
WELCOME
TO
15-213
14-513
18-213
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Floating Point

18-213/18-613: Introduction to Computer Systems
3rd Lecture, September 6th, 2022

Announcements

- **Lab 0 and Lab 1 are live (on Autolab)**
  - Lab 0's first deadline is today (Be warned if you miss it)
  - Lab 0's final deadline is Mon, Sep 26th (Cachelab out that Thurs)
- **Homework #1 is at its usual cadence**
  - HW #1 is due this Thursday
  - HW #2 goes out today, is due 1 week from this Thursday, and covers this week's material (today and Thursday)
- **Roster (Canvas, OH, Autolab) update nightly**
  - OH and Autolab may lag Canvas by a day.
- **Small groups should be meeting this week, just at an alternative time and location**
  - We **do** have real space to meet physically starting next week.

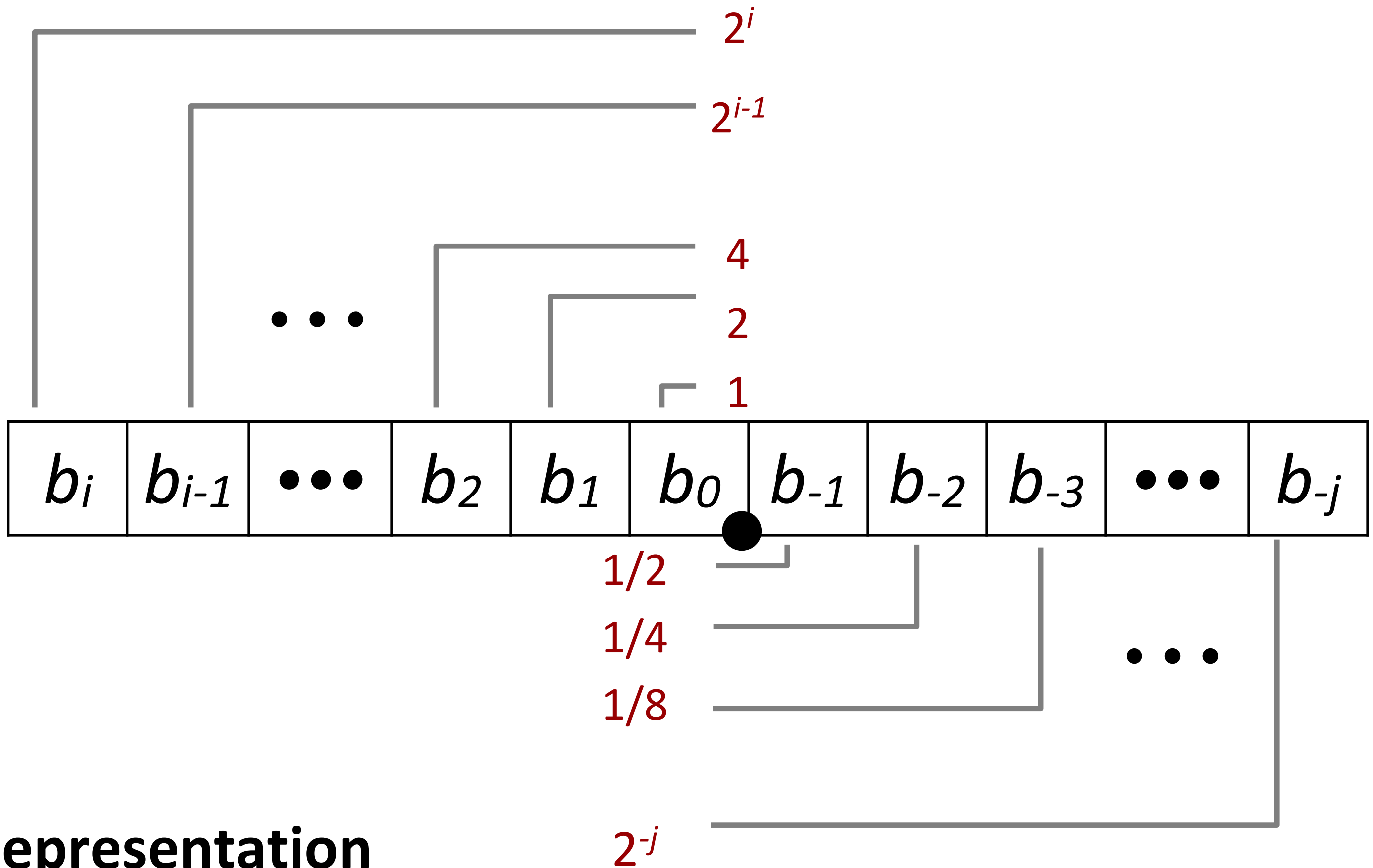
Today: Floating Point

- Background: Fractional binary numbers — CSAPP 2.4.1
- IEEE floating point standard: Definition — CSAPP 2.4.2
- Example and properties — CSAPP 2.4.3
- Rounding, addition, multiplication — CSAPP 2.4.4-2.4.5
- Floating point in C — CSAPP 2.4.6
- Summary

Fractional binary numbers

- What is 00101.110_2?

