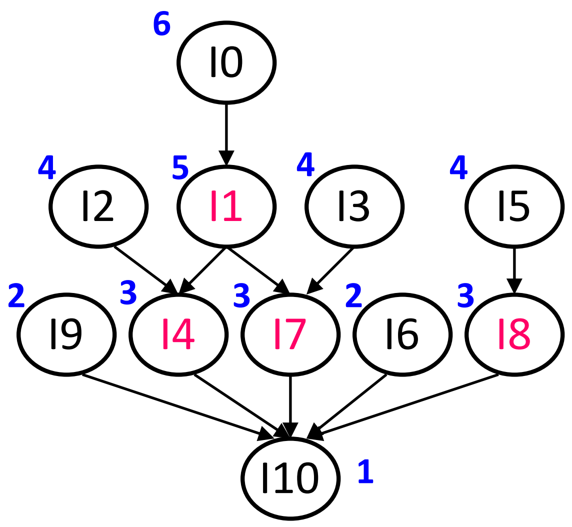
Contrasting the Two Schedules



		Cycle
I0	I2	0
I1	I3	1
I5	I6	2
I4	I7	3
I8	I9	4
--	--	5
I10	--	6

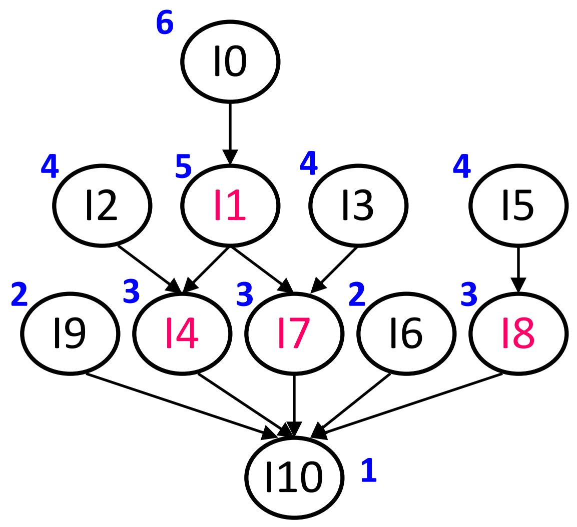
		Cycle
I0	I2	0
I1	I5	1
I3	I8	2
I4	I7	3
I6	I9	4
I10	--	5

- Breaking ties arbitrarily may not be the best approach

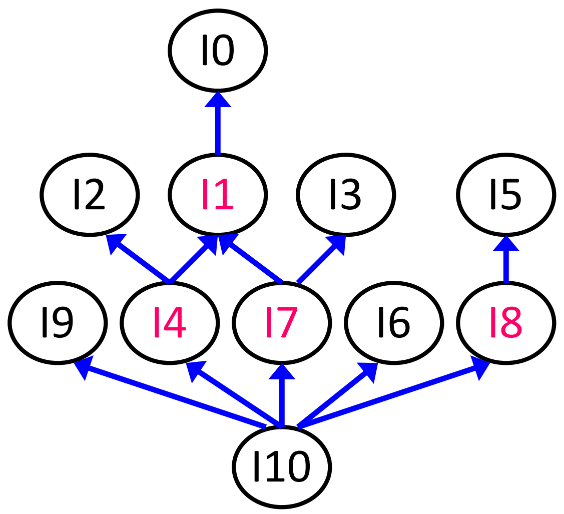
Backward List Scheduling

Modify the algorithm as follows:

- reverse the direction of all edges in the DPG
- schedule the *finish times* of each operation
  - start times must still be used to ensure Functional Unit availability



Forward Scheduling Priorities
(build up priorities upwards, schedule downwards)



Backward Scheduling Priorities
(build up priorities downwards, schedule upwards)

