Abstract Syntax

Types	τ	$::=$	$\tau_1 \multimap \tau_2 \mid \&_{i\in I}(i:\tau_i) \mid \tau_1 \otimes \tau_2 \mid 1 \mid \oplus_{i\in I}(i:\tau_i) \mid \rho\alpha.\,\tau$	
Contexts	Δ	$::=$	$\cdot \mid \Delta, x:\tau$	(all variables distinct)
Processes	P	$::=$	$x \leftarrow P \;;\; Q$	allocate/spawn
		$\mid$	$x^w \leftarrow y^R$	move
		$\mid$	$x^W.V \mid \mathsf{case}\ x^R\ K$	$(1, \otimes, \oplus, \rho)$
		$\mid$	$x^R.V \mid \mathsf{case}\ x^W\ K$	$(\multimap, \&)$
Small values	V	$::=$	$\langle\,\rangle \mid \langle a_1, a_2\rangle \mid i\cdot a \mid \mathsf{fold}\ a$	
Continuations	K	$::=$	$(\langle\,\rangle \Rightarrow P) \mid (\langle x_1, x_2\rangle \Rightarrow P) \mid (i\cdot x_i \Rightarrow P_i)_{i\in I} \mid (\mathsf{fold}\ x \Rightarrow P)$	
Cell contents	W	$::=$	$V \mid K$	
Configurations	$\mathcal{C}$	$::=$	$\cdot \mid \mathcal{C}_1, \mathcal{C}_2 \mid \mathsf{proc}\ d\ P \mid \mathsf{cell}\ c\ W$	

Judgments

$\Delta \Vdash P :: (z:\sigma)$	process P reads from Δ and writes to $z:\sigma$
$\Delta \Vdash \mathcal{C} :: \Delta'$	configuration $\mathcal{C}$ reads from Δ and writes to Δ'
$\mathcal{C}$ *final*	configuration $\mathcal{C}$ is final (consists only of cells)

Theorems

Preservation. If $\Delta \Vdash \mathcal{C} :: \Delta'$ and $\mathcal{C} \mapsto \mathcal{C}'$ then $\Delta \Vdash \mathcal{C}' :: \Delta'$.

Progress. If $\cdot \Vdash \mathcal{C} :: \Delta$ then either $\mathcal{C} \mapsto \mathcal{C}'$ for some $\mathcal{C}'$ or $\mathcal{C}$ *final*.

Statics, Allocate and Move

$$\frac{\Delta \Vdash P :: (x:\tau) \quad \Delta', x:\tau \Vdash Q :: (z:\sigma)}{\Delta, \Delta' \Vdash (x \leftarrow P \;;\; Q) :: (z:\sigma)}\ \mathsf{tp/alloc} \qquad \frac{}{y:\tau \Vdash x^W \leftarrow y^R :: (x:\tau)}\ \mathsf{tp/move}$$

Statics, Positive Types

$$\frac{}{\cdot \Vdash x^W.\langle\,\rangle :: (x : 1)}\ \textsf{w/unit} \qquad \frac{\Delta \Vdash P :: (z : \sigma)}{\Delta, x : 1 \Vdash \textsf{case}\ x^R\ (\langle\,\rangle \Rightarrow P) :: (z : \sigma)}\ \textsf{r/unit}$$

$$\frac{}{y : \tau, z : \sigma \Vdash x^W.\langle y, z\rangle :: (x : \tau \otimes \sigma)}\ \textsf{w/pair} \qquad \frac{\Delta, x_1 : \tau_1, x_2 : \tau_2 \Vdash P :: (z : \sigma)}{\Delta, x : \tau_1 \otimes \tau_2 \Vdash \textsf{case}\ x^R\ (\langle x_1, x_2\rangle \Rightarrow P) : (z : \sigma)}\ \textsf{r/pair}$$

$$\frac{(j \in I)}{y : \tau_j \Vdash x^W.(j \cdot y) :: (x : \oplus_{i\in I}(i : \tau_i))}\ \textsf{w/tag} \qquad \frac{\Delta, y_i : \tau_i \Vdash P_i :: (z : \sigma) \quad (\text{for all } i \in I)}{\Delta, x : \oplus_{i\in I}(i : \tau_i) \Vdash \textsf{case}\ x^R\ (i \cdot y_i \Rightarrow P_i)_{i\in I} :: (z : \sigma)}\ \textsf{r/tag}$$

$$\frac{}{y : [\rho\alpha.\,\tau/\alpha]\tau \Vdash x^W.(\textsf{fold}\ y) :: (x : \rho\alpha.\,\tau)}\ \textsf{w/fold} \qquad \frac{\Delta, y : [\rho\alpha.\,\tau/\alpha]\tau \Vdash P :: (z : \sigma)}{\Delta, x : \rho\alpha.\,\tau \in \Delta \Vdash \textsf{case}\ x^R\ (\textsf{fold}\ y \Rightarrow P) :: (z : \sigma)}\ \textsf{r/fold}$$