Fractional Binary Numbers



- **Representation**
  - Bits to right of "binary point" represent fractional powers of 2
  - Represents rational number:

$$\sum_{k=-j}^{i} b_k \times 2^k$$

Fractional Binary Numbers: Examples

- **Value** / **Representation**

Value		Representation	
23		10111.000_2	= 16 + 4 + 2 + 1
11 1/2	= 23/2	01011.100_2	= 8 + 2 + 1 + 1/2
5 3/4	= 23/4	00101.110_2	= 4 + 1 + 1/2 + 1/4
2 7/8	= 23/8	00010.111_2	= 2 + 1/2 + 1/4 + 1/8

- **Observations**
  - Divide by 2 by shifting right (unsigned)
  - Multiply by 2 by shifting left
  - Numbers of form $0.111111\ldots_2$ are just below 1.0
    - $1/2 + 1/4 + 1/8 + \ldots + 1/2^i + \ldots \rightarrow 1.0$
    - Use notation $1.0 - \varepsilon$

Representable Numbers

- **Limitation #1**
  - Can only exactly represent numbers of the form $x/2^k$
    - Other rational numbers have repeating bit representations
  - Value Representation
    - 1/3 `0.0101010101[01]`…2
    - 1/5 `0.001100110011[0011]`…2
    - 1/10 `0.0001100110011[0011]`…2
- **Limitation #2**
  - Just one setting of binary point within the *w* bits
    - Limited range of numbers (very small values? very large?)

(Binary) Scientific Notation

- What are the parts of a number in scientific notation?

$$1.1101101101101_2 \times 2^{13}$$

Significand

Exponent

- What value does the significand always begin with in scientific notation?

Floating Point Representation

Example:
15213_{10} = $(-1)^0$ x 1.1101101101101_2 x 2^{13}

- **Numerical Form:**

$$(-1)^s \cdot M \cdot 2^E$$

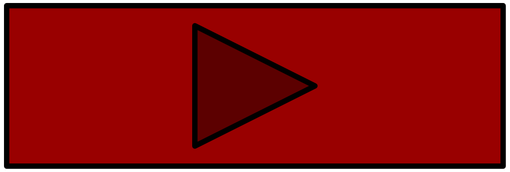
  - **Sign bit *s*** determines whether number is negative or positive
  - **Significand *M*** normally a fractional value in range [1.0,2.0).
  - **Exponent *E*** weights value by power of two

- **Encoding**
  - MSB s is sign bit ***s***
  - **exp** field encodes ***E*** (but is not equal to E)
  - **frac** field encodes ***M*** (but is not equal to M)

s	exp	frac

Floats can’t represent all real numbers!

- **Ariane 5 explodes on maiden voyage: $500 MILLION dollars lost**
  - 64-bit floating point number assigned to 16-bit integer (1996)
  - Legacy code from Ariane 4 with a lower top speed
  - Causes rocket to get incorrect value of horizontal velocity and crash



- **Patriot Missile defense system misses scud – 28 people die**
  - System tracks time in tenths of second
  - Converted from integer to floating point number.
  - Accumulated rounding error causes drift. 20% drift over 8 hours.
  - Eventually (on 2/25/1991 system was on for 100 hours) causes range mis-estimation sufficiently large to miss incoming missiles.

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IEEE Floating Point

- **IEEE Standard 754**
  - Established in 1985 as uniform standard for floating point arithmetic
    - Before that, many idiosyncratic formats
  - Supported by all major CPUs
    - Some specialized CPUs don't implement IEEE 754 in full

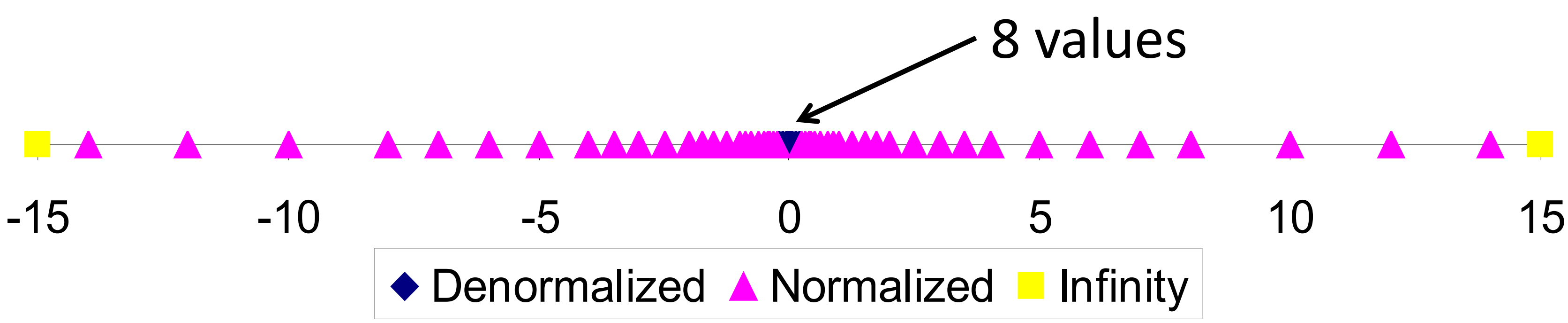
- **Driven by numerical concerns**
  - Nice standards for rounding, overflow, underflow
  - Hard to make fast in hardware
    - Numerical analysts predominated over hardware designers in defining standard