Backward List Scheduling

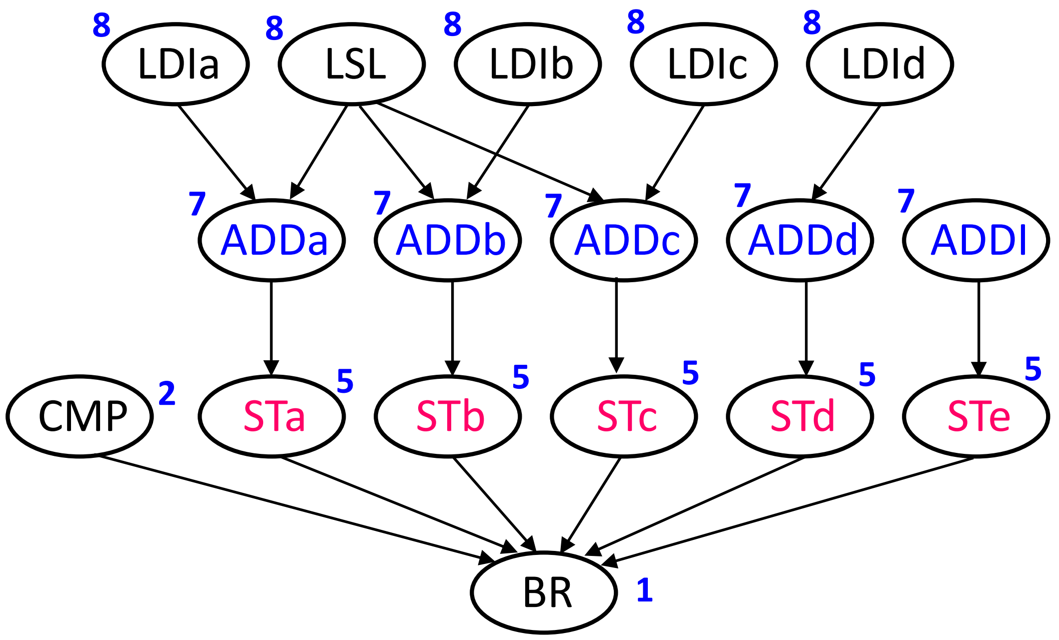
Modify the algorithm as follows:

- reverse the direction of all edges in the DPG
- schedule the *finish times* of each operation
  - start times must still be used to ensure FU availability

Impact of scheduling backwards:

- clusters operations near the end (vs. the beginning)
- may be either better or worse than forward scheduling

Backward List Scheduling Example: Let's Schedule it Forward First



INT	INT	MEM	Cycle
			0
			1
			2
			3
			4
			5
			6
			7
			8
			9
			10
			11
			12

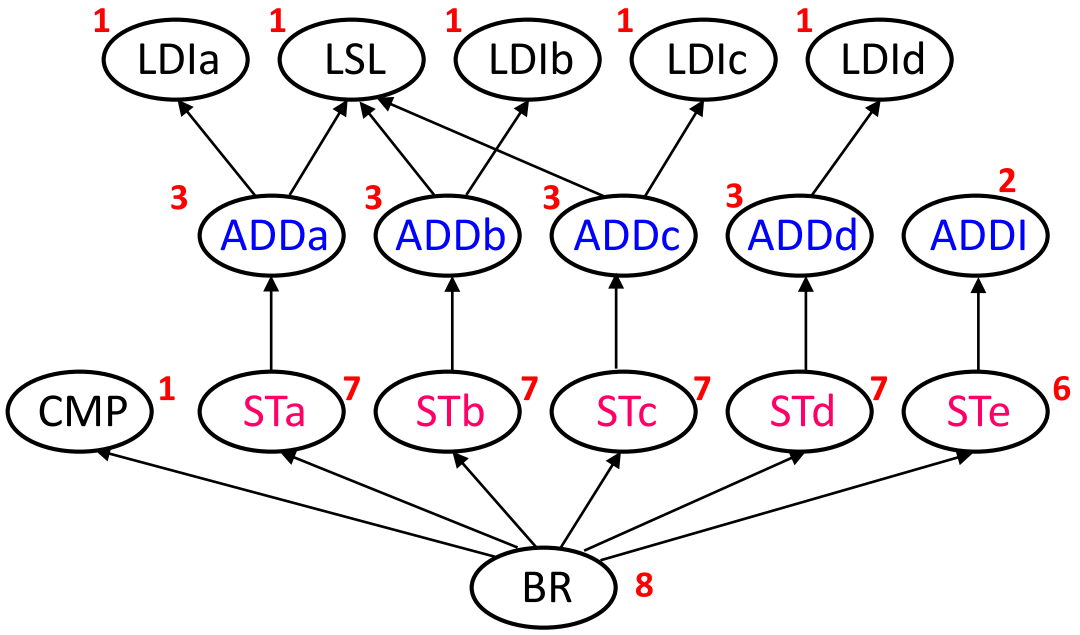
Hardware parameters:

