Analysis of Algorithms: Solutions 9

```
   X              X
   X  X     X     X
X  X  X  X  X     X     X
X  X  X  X  X     X  X  X
X  X  X  X  X     X  X  X
--------------------------
2  3  4  5  6  7  8  9  10
```

Problem 1

Suppose that you need to compute a product of five matrices, and their dimensions are $p[0..5] = \langle 3, 8, 5, 2, 20, 4 \rangle$. Give the optimal parenthesization of this product and the corresponding value of $m[0, 5]$.

5	312				
4	248	304			
3	128	400	200		
2	120	80	200	160	
1	0	0	0	0	0
	1	2	3	4	5

The optimal parenthesization is $(A_1 \cdot (A_2 \cdot A_3)) \cdot (A_4 \cdot A_5)$, and $m[0, 5]$ is 312.

Problem 2

Determine a longest common subsequence of $\langle a, b, a, b, a, a \rangle$ and $\langle b, a, a, b, b, a, a, b \rangle$. Using Figure 16.3 in the book as a model, draw the table constructed by LCS-Length.

8	b	0	1	2	3	4	4	5
7	a	0	1	2	3	3	4	5
6	a	0	1	2	3	3	4	4
5	b	0	1	2	2	3	3	3
4	b	0	1	2	2	3	3	3
3	a	0	1	1	2	2	3	3
2	a	0	1	1	2	2	2	2
1	b	0	0	1	1	1	1	1
0		0	0	0	0	0	0	0
i			a	b	a	b	a	a
	j	0	1	2	3	4	5	6

The longest common subsequence is $\langle b, a, b, a, a \rangle$.

Problem 3
Give pseudocode for a modified version of LCS-LENGTH that uses only $O(n)$ memory.

```
LINEAR-LCS(X, Y, m, n)
for j ← 0 to n
    do c[j] ← 0
for i ← 1 to m
    do old ← 0
       for j ← 1 to n
           do temp ← c[j]
              if X[i] = Y[j]
                then c[j] ← old + 1
                else c[j] ← max(temp, c[j − 1])
              old ← temp
return c[n]
```

Problem 4
Consider two recurrences:

$$A_0 = 0$$
$$A_n = A_{n-1} + n \quad (\text{where } n \geq 1)$$

$$B_0 = 0$$
$$B_n = B_{n-1} + A_n \quad (\text{where } n \geq 1)$$

Give a constant-time algorithm FIND-B(n) for computing B_n.

We can prove the following expressions by induction:

$$\begin{aligned} A_n &= \frac{n \cdot (n+1)}{2} \\ B_n &= \frac{n \cdot (n+1) \cdot (n+2)}{6} \end{aligned}$$

The constant-time algorithm uses the resulting expression for computing B_n:

FIND-B(n)
return $\frac{n(n+1)(n+2)}{6}$