IEEE 754 Floating Point: Goal

- **6-bit IEEE-like format**
  - e = 3 exponent bits
  - f = 2 fraction bits
  - Bias is $2^{3-1}-1 = 3$

s	exp	frac
1	3-bits	2-bits

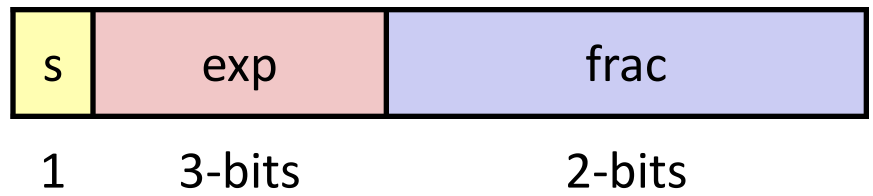
- **Notice how the distribution gets denser toward zero.**

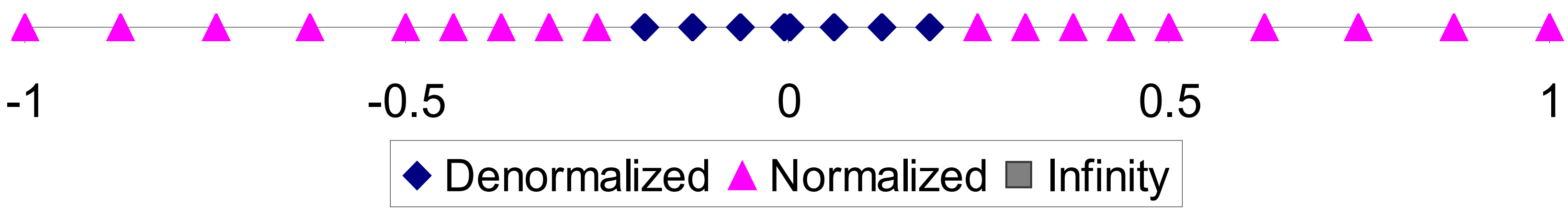


Wide range and precision where it is needed most

Distribution of Values (close-up view)

- **6-bit IEEE-like format**
  - e = 3 exponent bits
  - f = 2 fraction bits
  - Bias is 3