- 2 INT units: ADDs take 2 cycles; others take 1 cycle
- 1 MEM unit: stores (ST) take 4 cycles

Break ties left-to-right in above dag

LDI=load immediate, LSL=logical shift left

Now Let's Try Scheduling Backward



INT	INT	MEM	Cycle
			0
			1
			2
			3
			4
			5
			6
			7
			8
			9
			10
			11

Hardware parameters:

- 2 INT units: ADDs take 2 cycles; others take 1 cycle
- 1 MEM unit: stores (ST) take 4 cycles

Break ties left-to-right in above dag

Contrasting Forward vs. Backward List Scheduling

Forward

INT	INT	MEM	Cycle
LDIa	LSL	----	0
LDIb	LDIc	----	1
LDId	ADDa	----	2
ADDb	ADDc	----	3
ADDd	ADDI	STa	4
CMP	----	STb	5
----	----	STc	6
----	----	STd	7
----	----	STe	8
----	----	----	9
----	----	----	10
----	----	----	11
BR	----	----	**12**

Backward

INT	INT	MEM	Cycle
LDId	----	----	0
ADDI	LDIc	----	1
ADDd	LSL	----	2
ADDc	LDIb	STe	3
ADDb	LDIa	STd	4
ADDa	----	STc	5
----	----	STb	6
----	----	STa	7
----	----	----	8
----	----	----	9
CMP	----	----	10
BR	----	----	**11**

- backward scheduling clusters work near the end
- backward is better in this case, but this is not always true

Evaluation of List Scheduling

Cooper *et al.* propose "RBF" scheduling:

- schedule each block M times forward & backward
- break any priority ties randomly

For real programs:

- regular list scheduling works very well

For synthetic blocks:

- RBF wins when "available parallelism" (AP) is ~2.5
- for smaller AP, scheduling is too constrained
- for larger AP, any decision tends to work well

List Scheduling Wrap-Up

- The priority function can be arbitrarily sophisticated
  - e.g., filling branch delay slots in early RISC processors
- List scheduling is widely used for instruction scheduling on in-order processors, and it works fairly well
- However, it has two limitations:
  - It schedules only within a basic block
    - An upcoming lecture will cover global scheduling
  - Modern out-of-order processors perform their own dynamic scheduling
    - List scheduling can be used to feed the dynamic scheduler in a good order

List Scheduling Wrap-Up

"An Experimental Evaluation of List Scheduling", Cooper, Schielke, Subramanian.

"Despite the importance of scheduling, we know quite little about the behavior of list scheduling—the most widely used technique for instruction scheduling [1, 3]."

Today's Class: Instruction Scheduling

I. Hardware Support for Parallel Execution
II. Constraints on Scheduling
III. List Scheduling

Coming Attractions

- Wednesday: No class. Project discussions 11 am – 5 pm
- Friday: In-class discussion of Assignment 3
- Monday: No class. Project Proposals due midnight
- Wednesday: Region-based Analysis
- Friday: Instruction Scheduling – the sequel