Abstract Syntax

Types	τ	$::=$	$\tau_1 \to \tau_2 \mid \forall\alpha.\,\tau \mid \alpha$	
Terms	e	$::=$	$x \mid \lambda x.\,e \mid e_1\, e_2$	$(\to)$
		$\mid$	$\Lambda\alpha.\,e \mid e\,[\tau]$	$(\forall)$
Contexts	Γ	$::=$	$\cdot \mid \Gamma, \alpha\ \textit{type} \mid \Gamma, x : \tau$	(all variables distinct)

Judgments

$\Gamma\ \textit{ctx}$	Γ is a valid context	
$\Gamma \vdash \tau\ \textit{type}$	τ is a valid type	presupposes $\Gamma\ \textit{ctx}$
$\Gamma \vdash e : \tau$	expression e has type τ	presupposes $\Gamma\ \textit{ctx}$, ensures $\Gamma \vdash \tau\ \textit{type}$
$e\ \textit{normal}$	expression e is normal	
$e\ \textit{neutral}$	expression e is normal	
$e \longrightarrow e'$	expression e reduces to e'	

Theorems

Subject Reduction. If $\Gamma \vdash e : \tau$ and $e \mapsto e'$ then $\Gamma \vdash e' : \tau$.

Progress. For every expression e either $e \longrightarrow e'$ for some e' or $e\ \textit{normal}$.

Finality of Normal Forms. There is no e such that $e \longrightarrow e'$ for some e' and $e\ \textit{normal}$.

Contexts Γ

$$\frac{}{(\cdot)\ \textit{ctx}}\ \mathsf{ctx/emp} \qquad \frac{\Gamma\ \textit{ctx}}{(\Gamma, \alpha\ \textit{type})\ \textit{ctx}}\ \mathsf{ctx/tpvar} \qquad \frac{\Gamma\ \textit{ctx} \quad \Gamma \vdash \tau\ \textit{type}}{(\Gamma, x : \tau)\ \textit{ctx}}\ \mathsf{ctx/var}$$

Functions $\tau_2 \to \tau_1$

$$\frac{\Gamma \vdash \tau_2 \; \textit{type} \quad \Gamma \vdash \tau_1 \; \textit{type}}{\Gamma \vdash \tau_2 \to \tau_1 \; \textit{type}} \; \mathsf{tp/arrow}$$

$$\frac{\Gamma, x_1 : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \lambda x_1{:}\tau_1.\, e_2 : \tau_1 \to \tau_2} \; \mathsf{tp/lam} \qquad \frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \; \mathsf{tp/var} \qquad \frac{\Gamma \vdash e_1 : \tau_2 \to \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1\, e_2 : \tau_1} \; \mathsf{tp/app}$$

$$\frac{e \; \textit{normal}}{\lambda x.\, e \; \textit{normal}} \; \mathsf{norm/lam} \qquad \frac{e \; \textit{neutral}}{e \; \textit{normal}} \; \mathsf{norm/neut}$$

$$\frac{}{x \; \textit{neutral}} \; \mathsf{neut/var} \qquad \frac{e_1 \; \textit{neutral} \quad e_2 \; \textit{normal}}{e_1\, e_2 \; \textit{neutral}} \; \mathsf{neut/app}$$

$$\frac{}{(\lambda x.\, e_1)\, e_2 \longrightarrow [e_2/x]e_1} \; \mathsf{red/beta}$$

$$\frac{e \longrightarrow e'}{\lambda x.\, e \longrightarrow \lambda x.\, e'} \; \mathsf{red/lam} \qquad \frac{e_1 \longrightarrow e_1'}{e_1\, e_2 \longrightarrow e_1'\, e_2} \; \mathsf{red/app_1} \qquad \frac{e_2 \longrightarrow e_2'}{e_1\, e_2 \longrightarrow e_1\, e_2'} \; \mathsf{red/app_2}$$

Polymorphic Types $\forall \alpha.\, \tau$

$$\frac{\alpha \; \textit{type} \in \Gamma}{\Gamma \vdash \alpha \; \textit{type}} \; \mathsf{tp/tpvar} \qquad \frac{\Gamma, \alpha \; \textit{type} \vdash \tau \; \textit{type}}{\Gamma \vdash \forall \alpha.\, \tau \; \textit{type}} \; \mathsf{tp/forall}$$

$$\frac{\Gamma, \alpha \; \textit{type} \vdash e : \tau}{\Gamma \vdash \Lambda \alpha.\, e : \forall \alpha.\, \tau} \; \mathsf{tp/tplam} \qquad \frac{\Gamma \vdash e : \forall \alpha.\, \tau \quad \Gamma \vdash \sigma \; \textit{type}}{\Gamma \vdash e\,[\sigma] : [\sigma/\alpha]\tau} \; \mathsf{tp/tpapp}$$

$$\frac{e \; \textit{normal}}{\Lambda \alpha.\, e \; \textit{normal}} \; \mathsf{norm/tplam} \qquad \frac{e \; \textit{normal}}{e\,[\sigma] \; \textit{normal}} \; \mathsf{norm/tpapp}$$

$$\frac{}{(\Lambda \alpha.\, e)\,[\tau] \longrightarrow [\tau/\alpha]e} \; \mathsf{red/tpbeta} \qquad \frac{e \longrightarrow e'}{\Lambda \alpha.\, e \longrightarrow \Lambda \alpha.\, e'} \; \mathsf{red/tplam} \qquad \frac{e \longrightarrow e'}{e\,[\tau] \longrightarrow e'\,[\tau]} \; \mathsf{red/tpapp}$$