Increasingly bigger steps for large numbers

Dense packing for small numbers

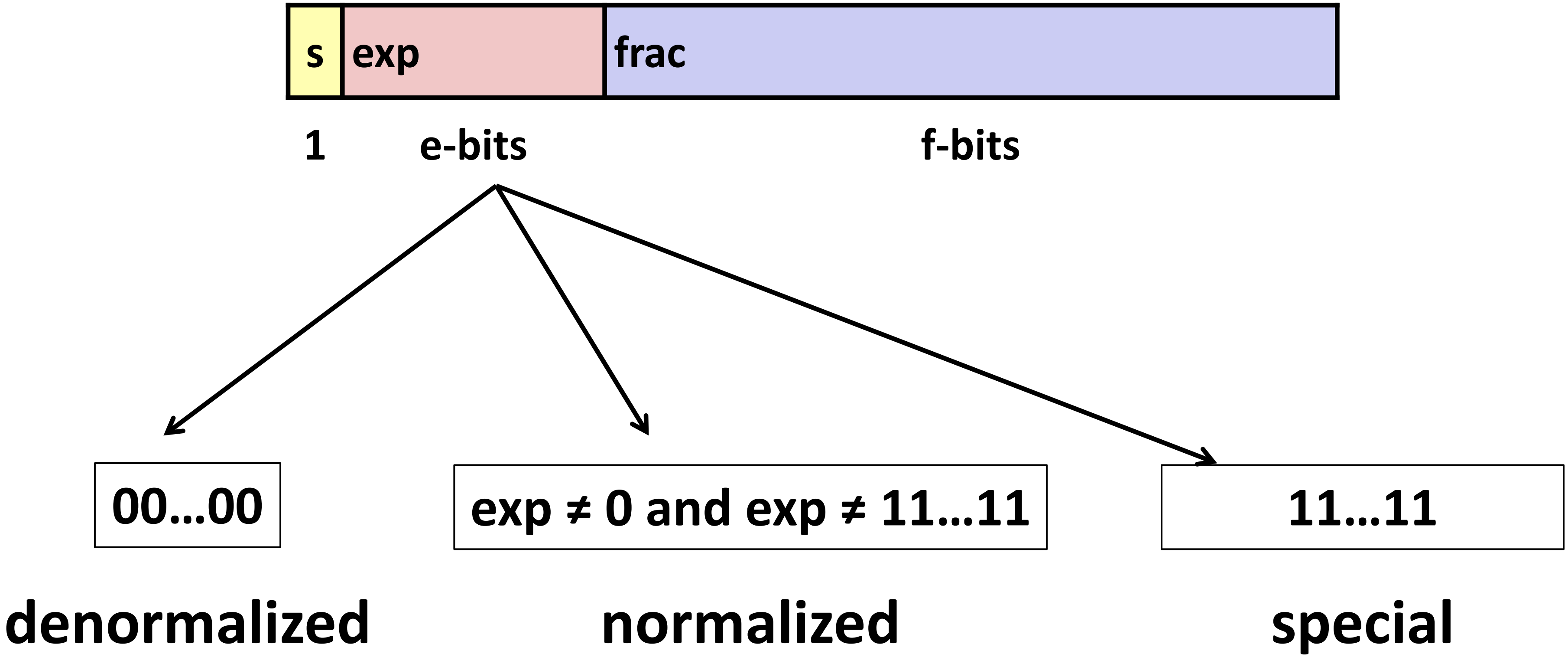
Precision options

- **Single precision: 32 bits**
  ≈ 7 decimal digits, $10^{\pm 38}$

- **Double precision: 64 bits**
  ≈ 16 decimal digits, $10^{\pm 308}$

- **Other formats: half precision, quad precision**

Three “kinds” of floating point numbers



"Normalized" Values

$v = (-1)^s \, M \, 2^E$

- **When: `exp` ≠ 000…0 and `exp` ≠ 111…1**

- **Exponent coded as a *biased* value: E = `exp` – *Bias***
  - *exp*: unsigned value of exp field
  - *Bias* = $2^{k-1} - 1$, where k is number of exponent bits
    - Single precision: 127 (**`exp`**: 1…254, E: -126…127)
    - Double precision: 1023 (**`exp`**: 1…2046, E: -1022…1023)

- **Significand coded with implied leading 1: M = $1.xxx…x_2$**
  - xxx…x: bits of frac field
  - Minimum when **`frac`**=000…0 (M = 1.0)
  - Maximum when **`frac`**=111…1 (M = 2.0 – ε)
  - Get extra leading bit for "free"

Normalized Encoding Example

$v = (-1)^s M\, 2^E$
$E = \texttt{exp} - Bias$

- **Value:** `float F = 15213.0;`
  - $15213_{10} = 11101101101101_2$
    $= 1.1101101101101_2 \times 2^{13}$

- **Significand**

  M = `1.`$\underline{\texttt{1101101101101}}_2$

  `frac`= $\underline{\texttt{1101101101101}}\texttt{0000000000}_2$

- **Exponent**

  E = 13

  $Bias$ = 127

  `exp` = 140 = $\texttt{10001100}_2$

- **Result:**

s	exp	frac
0	10001100	11011011011010000000000

Denormalized Values

$v = (-1)^s\, M\, 2^E$
$E = 1 - Bias$

- **Condition:** exp = 000…0
- **Exponent value:** $E = 1 - \text{Bias}$ **(instead of `exp` –** ***Bias*****)** (why?)
- **Significand coded with implied leading 0:** $M = 0.\text{xxx}\ldots\text{x}_2$
  - `xxx`…`x`: bits of `frac`
- **Cases**
  - `exp` = 000…0, `frac` = 000…0
    - Represents zero value
    - Note distinct values: +0 and –0 (why?)
  - `exp` = 000…0, `frac` ≠ 000…0
    - Numbers closest to 0.0
    - Equispaced

Special Values

- **Condition: `exp` = `111`…`1`**

- **Case: `exp` = `111`…`1`, `frac` = `000`…`0`**
  - **Represents value ∞ (infinity)**
  - Operation that overflows
  - Both positive and negative
  - E.g., $1.0/0.0 = -1.0/-0.0 = +\infty$, $1.0/-0.0 = -\infty$

- **Case: `exp` = `111`…`1`, `frac` ≠ `000`…`0`**
  - **Not-a-Number (NaN)**
  - Represents case when no numeric value can be determined
  - E.g., sqrt(–1), $\infty - \infty$, $\infty \times 0$

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C float Decoding Example #1

$v = (-1)^s\, M\, 2^E$

$E = \mathtt{exp} - \mathit{Bias}$

float: `0xC0A00000`

binary: `1100 0000 1010 0000 0000 0000 0000 0000`

1	1000 0001	010 0000 0000 0000 0000 0000
1	8-bits	23-bits

E =

S =

M = 1 .

v = $(-1)^s$ M 2^E =

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

C float Decoding Example #1

$v = (-1)^s\, M\, 2^E$

$E = \texttt{exp} - Bias$

$Bias = 2^{k-1} - 1 = 127$

float: `0xC0A00000`

binary: `1100 0000 1010 0000 0000 0000 0000 0000`

1	1000 0001	010 0000 0000 0000 0000 0000
1	8-bits	23-bits

E = exp – Bias = 129 – 127 = 2 (decimal)

S = 1 -> negative number

M = 1.010 0000 0000 0000 0000 0000

= 1 + 1/4 = 1.25

$v = (-1)^s\, M\, 2^E = (-1)^1 * 1.25 * 2^2 = -5$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

C float Decoding Example #2

$$v = (-1)^s M\, 2^E$$
$$E = 1 - Bias$$

float: 0x001C0000

binary: 0000 0000 0001 1100 0000 0000 0000 0000

0	0000 0000	001 1100 0000 0000 0000 0000
1	8-bits	23-bits

E =

S =

M = 0 .

