Analysis of Algorithms: Solutions 2

```
        X       X
        X       X
        X X X   X
        X X X   X
        X X X X X
        X X X X X
    X   X X X X X
    X X X X X X X
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    4 5 6 7 8 9 10
        grades
```

The histogram shows the distribution of grades for the homeworks submitted on time.

Problem 1
For each of the following functions, give an asymptotically tight bound (Θ-notation).

(a) $(n+1)^9 = (n+o(n))^9 = \Theta(n^9)$

(b) $(n+2)\cdot(2n+1)\cdot\sqrt{n+1} = \Theta(n)\cdot\Theta(n)\cdot\Theta(\sqrt{n}) = \Theta(n\cdot n\cdot\sqrt{n}) = \Theta(n^{5/2})$

(c) $n^9 + 9^n = o(9^n) + 9^n = \Theta(9^n)$

(d) $(n^{4/3} + n^{5/3} + \lg n)^{3/5} = (o(n^{5/3}) + n^{5/3} + o(n^{5/3}))^{3/5} = \Theta((n^{5/3})^{3/5}) = \Theta(n)$

(e) $2^n + n! + n^n = O(n^n) + O(n^n) + n^n = \Theta(n^n)$

(f) $2^{\left(2^{\lg\left(\frac{\log_3 n}{\log_3 2}\right)}\right)} = 2^{\left(2^{\lg(\lg n)}\right)} = 2^{\lg n} = n = \Theta(n)$

Problem 2
Give an example of functions $f(n)$ and $g(n)$ such that $f(n) \neq O(g(n))$ and $f(n) \neq \Omega(g(n))$.

Consider the following two functions:

$$f(n) = \begin{cases} n & \text{if } n \text{ is even;} \\ 1 & \text{if } n \text{ is odd.} \end{cases}$$

$$g(n) = \begin{cases} 1 & \text{if } n \text{ is even;} \\ n & \text{if } n \text{ is odd.} \end{cases}$$

For even n, $f(n)$ grows asymptotically faster than $g(n)$. On the other hand, for odd n, $f(n)$ grows asymptotically slower. Therefore, $g(n)$ is neither asymptotically lower bound nor asymptotically upper bound for $f(n)$.