Statics, Negative Types

$$\frac{\Delta, y : \tau \Vdash P :: (z : \sigma)}{\Delta \Vdash \textsf{case}\ x^W\ (\langle y, z\rangle \Rightarrow P) :: (x : \tau \multimap \sigma)}\ \textsf{w/fun} \qquad \frac{}{x : \tau \multimap \sigma, y : \tau \Vdash x^R.\langle y, z\rangle :: (z : \sigma)}\ \textsf{r/fun}$$

$$\frac{\Delta \Vdash P_i :: (z_i : \tau_i) \quad (\text{for all } i \in I)}{\Delta \Vdash \textsf{case}\ x^W\ (i \cdot z_i \Rightarrow P_i)_{i\in I} :: (x : \&_{i\in I}(i : \tau_i))}\ \textsf{w/record} \qquad \frac{(j \in I)}{x : \&_{i\in I}(i : \tau_i) \Vdash x^R.(j \cdot z) :: (z : \tau_j)}\ \textsf{r/record}$$

Statics, Configurations

$$\frac{\Delta \Vdash P :: (c : \tau)}{\Delta', \Delta \Vdash \textsf{proc}\ c\ P :: (\Delta', c : \tau)}\ \textsf{tp/proc}$$

$$\frac{\Delta \Vdash c^W.V :: (c : \tau)}{\Delta', \Delta \Vdash \textsf{cell}\ c\ V :: (\Delta', c : \tau)}\ \textsf{tp/cell/val} \qquad \frac{\Delta \Vdash \textsf{case}\ c^W\ K :: (c : \tau)}{\Delta', \Delta \Vdash \textsf{cell}\ c\ K :: (\Delta', c : \tau)}\ \textsf{tp/cell/cont}$$

$$\frac{}{\Delta \Vdash (\cdot) :: \Delta}\ \textsf{tp/empty} \qquad \frac{\Delta \Vdash \mathcal{C}_1 :: \Delta_1 \quad \Delta_1 \Vdash \mathcal{C}_2 :: \Delta_2}{\Delta \Vdash (\mathcal{C}_1, \mathcal{C}_2) :: \Delta_2}\ \textsf{tp/join}$$

$$\frac{}{(\cdot)\ \textit{final}}\ \textsf{fin/empty} \qquad \frac{\mathcal{C}\ \textit{final}}{(\mathcal{C}, \textsf{cell}\ c\ W)\ \textit{final}}\ \textsf{fin/cell}$$

Dynamics

$$
\begin{array}{lllll}
 & \mathsf{proc}\ d\ (x \leftarrow P\ ;\ Q) & \mapsto & \mathsf{proc}\ c\ ([c/x]P), \mathsf{proc}\ d\ ([c/x]Q) & \text{(alloc/spawn, } c \text{ fresh)} \\
\mathsf{cell}\ c\ W, & \mathsf{proc}\ d\ (d^W \leftarrow c^R) & \mapsto & \mathsf{cell}\ d\ W & \text{(move)} \\
 & & & & \\
 & \mathsf{proc}\ d\ (d^W.V) & \mapsto & \mathsf{cell}\ d\ V & \text{(write: } \otimes, 1, \oplus, \rho) \\
\mathsf{cell}\ c\ V, & \mathsf{proc}\ d\ (\mathsf{case}\ c^R\ K) & \mapsto & \mathsf{proc}\ d\ (V \triangleright K) & \text{(read: } \otimes, 1, \oplus, \rho) \\
 & & & & \\
 & \mathsf{proc}\ d\ (\mathsf{case}\ d^W\ K) & \mapsto & \mathsf{cell}\ d\ K & \text{(write: } \multimap, \&) \\
\mathsf{cell}\ c\ K, & \mathsf{proc}\ d\ (c^R.V) & \mapsto & \mathsf{proc}\ d\ (V \triangleright K) & \text{(read: } \multimap, \&)
\end{array}
$$

$$
\begin{array}{rcll}
\langle\,\rangle & \triangleright & (\langle\,\rangle \Rightarrow P) & = & P \\
\langle c_1, c_2\rangle & \triangleright & (\langle x_1, x_2\rangle \Rightarrow P) & = & [c_1/x_1, c_2/x_2]P \\
j \cdot c & \triangleright & (i \cdot x_i \Rightarrow P_i)_{i \in I} & = & [c/x_j]P_j \\
\mathsf{fold}\ c & \triangleright & (\mathsf{fold}\ x \Rightarrow P) & = & [c/x]P
\end{array}
$$