$v = (-1)^s M\, 2^E =$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

C float Decoding Example #2

$v = (-1)^s M\, 2^E$
$E = 1 - Bias$

$Bias = 2^{k-1} - 1 = 127$

float: 0x001C0000

binary: 0000 0000 0001 1100 0000 0000 0000 0000

0	0000 0000	001 1100 0000 0000 0000 0000
1	8-bits	23-bits

E = 1 – Bias = 1 – 127 = –126 (decimal)

S = 0 -> positive number

M = 0.001 1100 0000 0000 0000 0000

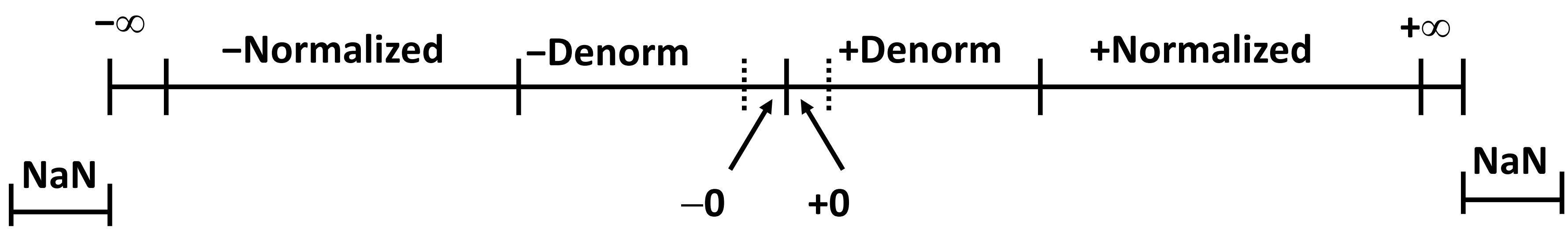
= 1/8 + 1/16 + 1/32 = 7/32 = $7*2^{-5}$

$v = (-1)^s M\, 2^E = (-1)^0 * 7*2^{-5} * 2^{-126} = 7*2^{-131}$

$\approx 2.571393892 \times 10^{-39}$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Visualization: Floating Point Encodings



Special Properties of the IEEE Encoding

- **FP Zero Same as Integer Zero**
  - All bits = 0
- **Can (Almost) Use Unsigned Integer Comparison**
  - Must first compare sign bits
  - Must consider −0 = 0
  - NaNs problematic
    - Will be greater than any other values
    - What should comparison yield? The answer is complicated.
  - Otherwise OK
    - Denorm vs. normalized
    - Normalized vs. infinity

Interesting Numbers `{single,double}`

Description	*exp*	*frac*	*Numeric Value*
■ **Zero**	**00…00**	**00…00**	**0.0**
■ **Smallest Pos. Denorm.**	**00…00**	**00…01**	$2^{-\{23,52\}} \times 2^{-\{126,1022\}}$
▪ Single ≈ 1.4×10^{-45}			
▪ Double ≈ 4.9×10^{-324}			
■ **Largest Denormalized**	**00…00**	**11…11**	$(1.0 - \varepsilon) \times 2^{-\{126,1022\}}$
▪ Single ≈ 1.18×10^{-38}			
▪ Double ≈ 2.2×10^{-308}			
■ **Smallest Pos. Normalized**	**00…01**	**00…00**	$1.0 \times 2^{-\{126,1022\}}$
▪ Just larger than largest denormalized			
■ **One**	**01…11**	**00…00**	**1.0**
■ **Largest Normalized**	**11…10**	**11…11**	$(2.0 - \varepsilon) \times 2^{\{127,1023\}}$
▪ Single ≈ 3.4×10^{38}			
▪ Double ≈ 1.8×10^{308}			

Dynamic Range (s=0 only)

