Assignment 8: Radiosity and Image Processing

15-462 Graphics I
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Sample Solution

1 Form Factors (20 pts)

In radiosity, the *form factor* F_{ij} is the proportion of the total power leaving patch P_i received by patch P_j.

1. Assume we have disjoint patches P_i, P_j, and P_k. Derive an equation for the form factor $F_{i(j\cup k)}$ from P_i to $P_j \cup P_k$ in terms of simpler form factors and areas, if needed. Explain your reasoning.

   Since P_j and P_k are disjoint, the proportion of the total power leaving P_i reaching P_j and P_k is equal to the sum of the individual proportions. That is,

   $$F_{i(j\cup k)} = F_{ij} + F_{ik}$$

   We can also derive this more formally using the mathematical definition of the form factor and simple properties of the integral, namely that $\int_{y\in P_j\cup P_k} f(y)dy = \int_{y\in P_j} f(y)dy + \int_{y\in P_k} f(y)dy$ if P_j and P_k are disjoint.

2. Again, assume we have disjoint patches P_i, P_j, and P_k. Derive an equation for the form factor $F_{(i\cup j)k}$ from $P_i \cup P_j$ to P_k in terms of simpler form factors and areas, if needed. Explain your reasoning.

   Here we use reciprocity, which means here that

   $$(A_i + A_j)F_{(i\cup j)k} = A_k F_{k(i\cup j)}$$

   Then

   $$\begin{aligned} F_{(i\cup j)k} &= \frac{A_k}{A_i + A_j} F_{k(i\cup j)} \\ &= \frac{A_k}{A_i + A_j}(F_{ki} + F_{kj}) && \textit{By part (1)} \\ &= \frac{A_k}{A_i + A_j}\left(\frac{A_i}{A_k} F_{ik} + \frac{A_j}{A_k} F_{jk}\right) && \textit{By reciprocity} \\ &= \frac{A_i F_{ik} + A_j F_{jk}}{A_i + A_j} \end{aligned}$$

3. Let d be the minimal distance between any two points on two patches P_i and P_j. Calculate a simple upper bound for F_{ij} that no longer involves an integral. Explain your steps.

We have

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos\theta \ \cos\theta'}{\pi r^2} V(x,y)\, dy\, dx$$

We estimate an upper bound, using

$$\begin{aligned} &-1 \leq \cos\theta, \cos\theta' \leq 1 \\ &0 \leq r \leq d \\ &0 \leq V(x,y) \leq 1 \end{aligned}$$

which yields

$$A_i F_{ij} \leq \int_{x \in P_i} \int_{y \in P_j} \frac{1}{\pi d^2}\, dy\, dx = \frac{A_i\, A_j}{\pi d^2}.$$

Hence $F_{ij} \leq \frac{A_j}{\pi d^2}$.

2 Point Processing (10 pts)

Suppose you are given a low-contrast grayscale image whose smallest pixel value is a and whose largest pixel value is b. Give the formulas for and a sketch of, a piecewise linear function to be applied at each pixel which would increase contrast maximally without losing information. The domain and range of this function should both be [0,255]. Map pixel values less than a to 0 and values greater than b to 255.

We just squeeze the whole interval [0,255] into the range from a to b linearly.

$$f(x) = \begin{cases} 0 & \text{if } x \leq a \\ 255\frac{x-a}{b-a} & \text{if } a < x < b \\ 255 & \text{if } b \leq x \end{cases}$$

3 Filters (20 pts)

In this problem you will design a filter to reduce interlace flicker.

Some background: The USA's NTSC broadcast TV standard displays one frame every 30th of a second, and each frame consists of 486 scan lines (rows) made of two *interlaced* fields. The two fields are spatially interleaved so that the first, or "odd" field contains the odd numbered rows of the frame and the second, "even" field contains the even numbered rows. This is done to reduce visual flicker of the image. Refreshing a screen at 30Hz shows objectionable "strobing", but the 60Hz field refresh rate is fast enough to be imperceptible to most people. (See Foley's book (index: interlace) for more info.) Unfortunately, if the source image being displayed has high vertical frequencies, these will still flicker. In particular, a frame where every even scan line is white and every odd scan line is black will flicker wildly, appearing to vibrate.

Design a filter that could be applied to an image to blur it vertically to eliminate this flicker without degrading the image too much. We'll tell you how to set up the constraints and then you figure out the coefficients.

- The filter should be 1 pixel wide and 5 pixels high. (In lecture we looked at some 3x3 filters; this one is 1x5.) This means you need to solve for five unknown coefficients.
- It should be a linear shift-invariant filter (the type that convolution yields).
- It should be symmetric about its center.
- If the input image (called a in the lecture notes) is constant, then after filtering, the output image (called b) should be the same constant.
- If the input image consists of alternating black and white scan lines, the output image should be constant, mid-gray.
- If the input image consists of horizontal stripes each two pixels wide (e.g. two lines black, two lines white, repeat) then the output image should be identical to the input.

Those are enough constraints to uniquely determine the five floating point filter coefficients. Compute their values and show the filter matrix (which is a column vector). **Show your work.**

- *The filter H should be 1×5, so we set*

$$H = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

- *The filter should be shift-invariant, so we cannot take the x and y values into account.*
- *The filter should be symmetric about its center, so*

$$a = e \quad \textit{and} \quad b = d$$

- *If the input image is constant, the output image should be constant, so, using white,*

$$a + b + c + d + e = 1.$$

  Hence

$$2a + 2b + c = 1.$$

- *If the input image consists of alternating black and white scan lines, the output image should be constant gray, so using black/white/black/white/black:*

$$b + d = 2b = 1/2$$

  which yields $b = 1/4$

- *If the input image consists of horizontal stripes each two pixels wide then the output image should be identical to the input, so (using black/black/white/white/black):*

$$1 = c + d = c + b = c + 1/4$$

which yields $c = 3/4$. *Plugging this into the first equation we get*

$$1 = 2a + 2b + c = 2a + 2/4 + 3/4 = 2a + 5/4$$

From the last equation we get $a = -1/8$.

Putting it all together we obtain

$$H = \frac{1}{8} \begin{bmatrix} -1 \\ 2 \\ 6 \\ 2 \\ -1 \end{bmatrix}$$

We only used part of the given information, but it is easy to verify now that all specified properties hold.