Analysis of Algorithms: Solutions 1

```
                                            X
                                X           X
                                X           X
                                X           X
                                X           X
                                X           X
number of                       X           X
homeworks                       X           X
                                X   X       X
                                X   X       X
                                X   X       X
                        X   X   X   X       X
        X           X   X   X   X   X   X   X
    ------------------------------------------
    0   1   2   3   4   5   6   7   8   9   10
                      grades
```

The histogram shows the distribution of grades, from 0 to 10.

Problem 1
Write an algorithm that finds the *most frequent element* in an integer array $A[1..n]$; that is, your algorithm must identify the element that occurs the greatest number of times.

MOST-FREQUENT(A, n)	*cost*	*times*
$max \leftarrow 0$	c_1	1
for $i \leftarrow 1$ **to** n	c_2	$n+1$
do $B[i] \leftarrow$ TRUE	c_3	n
for $i \leftarrow 1$ **to** n	c_4	$n+1$
do if $B[i]$	c_5	n
then $B[i] \leftarrow$ FALSE	c_6	$\leq n$
$count \leftarrow 1$	c_7	$\leq n$
for $j \leftarrow i+1$ **to** n	c_8	$\leq \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
do if $A[i] = A[j]$	c_9	$\leq \sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}$
then $B[j] \leftarrow$ FALSE	c_{10}	$\leq \sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}$
$count \leftarrow count + 1$	c_{11}	$\leq \sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}$
if $count > max$	c_{12}	$\leq n$
then $max \leftarrow count$	c_{13}	$\leq n$
$freq \leftarrow i$	c_{14}	$\leq n$
return $A[freq]$	c_{15}	1

Problem 2

Estimate the worst-case running time of your algorithm.

$$
\begin{aligned}
T(n) &\le c_1 + c_2(n+1) + c_3 n + c_4(n+1) + c_5 n + c_6 n + c_7 n + c_8 \frac{n(n+1)}{2} + c_9 \frac{n(n-1)}{2} \\
&\quad + c_{10} \frac{n(n-1)}{2} + c_{11} \frac{n(n-1)}{2} + c_{12} n + c_{13} n + c_{14} n + c_{15} \\
&= \left(\frac{c_8}{2} + \frac{c_9}{2} + \frac{c_{10}}{2} + \frac{c_{11}}{2} \right) n^2 \\
&\quad + \left(c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + \frac{c_8}{2} - \frac{c_9}{2} - \frac{c_{10}}{2} - \frac{c_{11}}{2} + c_{12} + c_{13} + c_{14} \right) n \\
&\quad + \left(c_1 + c_2 + c_4 + c_{15} \right) \\
&= \Theta(n^2)
\end{aligned}
$$