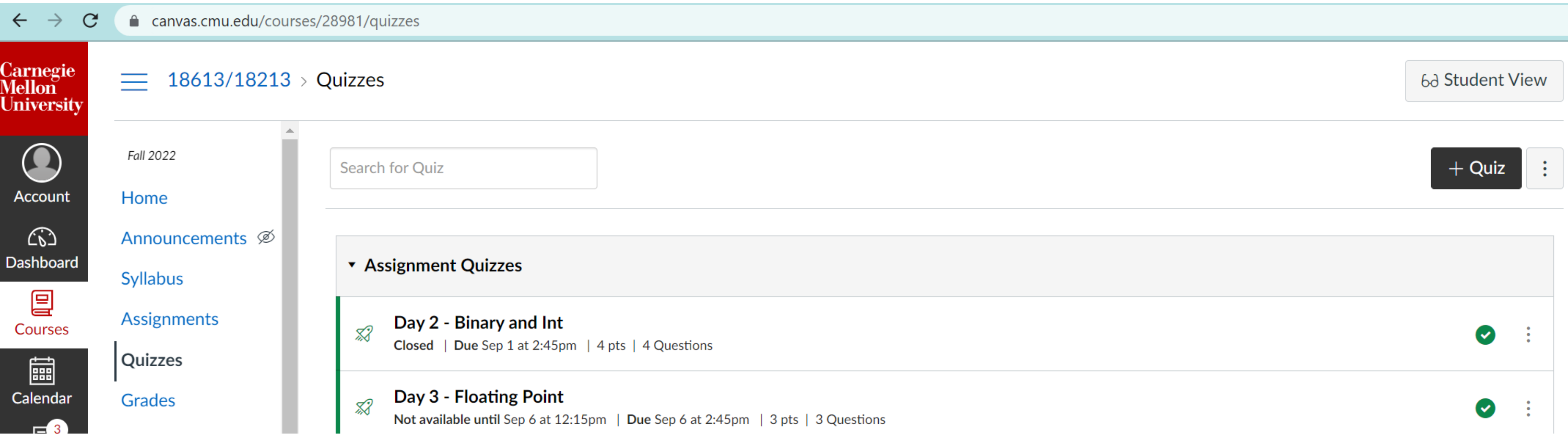
$v = (-1)^s M\, 2^E$

norm: $E = \texttt{exp} - Bias$

denorm: $E = 1 - Bias$

	s	exp	frac	E	Value	
	0	0000	000	-6	0	
	0	0000	001	-6	1/8*1/64 = 1/512	closest to zero
Denormalized numbers	0	0000	010	-6	2/8*1/64 = 2/512	(-1)^0(0+1/4)*2^{-6}
	…					
	0	0000	110	-6	6/8*1/64 = 6/512	
	0	0000	111	-6	7/8*1/64 = 7/512	largest denorm
	0	0001	000	-6	8/8*1/64 = 8/512	smallest norm
	0	0001	001	-6	9/8*1/64 = 9/512	(-1)^0(1+1/8)*2^{-6}
	…					
	0	0110	110	-1	14/8*1/2 = 14/16	
	0	0110	111	-1	15/8*1/2 = 15/16	closest to 1 below
Normalized numbers	0	0111	000	0	8/8*1 = 1	
	0	0111	001	0	9/8*1 = 9/8	closest to 1 above
	0	0111	010	0	10/8*1 = 10/8	
	…					
	0	1110	110	7	14/8*128 = 224	
	0	1110	111	7	15/8*128 = 240	largest norm
	0	1111	000	n/a	inf	

Quiz Time!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Assignment Quizzes
Day 2 - Binary and Int
Closed | Due Sep 1 at 2:45pm | 4 pts | 4 Questions
Day 3 - Floating Point
Not available until Sep 6 at 12:15pm | Due Sep 6 at 2:45pm | 3 pts | 3 Questions

Today: Floating Point

- Background: Fractional binary numbers
- IEEE floating point standard: Definition
- Example and properties
- **Rounding, addition, multiplication**
- Floating point in C
- Summary

Floating Point Operations: Basic Idea

- $x +_f y = \mathrm{Round}(x + y)$

- $x \times_f y = \mathrm{Round}(x \times y)$

- **Basic idea**
  - First compute exact result
  - Make it fit into desired precision
    - Possibly overflow if exponent too large
    - Possibly round to fit into **`frac`**

Rounding

- **Rounding Modes (illustrate with \$ rounding)**

	\$1.40	**\$1.60**	**\$1.50**	**\$2.50**	**–\$1.50**
Towards zero	\$1 ↓	\$1 ↓	\$1 ↓	\$2 ↓	–\$1 ↑
Round down (−∞)	\$1 ↓	\$1 ↓	\$1 ↓	\$2 ↓	–\$2 ↓
Round up (+∞)	\$2 ↑	\$2 ↑	\$2 ↑	\$3 ↑	–\$1 ↑
Nearest Even* (default)	\$1 ↓	\$2 ↑	\$2 ↑	\$2 ↓	–\$2 ↓

*Round to nearest, but if half-way in-between then round to nearest even

Closer Look at Round-To-Even

- **Default Rounding Mode**
  - Hard to get any other kind without dropping into assembly
    - C99 has support for rounding mode management
  - All others are statistically biased
    - Sum of set of positive numbers will consistently be over- or under-estimated
- **Applying to Other Decimal Places / Bit Positions**
  - When exactly halfway between two possible values
    - Round so that least significant digit is even
  - E.g., round to nearest hundredth

7.8949999	7.89	(Less than half way)
7.8950001	7.90	(Greater than half way)
7.8950000	7.90	(Half way—round up)
7.8850000	7.88	(Half way—round down)

Rounding Binary Numbers

- **Binary Fractional Numbers**
  - "Even" when least significant bit is `0`
  - "Half way" when bits to right of rounding position = `100`…$_2$

- **Examples**
  - Round to nearest 1/4 (2 bits right of binary point)

Value	Binary	Rounded	Action	Rounded Value
2 3/32	10.00011_2	10.00_2	(<1/2—down)	2
2 3/16	10.00110_2	10.01_2	(>1/2—up)	2 1/4
2 7/8	10.11100_2	11.00_2	(1/2—up)	3
2 5/8	10.10100_2	10.10_2	(1/2—down)	2 1/2

Rounding

1.BBGRXXX

Guard bit: LSB of result

Sticky bit: OR of remaining bits

Round bit: 1st bit removed

- **Round up conditions**
  - Round = 1, Sticky = 1 → > 0.5
  - Guard = 1, Round = 1, Sticky = 0 → Round to even

Fraction	*GRS*	*Incr?*	*Rounded*
1.0000000	000	N	1.000
1.1010000	100	N	1.101
1.0001000	010	N	1.000
1.0011000	110	Y	1.010
1.0001010	011	Y	1.001
1.1111100	111	Y	10.000

FP Multiplication

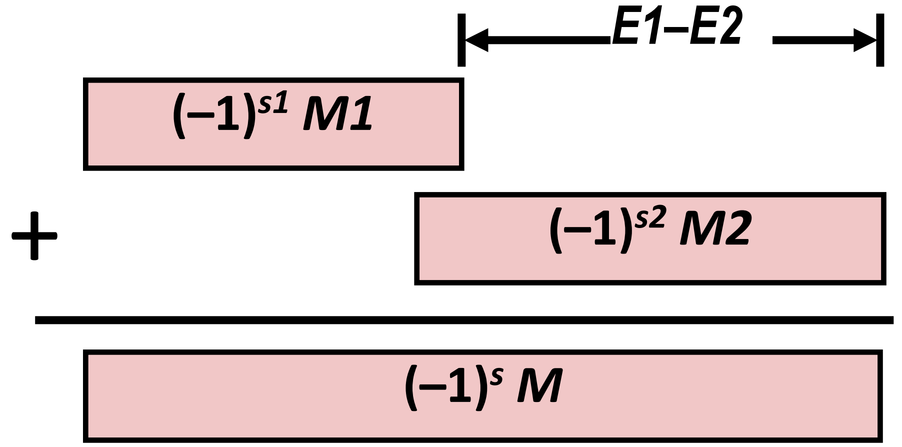
- $(-1)^{s1}\ M1\ \ 2^{E1}\ \ \mathbf{x}\ \ (-1)^{s2}\ M2\ \ 2^{E2}$
- **Exact Result:** $(-1)^{s}\ M\ \ 2^{E}$
  - Sign *s*: *s1* ^ *s2*
  - Significand *M*: *M1* x *M2*
  - Exponent *E*: *E1* + *E2*
- **Fixing**
  - If *M* ≥ 2, shift *M* right, increment *E*
  - If *E* out of range, overflow
  - Round *M* to fit `frac` precision
- **Implementation**
  - Biggest chore is multiplying significands

4 bit significand: $1.010*2^{2}$ **x** $1.110*2^{3} = 10.0011*2^{5}$
$= 1.00011*2^{6} = 1.001*2^{6}$

Floating Point Addition

- $(-1)^{s1}\,M1\;\;2^{E1}\;\;+\;\;(-1)^{s2}\,M2\;\;2^{E2}$
  - Assume $E1 > E2$
- **Exact Result:** $(-1)^{s}\,M\;\;2^{E}$
  - Sign s, significand M:
    - Result of signed align & add
  - Exponent E: $E1$
- **Fixing**
  - If $M \geq 2$, shift M right, increment E
  - if $M < 1$, shift M left k positions, decrement E by k
  - Overflow if E out of range
  - Round M to fit **`frac`** precision

Get binary points lined up




```
1.010*2^2 + 1.110*2^3 = (0.1010 + 1.1100)*2^3
= 10.0110 * 2^3 = 1.00110 * 2^4 = 1.010 * 2^4
```

Mathematical Properties of FP Add

- **Compare to those of Abelian Group**
  - Closed under addition? *Yes*
    - But may generate infinity or NaN
  - Commutative? *Yes*
  - Associative? *No*
    - Overflow and inexactness of rounding
    - `(3.14+1e10)-1e10 = 0, 3.14+(1e10-1e10) = 3.14`
  - 0 is additive identity? *Yes*
  - Every element has additive inverse? *Almost*
    - Yes, except for infinities & NaNs
- **Monotonicity**
  - $a \geq b \Rightarrow a+c \geq b+c$? *Almost*
    - Except for infinities & NaNs

Mathematical Properties of FP Mult

- **Compare to Commutative Ring**
  - Closed under multiplication? ***Yes***
    - But may generate infinity or NaN
  - Multiplication Commutative? ***Yes***
  - Multiplication is Associative? ***No***
    - Possibility of overflow, inexactness of rounding
    - Ex: `(1e20*1e20)*1e-20= inf, 1e20*(1e20*1e-20)= 1e20`
  - 1 is multiplicative identity? ***Yes***
  - Multiplication distributes over addition? ***No***
    - Possibility of overflow, inexactness of rounding
    - `1e20*(1e20-1e20)= 0.0, 1e20*1e20 – 1e20*1e20 = NaN`
- **Monotonicity**
  - $a \geq b \;\&\; c \geq 0 \Rightarrow a * c \geq b * c$? ***Almost***
    - Except for infinities & NaNs

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Floating Point in C

- **C Guarantees Two Levels**
  - **`float`** single precision
  - **`double`** double precision

- **Conversions/Casting**
  - Casting between **`int`**, **`float`**, and **`double`** changes bit representation
  - **`double`**/**`float`** → **`int`**
    - Truncates fractional part
    - Like rounding toward zero
    - Not defined when out of range or NaN: Generally sets to TMin
  - **`int`** → **`double`**
    - Exact conversion, as long as **`int`** has ≤ 53 bit word size
  - **`int`** → **`float`**
    - Will round according to rounding mode

Floating Point Puzzles

- **For each of the following C expressions, either:**
  - Argue that it is true for all argument values
  - Explain why not true

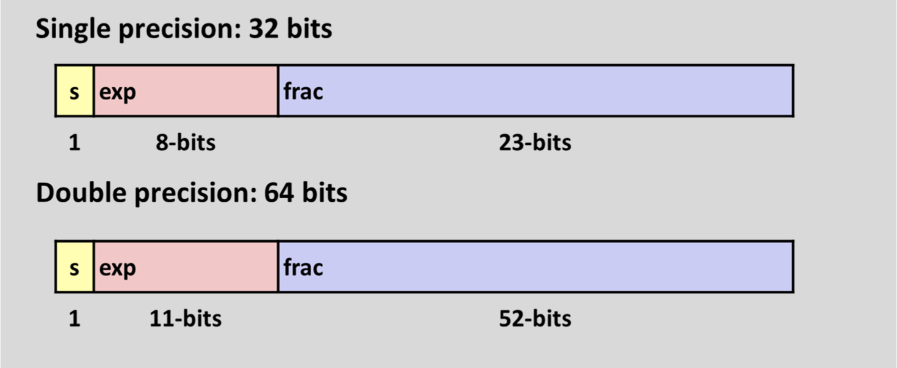
```
int x = …;
float f = …;
double d = …;
```

Assume neither `d` nor `f` is NaN

- `x == (int)(float) x` ✗
- `x == (int)(double) x` ✓
- `f == (float)(double) f` ✓
- `d == (double)(float) d` ✗
- `f == -(-f);` ✓
- `2/3 == 2/3.0` ✗
- `d < 0.0` ⇒ `((d*2) < 0.0)` ✓
- `d > f` ⇒ `-f > -d` ✓
- `d * d >= 0.0` ✓
- `(d+f)-d == f` ✗

Summary

- **IEEE Floating Point has clear mathematical properties**
- **Represents numbers of form M x 2^E**
- **One can reason about operations independent of implementation**
  - As if computed with perfect precision and then rounded
- **Not the same as real arithmetic**
  - Violates associativity/distributivity
  - Makes life difficult for compilers & serious numerical applications programmers



Additional Slides

Creating Floating Point Number

s	exp	frac
1	4-bits	3-bits

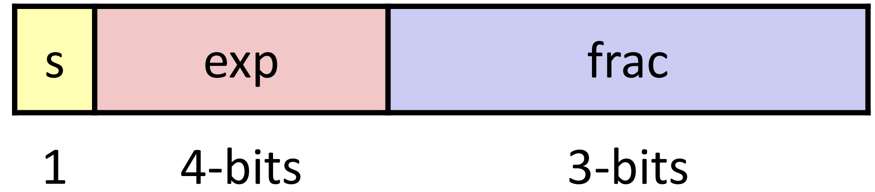
- **Steps**
  - Normalize to have leading 1
  - Round to fit within fraction
  - Postnormalize to deal with effects of rounding

- **Case Study**
  - Convert 8-bit unsigned numbers to tiny floating point format

Example Numbers

`128`	`10000000`
`15`	`00001101`
`33`	`00010001`
`35`	`00010011`
`138`	`10001010`
`63`	`00111111`

Normalize



- **Requirement**
  - Set binary point so that numbers of form 1.xxxxx
  - Adjust all to have leading one
    - Decrement exponent as shift left

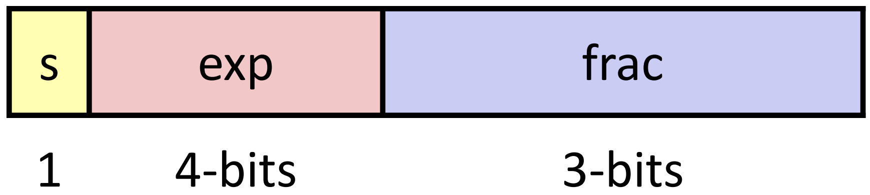
Value	*Binary*	*Fraction*	*Exponent*
128	10000000	1.0000000	7
15	00001101	1.1010000	3
17	00010001	1.0001000	4
19	00010011	1.0011000	4
138	10001010	1.0001010	7
63	00111111	1.1111100	5

Postnormalize

- **Issue**
  - Rounding may have caused overflow
  - Handle by shifting right once & incrementing exponent

Value	*Rounded*	*Exp*	*Adjusted*	*Numeric Result*
`128`	`1.000`	`7`		`128`
`15`	`1.101`	`3`		`15`
`17`	`1.000`	`4`		`16`
`19`	`1.010`	`4`		`20`
`138`	`1.001`	`7`		`134`
`63`	`10.000`	`5`	`1.000/6`	`64`

Tiny Floating Point Example



- **8-bit Floating Point Representation**
  - the sign bit is in the most significant bit
  - the next four bits are the **`exp`**, with a bias of 7
  - the last three bits are the **`frac`**

- **Same general form as IEEE Format**
  - normalized, denormalized
  - representation of 0, NaN, infinity